Failure Trace Semantics for a Process Algebra with Time-Outs

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September 2023

Context

Classic process algebra

Classic semantics in terms of labelled transition systems

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No probabilities, no quantified time, no name binding, no nothing

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Goal: *find the coarsest reasonable semantics for LTSs with* t.

The passage of time in labelled transition systems

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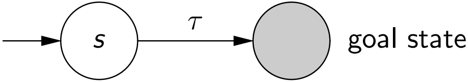

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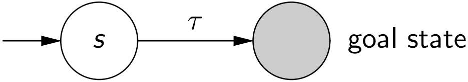

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(This assumption is called *progress* in [GH19].)
Without this assumption, no useful liveness properties can be established.

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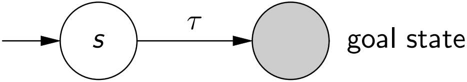

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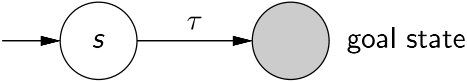

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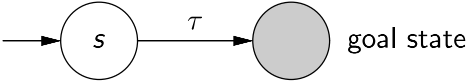

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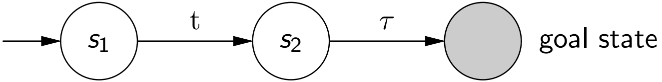
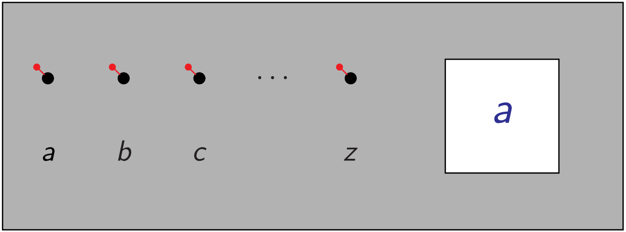
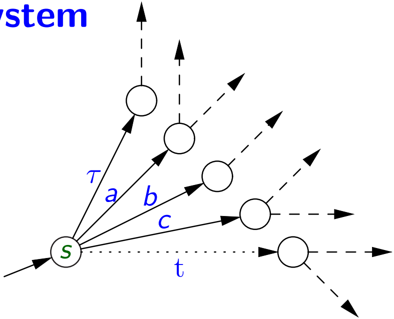


The environment



Right blocks

The system



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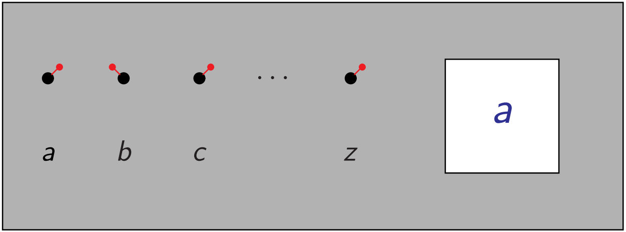

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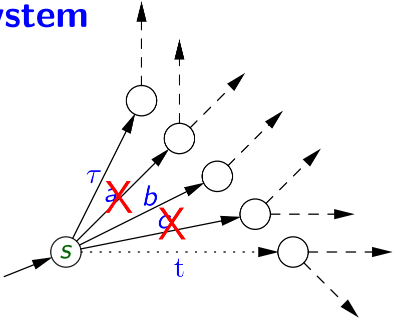

User blocks *a* and *c*:
System makes nondeterministic choice between τ and *b*.

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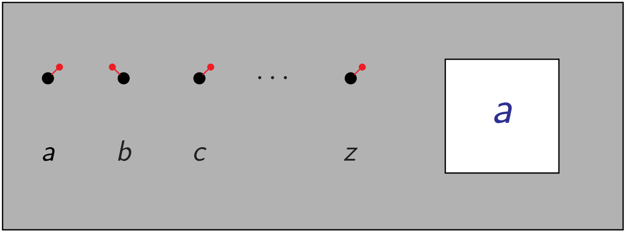

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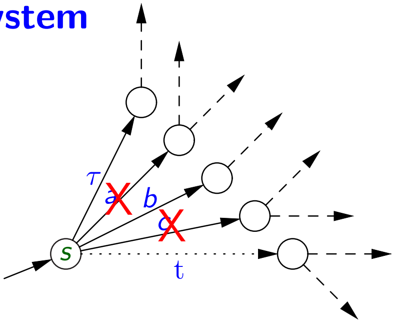

User blocks *a* and *c*:
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Will not choose t.

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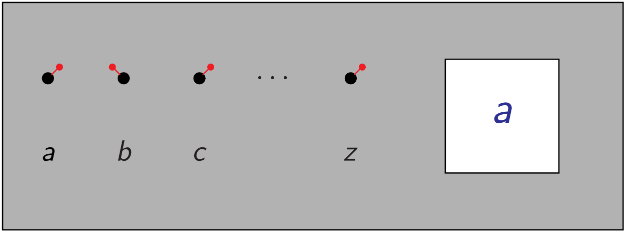

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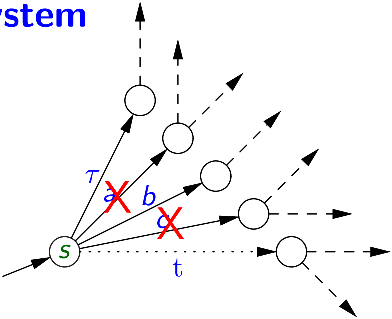
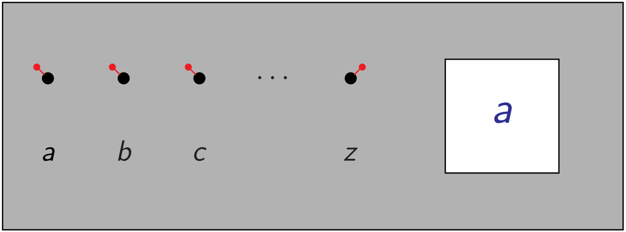


User blocks a and c:
System makes nondeterministic choice between τ and b.
Will not choose t.
No time spend in state s.

The environment



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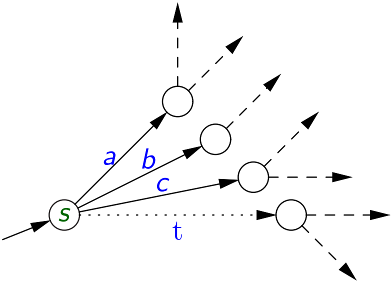
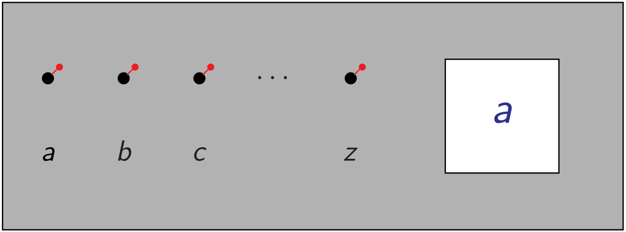


System chooses a, b or c.

The environment



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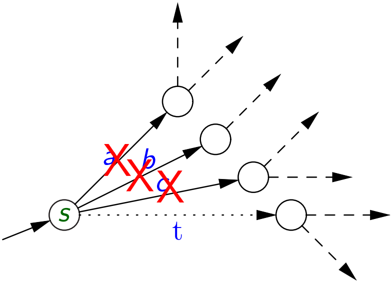

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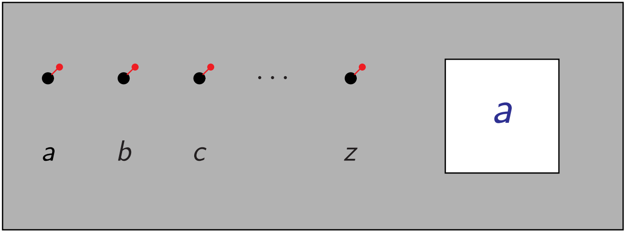

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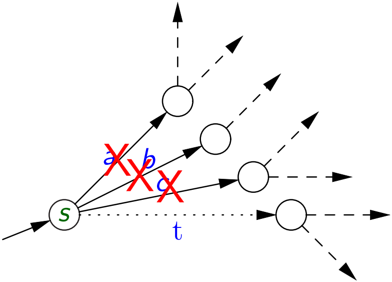

User blocks a, b, and c:
System stays in state s.

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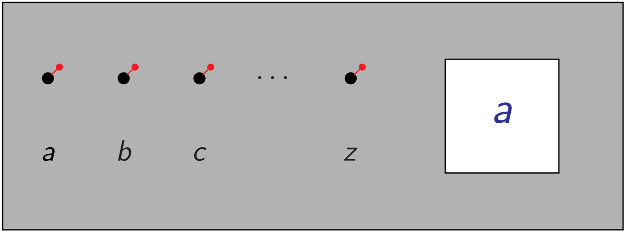

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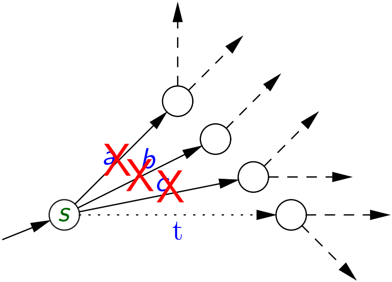

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System stays in state s.
User may change mind.

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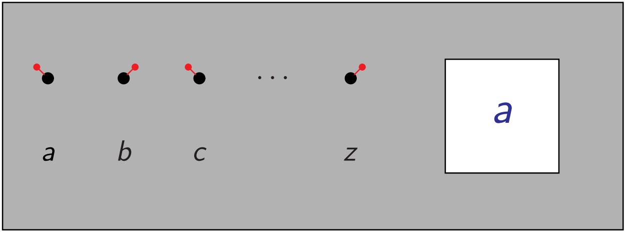

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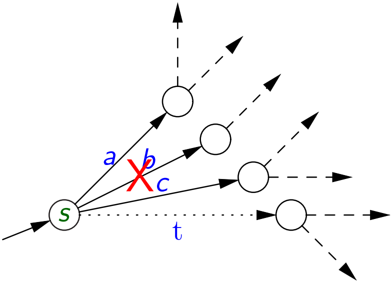

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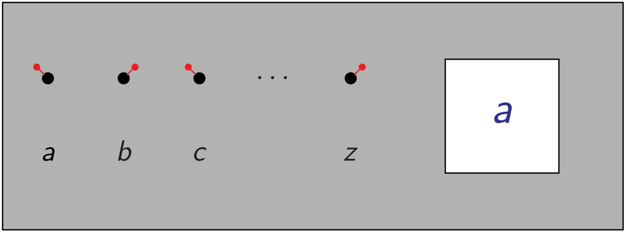

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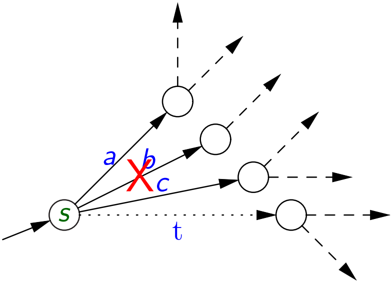

User blocks *a*, *b*, and *c*:
System stays in state *s*.
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System chooses *a* or *c*.

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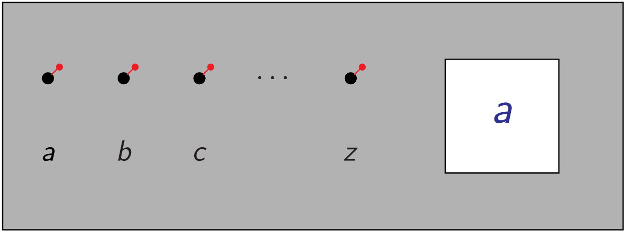

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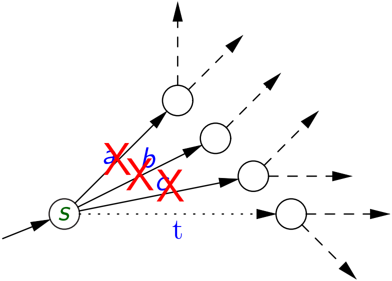

User blocks *a*, *b*, and *c*:
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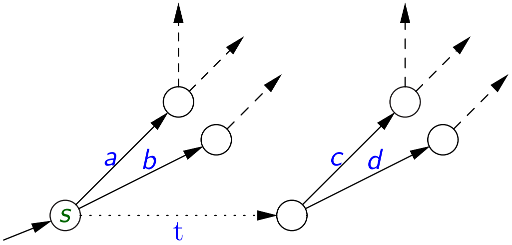
User may change mind.

Or time-out occurs first and system takes t.

More expressiveness



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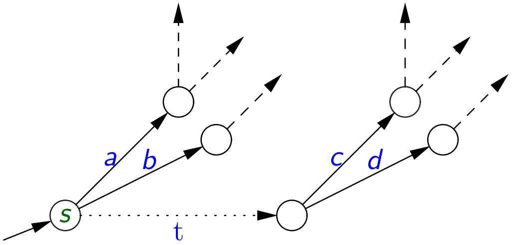

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It has been shown [GH15] that mutual exclusion cannot be adequately expressed in CCS-like languages. We need to
(a) extend the language, e.g. with "signals", and
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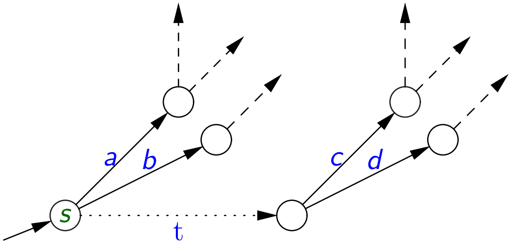

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In the presence of time-out transitions mutual exclusion can be correctly expressed in CCS, even without assuming justness.

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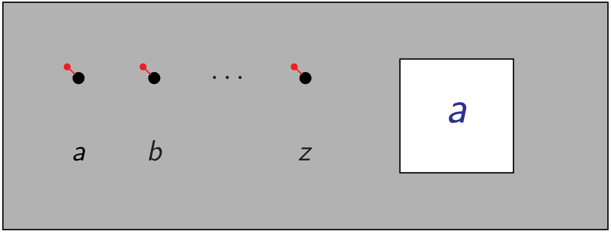
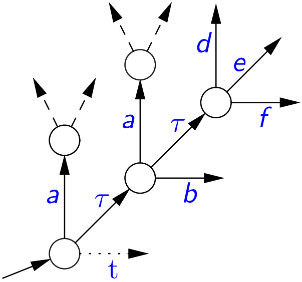
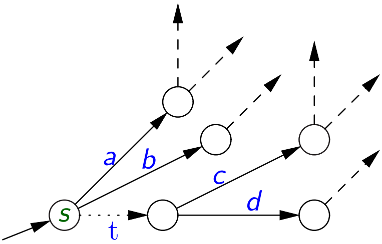


Environment $\|_A$ System



$\|_A$



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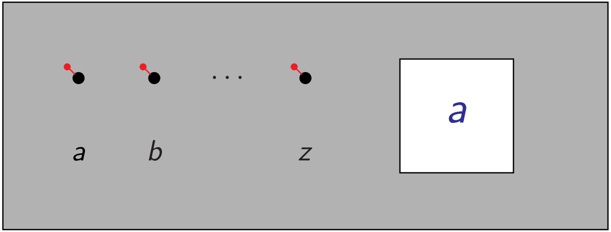

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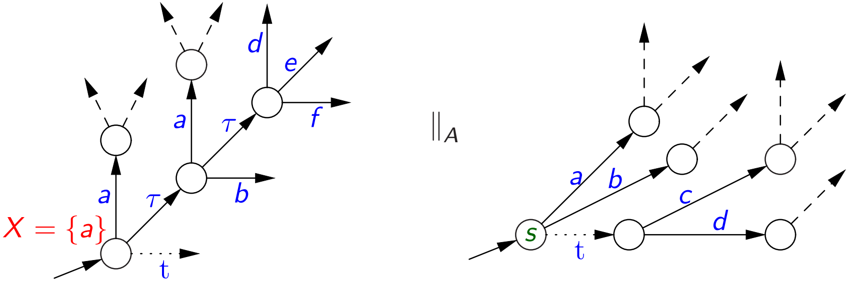

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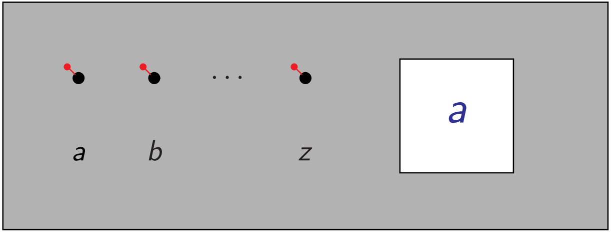

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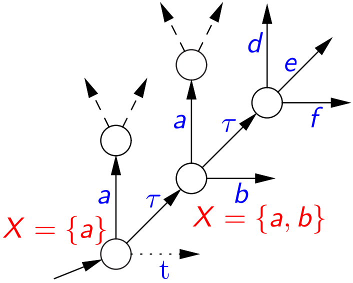

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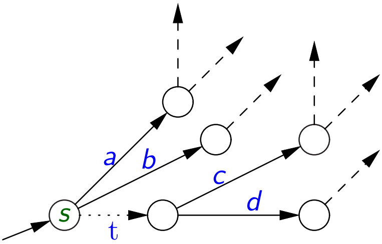

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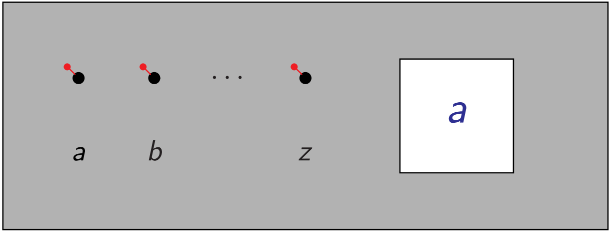

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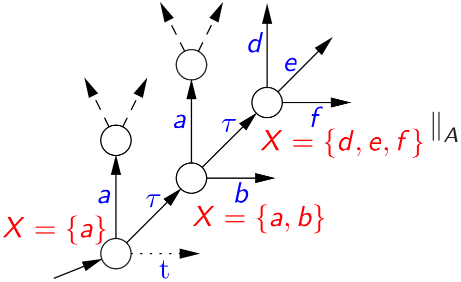

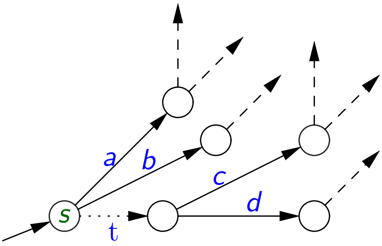

The process algebra $\mathrm{CCSP_t}$

$$\begin{array}{rl} E ::= & 0 \;\big|\; \alpha.E \;\big|\; E+E \;\big|\; E \,\|_S\, E \;\big|\; \tau_I(E) \;\big|\; \mathcal{R}(E) \;\big| \\ & x \;\big|\; \langle x|\mathcal{S}\rangle \text{ (with } x \in V_\mathcal{S}) \end{array}$$

with $\alpha \in Act := A \uplus \{\tau, \mathrm{t}\}$, $S, I \subseteq A$, $\mathcal{R} \subseteq A \times A$, $x \in Var$ and $\mathcal{S}$ a *recursive specification*:
a set of equations $\{y = \mathcal{S}_y \mid y \in V_\mathcal{S}\}$ with $V_\mathcal{S} \subseteq Var$ (the *bound variables* of $\mathcal{S}$) and each $\mathcal{S}_y$ a $\mathrm{CCSP_t}$ expression.

$$\alpha.x \xrightarrow{\alpha} x \qquad \frac{x \xrightarrow{\alpha} x'}{x+y \xrightarrow{\alpha} x'} \qquad \frac{y \xrightarrow{\alpha} y'}{x+y \xrightarrow{\alpha} y'} \qquad \frac{x \xrightarrow{\alpha} x'}{\mathcal{R}(x) \xrightarrow{\beta} \mathcal{R}(x')} \begin{pmatrix} \alpha=\beta=\tau \\ \vee\ \alpha=\beta=\mathrm{t} \\ \vee\ (\alpha,\beta)\in\mathcal{R} \end{pmatrix}$$

$$\frac{x \xrightarrow{\alpha} x'}{x \,\|_S\, y \xrightarrow{\alpha} x' \,\|_S\, y}\ (\alpha \notin S) \qquad \frac{x \xrightarrow{a} x' \quad y \xrightarrow{a} y'}{x \,\|_S\, y \xrightarrow{a} x' \,\|_S\, y'}\ (a \in S) \qquad \frac{y \xrightarrow{\alpha} y'}{x \,\|_S\, y \xrightarrow{\alpha} x \,\|_S\, y'}\ (\alpha \notin S)$$

$$\frac{x \xrightarrow{\alpha} x'}{\tau_I(x) \xrightarrow{\alpha} \tau_I(x')}\ (\alpha \notin I) \qquad \frac{x \xrightarrow{a} x'}{\tau_I(x) \xrightarrow{\tau} \tau_I(x')}\ (a \in I) \qquad \frac{\langle \mathcal{S}_X|\mathcal{S}\rangle \xrightarrow{\alpha} y}{\langle X|\mathcal{S}\rangle \xrightarrow{\alpha} y}$$

Three crucial laws

$$\tau.P + \mathrm{t}.Q = \tau.P$$

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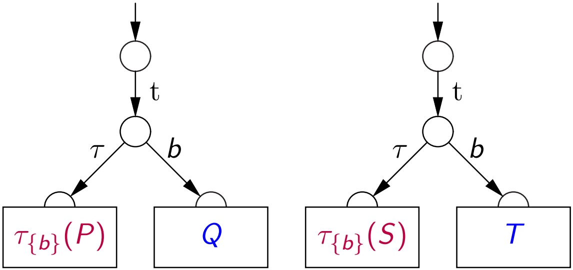
$$\mathrm{t}.(\tau.\tau_{\{b\}}(P) + b.Q) \|_{\{b\}} \mathrm{t}.(\tau.\tau_{\{b\}}(S) + b.T) = \mathrm{t}.\tau.\tau_{\{b\}}(P) \|_{\{b\}} \mathrm{t}.\tau.\tau_{\{b\}}(S)$$



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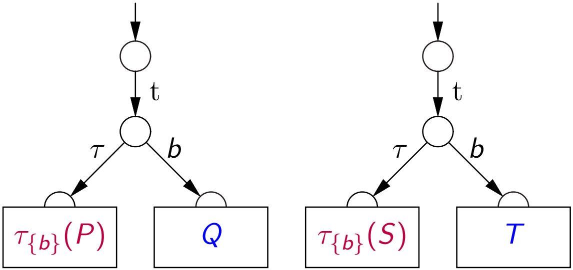
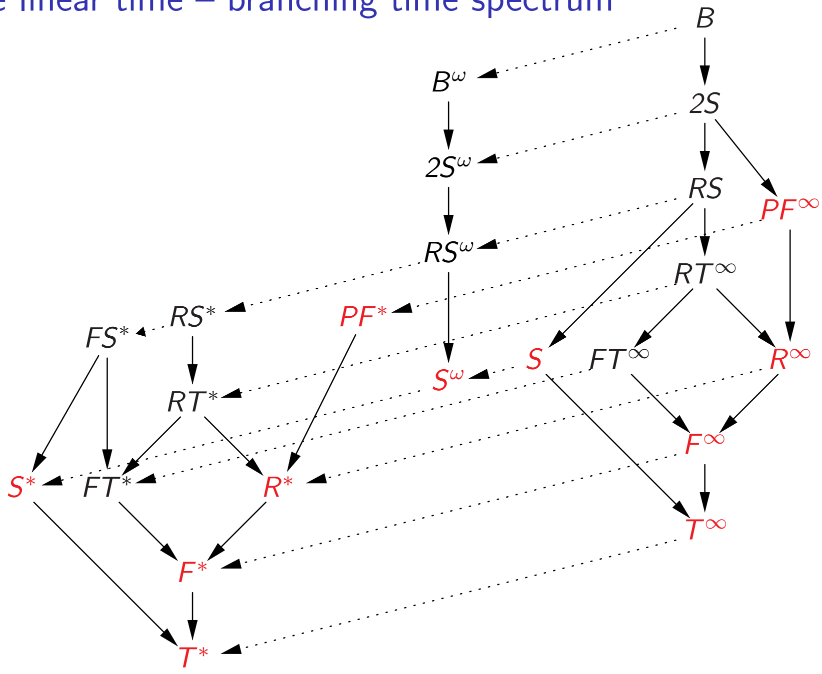
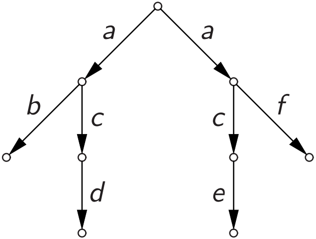


$$a.P + \mathrm{t}.(Q + \tau.R + a.S) = a.P + \mathrm{t}.(Q + \tau.R)$$

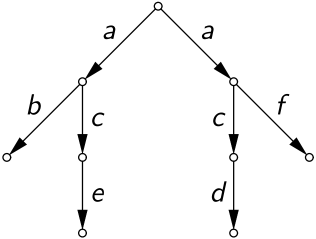
The linear time – branching time spectrum



Trace and failures equivalence fail to be a congruence



$=_F$
$\neq_{FT}$
$=_R$
$\neq_{RT}$



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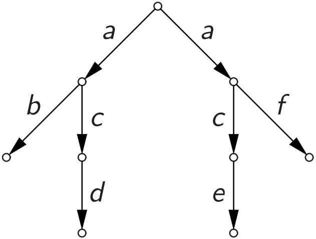

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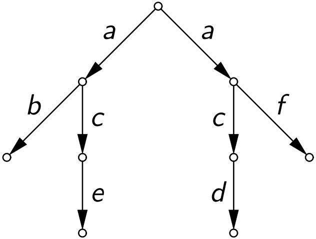

$a.(b + c.d) + a.(f + c.e)$ $\qquad$ $a.(b + c.e) + a.(f + c.d)$

Use the context $\mathcal{C}[\_] := \tau_{\{a,b,c\}}(a.(b + \mathrm{t}.c) \parallel_{\{a,b,c,f\}} \_)$.

This context implements a priority of b over c.

Only the RHS can ever reach d.

Simulation equivalence fails to be a congruence

Take $P := a.b + a$ and $Q = a.b$, and use the context
$\mathcal{C}[\_] := \tau_{\{a,b\}}(a.(b + \mathrm{t}.d) \parallel_{\{a,b\}} \_)$.
Then only $\mathcal{C}[P]$ can ever perform the action d.

Operational def. of failure trace semantics

Each execution of a system generates a *failure trace*, such as

$$a\, b\, X\, c\, Y\, d\, e\, Z\, W \top$$

a sequence of actions $a \in A$ and sets of refused actions $X \subseteq A$. It is the observation of a sequence of instantaneous actions $a\, b\, c\, d\, e$ interspersed with periods of idling. Each period of idling is denoted by the set $X \subseteq A$ of actions that are offered by the environment during this period. The sequence ends with $\top$, the act of the observer of ending the observation.

$$\top \in FT^*(x) \qquad \frac{x \stackrel{a}{\longrightarrow} y \quad \rho \in FT^*(y)}{a\rho \in FT^*(x)} \qquad \frac{x \stackrel{\tau}{\longrightarrow} y \quad \rho \in FT^*(y)}{\rho \in FT^*(x)}$$

$$\frac{x \stackrel{\alpha}{\not\longrightarrow} \text{ for all } \alpha \in X \cup \{\tau\} \quad \rho \in FT^*(x)}{X\rho \in FT^*(x)} \qquad \frac{x \stackrel{\alpha}{\not\longrightarrow} \text{ for all } \alpha \in X \cup \{\tau\} \quad x \stackrel{\mathrm{t}}{\longrightarrow} y \quad X\rho \in FT^*(y)}{X\rho \in FT^*(x)} \qquad \frac{x \stackrel{\alpha}{\not\longrightarrow} \text{ for all } \alpha \in X \cup \{\tau\} \quad x \stackrel{\mathrm{t}}{\longrightarrow} y \quad a\rho \in FT^*(y)}{Xa\rho \in FT^*(x)}\ (a \in X)$$

Congruence

Systems P, Q are *failure trace equivalent*, $P \equiv^*_{FT} Q$, if $FT^*(P) = FT^*(Q$

Theorem: $\equiv^*_{FT}$ is a congruence for the operators of $\mathrm{CCSP_t}$, except $+$.

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Write $P \sqsubseteq^*_{FT} Q$ iff $FT^*(P) \supseteq FT^*(Q)$.

The coarsest preorder respecting safety properties

Assume that the alphabet A of visible actions contains one specific action b, whose occurrence is *bad*.
The *canonical safety property* says that b **will never happen**.

A process P satisfies this property, notation $P \models \mathit{safety}(b)$, if no partial failure trace of P contains the action b.

A preorder $\sqsubseteq$ *respects* the canonical safety property if $P \sqsubseteq Q$ and $P \models \mathit{safety}(b)$ implies $Q \models \mathit{safety}(b)$.

Theorem: $\sqsubseteq^*_{FT}$ is the coarsest preorder that respects the canonical safety property.

In other words, if $P \not\sqsubseteq^*_{FT} Q$, then there is a context $\mathcal{C}[\_]$ such that $\mathcal{C}[P] \models \mathit{safety}(b)$, yet $\mathcal{C}[Q] \not\models \mathit{safety}(b)$.

May testing

Let $\omega \notin A$ be a special action, that does not occur in ordinary processes, but may be used in *testing contexts* $\mathcal{C}[\_]$.
Occurrence of ω denotes a successful test run.
We say that $\mathcal{C}[P]$ **may succeed** if it has a trace containing ω.

The *may-testing preorder* is defined by $P \sqsubseteq_{may} Q$ if

$$\forall \mathcal{C}[\_]. \qquad \mathcal{C}[P] \textbf{ may succeed} \Rightarrow \mathcal{C}[Q] \textbf{ may succeed}$$

Theorem: $\sqsupseteq^*_{FT}$ equals $\sqsubseteq_{may}$.

In other words, if $P \not\sqsupseteq^*_{FT} Q$, then there is a context $\mathcal{C}[\_]$ such that $\mathcal{C}[P]$ **may succeed**, yet $\mathcal{C}[Q]$ **may not**.

Concluding remarks

I added a time-out action to standard untimed process algebra.

Failure trace equivalence is now the coarsest reasonable congruence:
the coarsest that satisfies the canonical safety property,
the coarsest that satisfies all safety properties,
the congruence closure of trace equivalence,
and the equivalence generated by may testing.

Future work includes

- proving a congruence result for recursion
- finding complete axiomatisations
- and extending the approach from partial to complete failure traces, so that liveness properties will be respected.