Towards Efficient Verification of Population Protocols

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Joint work with
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Deaf Black Ninjas in the Dark

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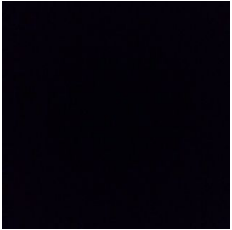

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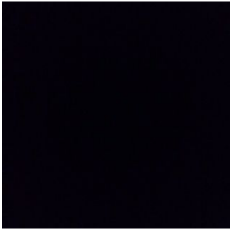
- How can they conduct the vote?



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- Ninjas follow this protocol:

(YA , NA)	$\rightarrow$	(NP , NP)	(opposite votes "cancel")
(YA , NP)	$\rightarrow$	(YA , YP)	(active "survivors" tell
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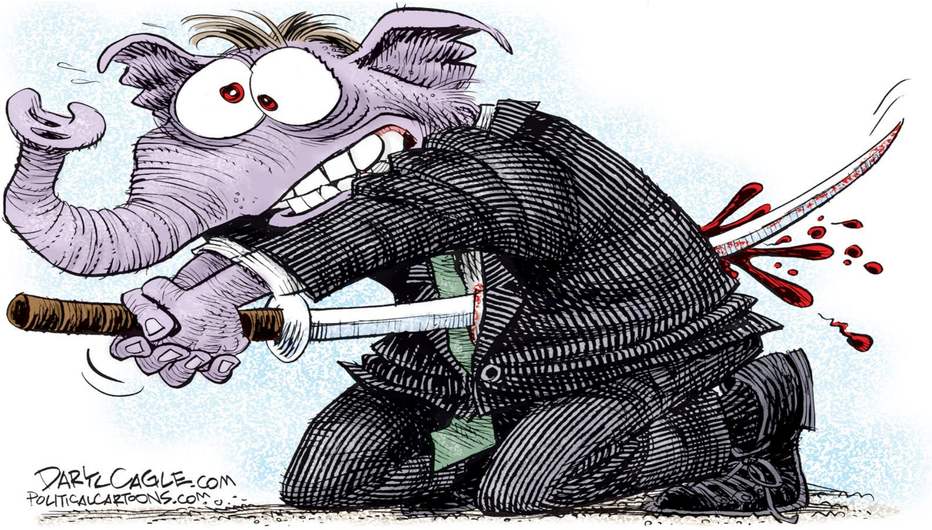

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Big Ninja's three questions:

- What is a protocol?
- When is a protocol correct?
- How can I decide if a protocol is correct?

Big Ninja's first question: What is a protocol?

Population protocols: Theoretical model for distributed computation proposed in 2004 by Yale group (Angluin, Fischer, Aspnes ...)

Designed to model collections of

identical, finite-state, and mobile agents

like

- ad-hoc networks of mobile sensors
- "soups" of interacting molecules (Chemical Reaction Networks)
- people in social networks

Syntax

PP-scheme: pair (Q, Δ), where Q is a finite set of states, and Δ is a set of interactions of the form $(q_1, q_2) \mapsto (q_3, q_4)$.

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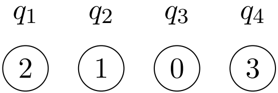

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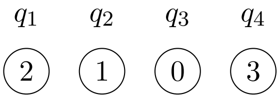

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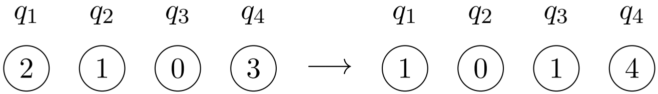

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Execution: infinite sequence $C_0 \to C_1 \to C_2 \to \cdots$ of steps.

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A PP computes $P(x_1, \ldots, x_n) \colon \mathbb{N}^n \to \{0, 1\}$ if it computes $P(n_1, \ldots, n_k)$ for every input $(n_1, \ldots, n_k)$

What predicates can PPs compute?

Theorem (Angluin *et al.* 2007): PPs compute exactly the Presburger predicates.

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- Threshold predicates: $\sum_i \alpha_i x_i > c$
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To show that PPs compute all Presburger predicates:

- Give protocols for the threshold and remainder predicates.
- Show that computable predicates are closed under negation and conjunction.

Big Ninja's second question: When is a protocol correct?

A protocol is well specified if it computes some predicate:

- for every input $(x_1, \ldots, x_n)$, the executions reach the same consensus (which depends on $(x_1, \ldots, x_n)$) with probability one.

A protocol is correct for a given predicate P if it is well specified and computes P.

Big Ninja's second question: When is a protocol correct?

A protocol is well specified if it computes some predicate:

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A protocol is correct for a given predicate P if it is well specified and computes P.

Well-specification problem: Given a protocol, decide if it is well-specified.

Correctness problem: Given a protocol and a Presburger predicate, decide if the protocol is well-specified and computes the predicate.

Big Ninja's third question: How can I decide correctness?

Theorem [E., Ganty, Leroux, Majumdar '15]: The well-specification and correctness problems can be reduced to the reachability problem for Petri nets, and are thus decidable.

But . . .

Theorem: The reachability problem for Petri nets is polynomially reducible to the well-specification problem.

The reachability problem for Petri nets is

- EXPSPACE-hard
- All known algorithms have non-primitive recursive complexity

Fighting complexity

Search for a subclass of the class WS of well-specified protocols that

- has a membership problem of reasonable complexity,
- still can compute all Presburger predicates, and
- contains many of the protocols in the literature.

Fighting complexity II: The class WS^2

Many protocols from the literature are silent: Executions end w.p.1 in terminal configurations that enable no transitions.

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Proposition : Petri net reachability is reducible to the membership problem for WS^2.

Fighting complexity III: The class WS^3

WS^2: Well-sp. silent

Termination

For every reachable configuration C there exists an execution leading from C to a terminal conf. $C_\perp$

Consensus

All terminal configurations reachable from a given initial configuration form the same consensus.

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WS^3: Well-sp. strongly silent

Layered Termination

For every configuration C there exists a layered execution leading from C to a terminal configuration $C_\perp$

Strong Consensus

All terminal configurations weakly reachable from a given initial configuration form the same consensus.

Layered Termination

A protocol is layered if there is a partition of the set T of transitions into layers $T_1, \dots T_n$ s.t. for every configuration C (reachable or not):

- all executions from C containing only transitions of a single layer are finite.
- if all transitions of T_i are disabled at C, then they cannot be re-enabled by any sequence of transitions of $T_{i+1}, \dots, T_n$.

An execution is layered if it "respects the layers", i.e., if it belongs to $T_1^* T_2^* \dots T_n^*$.

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Fact: For every configuration C (reachable or not) there exists a layered execution leading from C to a terminal configuration $C_\perp$.

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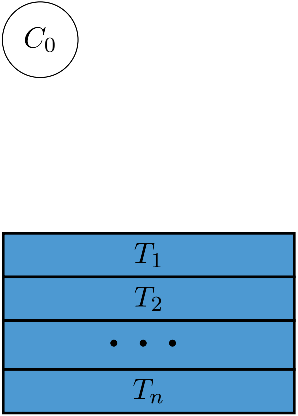

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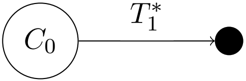

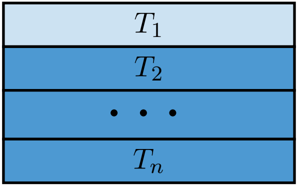

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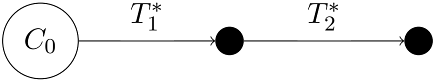

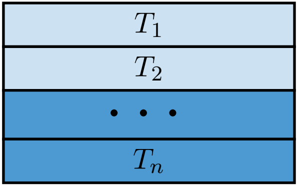

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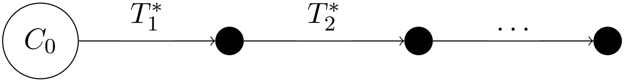

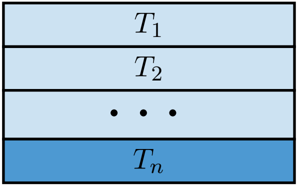

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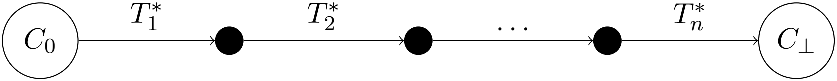

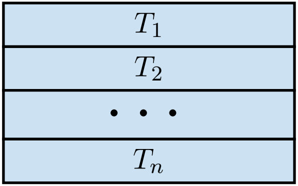

Complexity of checking Layered Termination

Lemma: Deciding Layered Termination is in NP.

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Proof sketch:

- Guess layers.
- Test that each individual layer terminates.
  Reducible to a Linear Programming Problem.
- Test that lower layers cannot re-enable higher layers.
  Simple syntactic check.

Strong Consensus: The Liquid Approximation

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Fluid agents in action

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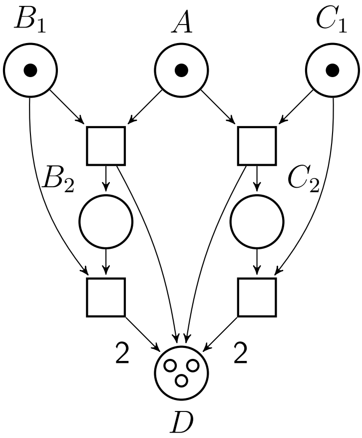

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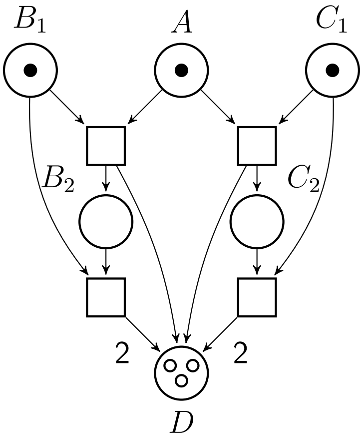

Theorem (Fraca, Haddad '15): Liquid reachability is in NP (P).

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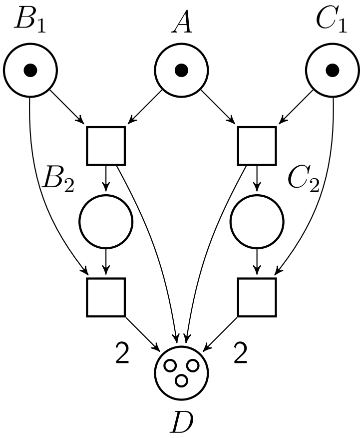

Theorem (Fraca, Haddad '15): Liquid reachability is in NP (P).

Lemma: Deciding Strong Consensus is in co-NP.

Completeness

Lemma: All well-specified population protocols can be represented by an equivalent population protocol satisfying Layered Termination and Strong Consensus.

- Give WS^3 protocols for Threshold and Remainder predicates
- Prove that WS^3 protocols are closed under conjunction and negation.

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- Give WS^3 protocols for Threshold and Remainder predicates
- Prove that WS^3 protocols are closed under conjunction and negation.

Fact: Protocols from the literature for Majority, Threshold, Modulo, etc. belong to WS^3.

Peregrine

- `Peregrine`: Haskell + SMT solver Z3

  `gitlab.lrz.de/i7/peregrine`

- `Peregrine` reads a protocol and constructs two sets of constraints:
  - The first is satisfiable iff Layered Termination holds.
  - The second is unsatisfiable iff Strong Consensus holds.

Experimental Results

Intel Core i7-4810MQ CPU and 16 GB of RAM.

Protocol	Predicate	$\|Q\|$	$\|T\|$	Time[s]
Majority [1]	$x \geq y$	4	4	0.1
Approx. Majority [2]	Not well-specified	3	4	0.1
Broadcast [3]	$x \geq 1$	2	1	0.1
Threshold [4]	$\Sigma_i \alpha_i x_i \geq c$	76	2148	2375.9
Modulo [5]	$\Sigma_i \alpha_i x_i \bmod 70 = 1$	72	2555	3176.5
Flock of birds [6]	$x \geq 50$	51	1275	181.6
Flock of birds [7]	$x \geq 325$	326	649	3470.8
Prime flock of birds	$x \geq 10^7$	37	155	18.91
Poly-log flock of birds	$x \geq 8 \cdot 10^4$	66	244	12.79

[1] Draief et al., 2012 [2] Angluin et al., 2007 [3] Clément et al., 2011

[4][5] Angluin et al., 2006 [6] Chatzigiannakis et al., 2010 [7] Clément et al., 2011

Conclusions

- The natural verification problems for population protocols are decidable.
- Efficient verification algorithms for the class WS^3.
- Implementation on top of SMT-solvers.

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- The natural verification problems for population protocols are decidable.
- Efficient verification algorithms for the class WS^3.
- Implementation on top of SMT-solvers.
- Many open questions:
  - Complexity for immediate observation and immediate transmission protocols.
  - Correctness problem and convergence speed for WS^3 protocols.
  - Minimal population protocols for given predicates.
  - Fault localization and repair.
  - Automatic synthesis of WS^3 protocols.
  - Theoretical and practical power of the liquid abstraction.
  - Expressive power of PPs in non-uniform computational models.
  - Applications to theoretical chemistry and systems biology.

Thank You