

An introduction to Quantitative Information Flow (QIF)

Michele Boreale

DiSIA - Università di Firenze

IFIP WG 2.2, FCT-UNL, Lisbon
23-27 September 2013

Two elusive concepts

Confidentiality (aka Secrecy)

Sensitive information is never leaked to unintended parties. Often pursued via encryption. Protection of 'high-entropy' secrets:

- PIN's, passwords, keys, credit card numbers
- memory content
- ...

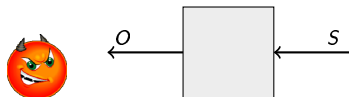
Privacy

Personal information about *individuals* is never disclosed. Often pursued via anonymization and aggregation of data. Protection of

- participation of an individual in a database
- value of an individual's sensitive (e.g. medical) attribute
- individual's purchase preferences
- ...

Attacker

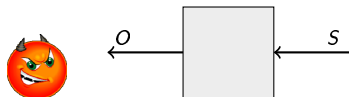
Despite a variety of concrete contexts and situations, the underlying paradigm is conceptually simple. We presuppose an **attacker** that gets to know certain **observable information** and, from this, tries her/his best to learn the **secret**.



- Attacker's task: **infer** the secret given the observable information.
- Our tasks:
 - 1 **quantify** the attacker's chances of success / necessary effort
 - 2 devise **tools and methods** to make chances as small as possible / effort as large as possible.

Attacker

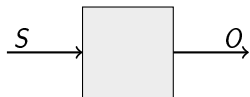
Despite a variety of concrete contexts and situations, the underlying paradigm is conceptually simple. We presuppose an **attacker** that gets to know certain **observable information** and, from this, tries her/his best to learn the **secret**.



- Attacker's task: **infer** the secret given the observable information.
- Our tasks:
 - 1 **quantify** the attacker's chances of success / necessary effort
 - 2 devise **tools and methods** to make chances as small as possible / effort as large as possible.
- **Two models**: Quantitative Information Leakage (QIF, confidentiality) and Differential Privacy (DP, privacy).

QIF: motivation and intuition

Let us consider a program/system operating taking as input a sensitive variable S and producing a public (observable) output O , as a 'black-box'.



Ideal situation: Noninterference (Goguen-Meseguer 1982). Value of O does not depend on the secret S .

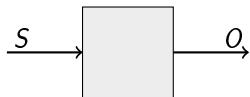
In practice, this is extremely hard to achieve, especially when the output O has to have some *utility*.

Example: PIN-checker

```
L=input()  
if S=L then O:=yes else O:=no
```

QIF: motivation and intuition

Let us consider a program/system operating taking as input a sensitive variable S and producing a public (observable) output O , as a 'black-box'.



Ideal situation: Noninterference (Goguen-Meseguer 1982). Value of O does not depend on the secret S .

In practice, this is extremely hard to achieve, especially when the output O has to have some *utility*.

Example: PIN-checker

```
L=input()
if S=L then O:=yes else O:=no
```

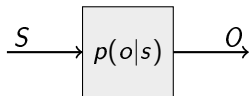
Realistic approach: measure the quantity of information (in bits) the attacker can learn about S by observing O . If this is very small - below a given threshold - decree the system *secure*.

A noisy channel model (e.g.[Chatzikokolakis, Palamidessi 2008])

(Probabilistic) programs or systems viewed as *noisy channels*:

- input S = sensitive information
- output O = observables

Noisy: fixed a given input, one can obtain different outputs each with a certain probability (probabilistic programs)

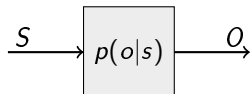


A noisy channel model (e.g.[Chatzikokolakis, Palamidessi 2008])

(Probabilistic) programs or systems viewed as *noisy channels*:

- input S = sensitive information
- output O = observables

Noisy: fixed a given input, one can obtain different outputs each with a certain probability (probabilistic programs)



Formally:

Randomization mechanism

A *randomization mechanism* is a triple $\mathcal{R} = (S, O, p(\cdot|\cdot))$, where:

- 1 S is a finite set of *secret inputs*, representing the sensitive information
- 2 O is a finite set of *observations*, representing the observable information
- 3 $p(\cdot|\cdot) \in [0, 1]^{S \times O}$ is a *conditional probability matrix*, where each row sums up to 1.

Note: a matrix with only 0-1 entries defines an I/O function $f : S \rightarrow O$.

Simple examples/1

PIN-checker. Assume $0 \leq S < 4$, uniformly distributed. $\mathcal{O} = \{\text{yes}, \text{no}\}$.

Program:

Matrix:

```
\\ assume L=3
if S=L then
  0:=yes
else
  0:=no
```

$$p(\cdot|\cdot) = \begin{array}{c} \begin{array}{cc} & \text{yes} & \text{no} \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} \left[\begin{array}{cc} 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{array} \right] \end{array}$$

Simple examples/1

PIN-checker. Assume $0 \leq S < 4$, uniformly distributed. $\mathcal{O} = \{\text{yes}, \text{no}\}$.

Program:

Matrix:

```
\\ assume L=3
if S=L then
  0:=yes
else
  0:=no
```

$$p(\cdot|\cdot) = \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{cc} \text{yes} & \text{no} \\ \left[\begin{array}{cc} 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{array} \right] \end{array}$$

An interesting program (Smith '09). Assume $0 \leq S < 2^{32}$, uniformly distributed.

```
if S mod 8 = 0 then
  0 := S
else
  0 := 1
```

Simple examples/1

PIN-checker. Assume $0 \leq S < 4$, uniformly distributed. $\mathcal{O} = \{\text{yes}, \text{no}\}$.

Program:

Matrix:

```
\ \ assume L=3
if S=L then
  0:=yes
else
  0:=no
```

$$p(\cdot|\cdot) = \begin{array}{c} \begin{array}{cc} & \text{yes} & \text{no} \\ 0 & \begin{bmatrix} 0 & 1 \end{bmatrix} \\ 1 & \begin{bmatrix} 0 & 1 \end{bmatrix} \\ 2 & \begin{bmatrix} 0 & 1 \end{bmatrix} \\ 3 & \begin{bmatrix} 1 & 0 \end{bmatrix} \end{array} \end{array}$$

An interesting program (Smith '09). Assume $0 \leq S < 2^{32}$, uniformly distributed.

```
if S mod 8 = 0 then
  0 := S
else
  0 := 1
```

The entire secret is leaked $\frac{1}{8}$ of the times. Is it a big leak or not? We will see.

Simple examples/2

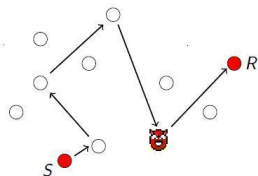
Crowds, a probabilistic anonymity protocol (Reiter, Rubin 1998).

A node is detected: is it the true sender or just a forwarder?

With three honest nodes and one corrupted, we have

$\mathcal{S} = \{n_1, n_2, n_3\}$, $\mathcal{O} = \{d_1, d_2, d_3\}$ and

$$p(\cdot|\cdot) = \begin{matrix} & d_1 & d_2 & d_3 \\ n_1 & \begin{bmatrix} \frac{7}{8} & \frac{1}{16} & \frac{1}{16} \end{bmatrix} \\ n_2 & \begin{bmatrix} \frac{1}{16} & \frac{7}{8} & \frac{1}{16} \end{bmatrix} \\ n_3 & \begin{bmatrix} \frac{1}{16} & \frac{1}{16} & \frac{7}{8} \end{bmatrix} \end{matrix}$$



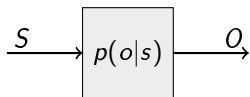
Many more examples

- in databases, queries may leak information about 'sensitive' fields
- side-channel attacks against smart-cards: exploit correlation between secret key and execution time, power consumption,...
- ...

Quantifying flow of information/1

Consider a randomization mechanism \mathcal{R} .

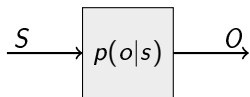
- Adversary knows prior probability distribution $p_S(\cdot)$ on \mathcal{S} : this also incorporates his own **background knowledge**.
- Secret and observable information form then a pair of random variables (S, O) , distributed according to $p_{S,O}(s, o) = p_S(s) \cdot p(o|s)$.



Quantifying flow of information/1

Consider a randomization mechanism \mathcal{R} .

- Adversary knows prior probability distribution $p_S(\cdot)$ on \mathcal{S} : this also incorporates his own **background knowledge**.
- Secret and observable information form then a pair of random variables (S, O) , distributed according to $p_{S,O}(s, o) = p_S(s) \cdot p(o|s)$.

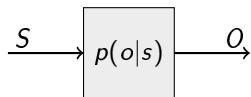


Assume we have an **uncertainty** measure $H(\cdot)$ for random variables.

Quantifying flow of information/1

Consider a randomization mechanism \mathcal{R} .

- Adversary knows prior probability distribution $p_S(\cdot)$ on \mathcal{S} : this also incorporates his own **background knowledge**.
- Secret and observable information form then a pair of random variables (S, O) , distributed according to $p_{S,O}(s, o) = p_S(s) \cdot p(o|s)$.



Assume we have an **uncertainty** measure $H(\cdot)$ for random variables.

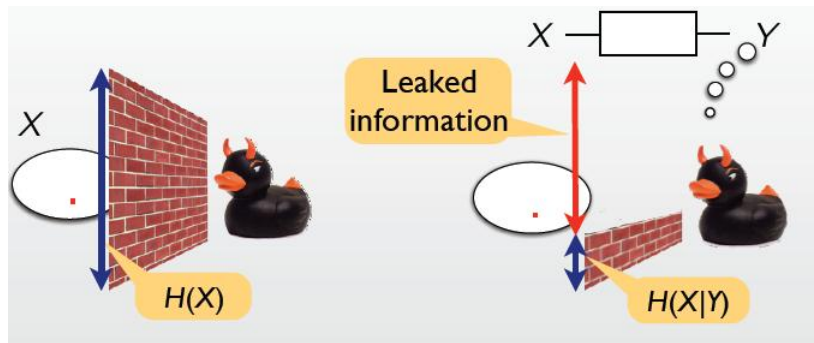
Information flow = reduction in uncertainty

$$\begin{aligned} \text{Information Flow} &= \text{prior uncertainty} - \text{posterior uncertainty} \\ &\stackrel{\text{def}}{=} H(S) - H(S|O) \end{aligned}$$

Note:

- $I(S; O) \stackrel{\text{def}}{=} H(S) - H(S|O)$ is often named *mutual information* in Information Theory.
- $H(S|O)$ represents *average* posterior uncertainty. E.g. $\sum_o p(o)H(S|O = o)$.
- The flow is 0 precisely when S and O are independent: only in this case $H(S|O) = H(S)$, hence $I(S; O) = 0$ (Non-Interference).
- If $H(S) \approx 0$, then $I(S; O) \approx 0$.
Alas, there is little we can do if passwords are badly chosen!

Quantifying flow of information - pictorially



Uncertainty $H(X)$ captures "height of wall", in terms of chances of success of guessing, or expected effort for learning, the secret X .

(courtesy of Boris Köpf)

But what is 'Uncertainty' ?

Several proposals for $H(\cdot)$. First (obvious) attempt:

Shannon entropy (Shannon 1948)

$$\begin{aligned} H_{\text{Sh}}(S) &\stackrel{\text{def}}{=} - \sum_s p(s) \log p(s) \\ &= \text{Average n. of binary questions necessary to learn } S \end{aligned}$$

But what is 'Uncertainty'?

Several proposals for $H(\cdot)$. First (obvious) attempt:

Shannon entropy (Shannon 1948)

$$\begin{aligned} H_{\text{Sh}}(S) &\stackrel{\text{def}}{=} -\sum_s p(s) \log p(s) \\ &= \text{Average n. of binary questions necessary to learn } S \end{aligned}$$

PIN-checking example. Let S be a 5-digits PIN, chosen at random.

if $S=L$ then $O:=\text{yes}$ else $O:=\text{no}$

- Prior uncertainty: $H(S) = \log 10^5 \approx 16.6096$ bits
 - Posterior uncertainty.
 - $H(S|O = \text{'yes'}) = 0$
 - $H(S|O = \text{'no'}) = \log(10^5 - 1)$
- On average: $H(S|O) = \left(\frac{10^5 - 1}{10^5}\right) \log(10^5 - 1) \approx 16.6094$
- Information flow = $H(S) - H(S|O) \approx 0.0002$ bits

So my PIN is safe, after all...

Does Shannon entropy properly reflect 'how difficult' is to guess?

Assume $0 \leq S < 2^{32}$, uniformly distributed.

if $S \bmod 8 = 0$ then

$O := S$

else

$O := 1$

- Prior uncertainty: $H(S) = 32$ bits
 - Posterior uncertainty.
 - $H(S|O = y) = 0$ for $y \neq 1$, happens $\frac{1}{8}$ of the times;
 - $H(S|O = 1) = 32 - \log \frac{8}{7}$, happens $\frac{7}{8}$ of the times;
- On average: $H(S|O) = \frac{7}{8} \times (32 - \log \frac{8}{7}) \approx 28 - 0.169$ bits
- Information flow = $H(S) - H(S|O) \approx 4.169$ bits

Does Shannon entropy properly reflect 'how difficult' is to guess?

Assume $0 \leq S < 2^{32}$, uniformly distributed.

if $S \bmod 8 = 0$ then

$0 := S$

else

$0 := 1$

- Prior uncertainty: $H(S) = 32$ bits
- Posterior uncertainty.
 - $H(S|O = y) = 0$ for $y \neq 1$, happens $\frac{1}{8}$ of the times;
 - $H(S|O = 1) = 32 - \log \frac{8}{7}$, happens $\frac{7}{8}$ of the times;

On average: $H(S|O) = \frac{7}{8} \times (32 - \log \frac{8}{7}) \approx 28 - 0.169$ bits

- Information flow = $H(S) - H(S|O) \approx 4.169$ bits

Suggests that $\approx 7/8$ of the secret bits remain leaked. However, adversary can guess the **whole** 32 bits of the secret $\frac{1}{8}$ of the times!

Min-entropy (Renyi 1961)

$$\begin{aligned} H_{\infty}(S) &\stackrel{\text{def}}{=} -\log \max_s p(s) \\ &= -\log (\text{chances of successfully guess } S \text{ in one try}) \end{aligned}$$

Min-entropy (Renyi 1961)

$$\begin{aligned} H_\infty(S) &\stackrel{\text{def}}{=} -\log \max_s p(s) \\ &= -\log (\text{chances of successfully guess } S \text{ in one try}) \end{aligned}$$

$$H_\infty(S|O) \stackrel{\text{def}}{=} -\log \left(\underbrace{\sum_o p(o) \max_s p(s|o)}_{=\text{avg. a posteriori chances of success}} \right)$$

- Proposed by Smith in 2009 as an alternative to Shannon for QIF
- Clear operational significance:

$$\text{Leakage} = H(S) - H(S|O) = \log \frac{p(\text{success a posteriori})}{p(\text{success a priori})}$$

1 bit gained by attacker = success probability doubled!

Some results/1

For a **deterministic** program and a **uniform prior**

$$\text{Leakage} = \log(\# \text{ distinct output values of the program })$$

(Smith 2009) In other words, leakage only depends on $|\text{Im}(f)|$, where $f : \mathcal{S} \rightarrow \mathcal{O}$ (termination considered observable).

Some results/1

For a **deterministic** program and a **uniform prior**

$$\text{Leakage} = \log(\# \text{ distinct output values of the program })$$

(Smith 2009) In other words, leakage only depends on $|\text{Im}(f)|$, where $f : \mathcal{S} \rightarrow \mathcal{O}$ (termination considered observable).

Example:

```
if S mod 8 = 0 then
  0 := S
else
  0 := 1
```

This leaks $\log 2^{29} = 29$ bits of min-entropy (vs. ≈ 4 of Shannon) about S .

Some results/1

For a **deterministic** program and a **uniform prior**

Leakage = $\log(\# \text{ distinct output values of the program})$

(Smith 2009) In other words, leakage only depends on $|\text{Im}(f)|$, where $f : \mathcal{S} \rightarrow \mathcal{O}$ (termination considered observable).

Example:

if $S \bmod 8 = 0$ then

$0 := S$

else

$0 := 1$

Proof:

$$\begin{aligned} H(S|O) &= -\log\left(\sum_{o:p(o)>o} p(o) \max_s p(s|o)\right) \\ &= -\log\left(\sum_{o:p(o)>o} p(o) \max_s \frac{p(o|s)p(s)}{p(o)}\right) && \text{(Bayes)} \\ &= -\log\left(\sum_{o:p(o)>o} \max_s p(o|s)p(s)\right) \\ &= -\log\left(\frac{1}{|\mathcal{S}|} \sum_{o:p(o)>o} \max_s p(o|s)\right) && \text{(uniform prior)} \\ &= -\log\left(\frac{1}{|\mathcal{S}|} \sum_{o:p(o)>o} 1\right) && \text{(determinism)} \\ &= -\log\left(\frac{|\text{Im}(f)|}{|\mathcal{S}|}\right) \\ H(S) - H(S|O) &= -\log \frac{1}{|\mathcal{S}|} + \log \frac{|\text{Im}(f)|}{|\mathcal{S}|} = \log\left(|\mathcal{S}| \frac{|\text{Im}(f)|}{|\mathcal{S}|}\right) \\ &= \log |\text{Im}(f)| \end{aligned}$$

This leaks $\log 2^{29} = 29$ bits of min-entropy (vs. ≈ 4 of Shannon) about S .

For a general **probabilistic** program and **repeated observations**;
conditional independence of O_1, \dots, O_n given S is typically assumed:

$$p(o_1, \dots, o_n | s) = \prod_j p(o_j | s)$$

$Leakage(n) \stackrel{\text{def}}{=} I(S; O^n)$. Under a uniform prior, as $n \rightarrow +\infty$

$Leakage(n) \rightarrow \log(\# \text{ distinct } \textit{indistinguishability} \text{ classes of the program})$

((Boreale et al. 2011) for H_∞ ; for generic uncertainty measures, (Boreale and Pampaloni 2013). Note: exact rate of convergence can be determined from the matrix.)

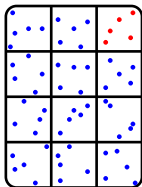
What does that mean?

Indistinguishability

Given $s, s' \in \mathcal{S}$, we let $s \equiv s'$ iff for each o : $p(o|s) = p(o|s')$.

This means rows s and s' in matrix $p(\cdot|\cdot)$ are equal.

Intuition: with infinitely many observations, precisely the *indistinguishability class* of the secret will be learned by the attacker.



In the case of uniform prior distribution, $I(S; [S]_{\equiv}) = \log K$, where K is the number of classes.

Example:

```
if S mod 8 = rnd[0..7] then
  0 := S
else
  0 := 1
```

In this case, $K = |\mathcal{S}| = 2^{32}$, hence asymptotic leakage is 32 bits.

Further research/1: Compositionality

- *Non-expansiveness* for sequential and parallel composition (Köpf et al., Smith et al. 2012), also in a process-algebraic setting (Boreale 2006):

$$\text{Leakage}(P_1 \circ P_2) \leq \text{Leakage}(P_1) + \text{Leakage}(P_2)$$

Further research/1: Compositionality

- *Non-expansiveness* for sequential and parallel composition (Köpf et al., Smith et al. 2012), also in a process-algebraic setting (Boreale 2006):

$$\text{Leakage}(P_1 \circ P_2) \leq \text{Leakage}(P_1) + \text{Leakage}(P_2)$$

- In the case of Shannon entropy:

- $\text{Leakage} = I(S; O) = I(O; S) = H(O) - \underbrace{H(O|S)}_{=0, \text{ if } P \text{ det.}} = H(O)$

- Chain rule (provided ϕ depends only on O):

$$H(O) = H(\phi) + H(O|\phi)$$

- Hence for if-then-else

$$\text{Leakage}(\text{if } b \text{ then } c_1 \text{ else } c_2) = H(b) + p(b)H(c_1|b) + p(\neg b)H(c_2|\neg b)$$

(provided final value of 0 determines initial value of b .)

- Extensible to looping constructs, cf. Malacaria, POPL'07.

Example (if-then-else)

$$\begin{aligned} \text{Leakage}(\text{if } b \text{ then } c_1 \text{ else } c_2) &= H(b) + p(b)H(c_1|b) \\ &\quad + p(\neg b)H(c_2|\neg b) \end{aligned}$$

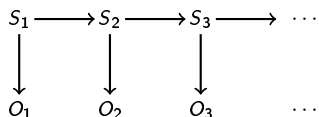
```
if S%8 =0 then
  0 := S
else
  0 := 1
```

$$\begin{aligned} \text{Leakage} &= H(S\%8 =0) + p(S\%8 =0)H(0:=S|S\%8 =0) \\ &\quad + p(S\%8 \neq 0)H(0:=1|S\%8 \neq 0) \\ &= H\left(\frac{1}{8}, \frac{7}{8}\right) + \frac{1}{8} \times 29 \\ &\quad + \frac{7}{8} \times 0 \\ &\approx 4.169 \end{aligned}$$

- **Trace-based observations?** Systems go through several states before producing a result, if any. At each step, attacker detects a (noisy) observation of the current state, like in *Hidden Markov Models*.

Observation = trace, hence set of observables is now \mathcal{O}^* .

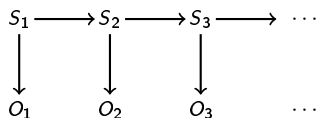
Much of the theory extends smoothly (Boreale et al. 2011).



- **Trace-based observations?** Systems go through several states before producing a result, if any. At each step, attacker detects a (noisy) observation of the current state, like in *Hidden Markov Models*.

Observation = trace, hence set of observables is now O^* .

Much of the theory extends smoothly (Boreale et al. 2011).



- **Adaptive attackers?** $O = f(S, q)$, for query $q \in \mathcal{Q}$. Attacker can repeatedly choose and submit queries q , based on previous observations, hence play a *strategy* $\sigma : O^* \rightarrow \mathcal{Q}$. Complete observation is O_σ . Leakage is

$$H(S) - \inf_{\sigma} H(S|O_\sigma)$$

Optimal strategy computable via *MDP*-based algorithms. Non-adaptive, brute force strategies are as efficient as adaptive ones, up to a length expansion of $\times |\mathcal{Q}|$. (Boreale, Pampaloni 2013).

- **Relation with privacy.** Aim: protect information about any *individual* in a DB, independently of attacker's prior knowledge. Ideally, even *participation* of the individual in the DB should be hidden.
- QIF may not be adequate, because it is an *average* measure

	<2.20	≥2.20
i_1	0	1
i_2	1	0
i_3	1	0
i_4	1	0
i_5	1	0
\vdots	\vdots	\vdots
i_{10^9}	1	0

Individual i_1 is the only one with height ≥ 2.20 m.

Yet (min-entropy): $I(S; O) = H(S) - H(S|O) = 1$ bit, out of 30 bits.

- **Relation with privacy.** Aim: protect information about any *individual* in a DB, independently of attacker's prior knowledge. Ideally, even *participation* of the individual in the DB should be hidden.
- QIF may not be adequate, because it is an *average* measure

	<2.20	≥2.20
i_1	0	1
i_2	1	0
i_3	1	0
i_4	1	0
i_5	1	0
\vdots	\vdots	\vdots
i_{10^9}	1	0

Individual i_1 is the only one with height ≥ 2.20 m.

Yet (min-entropy): $I(S; O) = H(S) - H(S|O) = 1$ bit, out of 30 bits.

- Also, answers of the mechanism should not be deterministic. E.g. query gives exact *average height*: attacker could make a query *before* and *after* insertion of individual i , and learn i 's height.

Definition (Dwork 2006). Let $\epsilon > 0$, assume \mathcal{S} is a set of DB instances. A randomization mechanism is ϵ -*differentially private* if for any two DB instances s and s' which differ by exactly *one* individual, for each $o \in \mathcal{O}$:

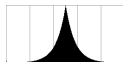
$$2^{-\epsilon} \leq \frac{p(o|s)}{p(o|s')} \leq 2^{\epsilon}$$

Definition (Dwork 2006). Let $\epsilon > 0$, assume \mathcal{S} is a set of DB instances. A randomization mechanism is ϵ -*differentially private* if for any two DB instances s and s' which differ by exactly *one* individual, for each $o \in \mathcal{O}$:

$$2^{-\epsilon} \leq \frac{p(o|s)}{p(o|s')} \leq 2^{\epsilon}$$

Laplacian noise. Let $Q : \mathcal{S} \rightarrow \mathbb{R}$ be a query function. Let $\Delta = \max_{s \text{ adj. } s'} |Q(s) - Q(s')|$ be the *sensitivity* of Q (e.g., if Q is the counting query, $\Delta = 1$). The mechanism defined by

$$O = Q(S) + Y \text{ where } Y \sim \frac{2^{-|y|} \frac{\epsilon}{\Delta}}{Z}$$



is differentially private, whatever S .

- QIF: a model of confidentiality based on simple information-theoretic concepts
- Very active research area in Theoretical Computer Science. Strong relations with Differential Privacy and Data Base communities.
- Challenges:
 - 1 Incorporate QIF concepts and analysis in programming languages (type systems, tools,...). Promising work by Köpf and Rybalchenko on automated estimation of QIF; for DP, cf. McSherry's PINQ.
 - 2 Real-world applications. Promising work on CPU caches and timing leaks in RSA (cf. work by Köpf and Smith).

Some papers (personal take) / 1

- M. S. Alvim, M. E. Andrés, C. Palamidessi. Quantitative information flow in interactive systems. *Journal of Computer Security* 20(1): 3-50 (2012)
- M. Backes, B. Köpf. Formally Bounding the Side-Channel Leakage in Unknown-Message Attacks. *ESORICS 2008*: 517-532
- M. Backes, B. Köpf, A. Rybalchenko. Automatic Discovery and Quantification of Information Leaks. *IEEE Symposium on Security and Privacy* 2009: 141-153
- M. Boreale. Quantifying information leakage in process calculi. *Inf. Comput.* 207(6): 699-725 (2009)
- M. Boreale, F. Pampaloni, M. Paolini. Quantitative Information Flow, with a View. *ESORICS 2011*: 588-606
- M. Boreale, F. Pampaloni, M. Paolini. Asymptotic Information Leakage under One-Try Attacks. *FOSSACS 2011*: 396-410 (full version to appear in *MSCS*).
- M. Boreale, F. Pampaloni. On the limits of adaptive adversaries. Submitted, 2013.

Some papers (personal take) / 2

- K. Chatzikokolakis, C. Palamidessi, P. Panangaden. Anonymity protocols as noisy channels. *Inf. Comput.* 206(2-4): 378-401 (2008)
- D. Clark, S. Hunt, P. Malacaria. Quantitative Analysis of the Leakage of Confidential Data. *Electr. Notes Theor. Comput. Sci.* 59(3): 238-251 (2001)
- J. Heusser, P. Malacaria. Quantifying information leaks in software. *ACSAC 2010*: 261-269
- B. Köpf, L. Mauborgne, M. Ochoa. Automatic Quantification of Cache Side-Channels. *CAV 2012*: 564-580
- B. Köpf, G. Smith. Vulnerability Bounds and Leakage Resilience of Blinded Cryptography under Timing Attacks. *CSF 2010*: 44-56
- P. Malacaria. Assessing security threats of looping constructs. *POPL 2007*: 225-235
- G. Smith. On the Foundations of Quantitative Information Flow. *FOSSACS 2009*: 288-302