Network Conscious π-calculus

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Outline

1. Network Conscious π-calculus
   - Example
   - Syntax
   - Semantics

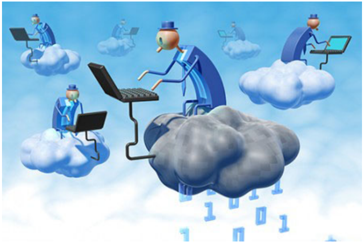
2. Presheaf semantics
   - Categorical environment
   - Semantics in $\mathbf{Set}^{\mathbf{G}_{\mathrm{I}}}$
   - Saturated semantics in $\mathbf{Set}^{\mathbf{G}}$

U. Montanari and M. Sammartino. Network conscious pi-calculus. Technical Report TR-12-01, Computer Science Department, University of Pisa, February 2012.

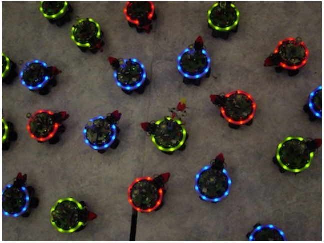
U. Montanari, and M. Sammartino, Network Conscious Pi-calculus: A Concurrent Semantics,

Proc. MFPS 2012, to appear in ENTCS, Elsevier.

A variety of resources



Storage, bandwidth, access rights...



Battery, signal strength...

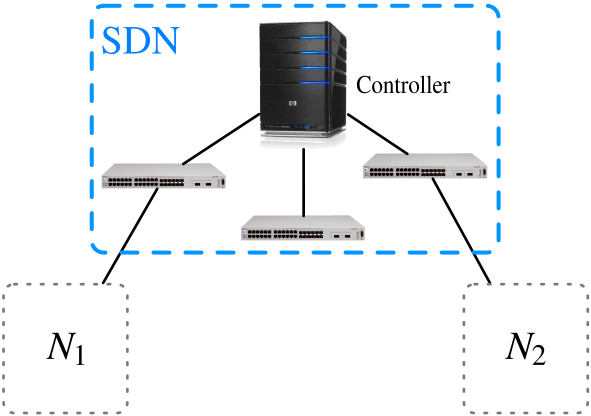
- Traditional formalisms abstract away from these details
- E.g. π-calculus

$\overline{a}b \mid a(x)$ just shared channels!

Network conscious π-calculus

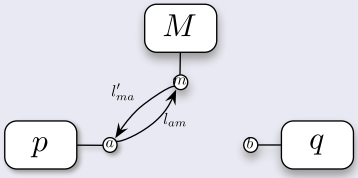
- Network-aware extension of the π-calculus
- Two kinds of names:
  - sites $a, b, c, \ldots$, i.e. network nodes
  - links $l_{ab}, l'_{ac}, l''_{cd} \ldots$, i.e. named connectors between nodes
- Two semantics:
  - Interleaving semantics: direct extension of the π-calculus one, observations are routing paths
  - Concurrent semantics: more natural in this context, observations are multisets of paths
- Main result: bisimilarity on the concurrent semantics is a congruence

Motivating example: Software Defined Networks



Formalization in NCPi

$$M = m(x).m(y).(l_{xy})(\overline{m}x l_{xy}.M)$$
$$p = \overline{a}ma.\overline{a}mb.a(l_{(xy)}).(L(l_{xy}) \mid \overline{a}bc.p')$$
$$q = b(x).q'$$
$$L(l_{xy}) = l_{xy}.L(l_{xy})$$



$$S = p \mid M \mid q \mid L(l_{am}) \mid L(l'_{ma})$$

$$M = m(x).m(y).(l_{xy})(\overline{m}x l_{xy}.M)$$
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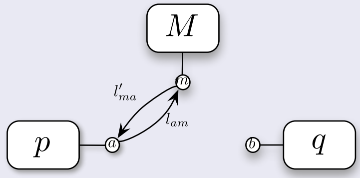

$$S = p \,|\, M \,|\, q \,|\, L(l_{am}) \,|\, L(l'_{ma})$$

Interleaving behavior

$$\left.\begin{array}{l} p \xrightarrow{\bullet;\overline{a}ma} \overline{a}mb.a(l_{(xy)}).(L(l_{xy}) \,|\, \overline{a}bc.p') \\ M \xrightarrow{mma;\bullet} m(y).(l_{ay})(\overline{m}al_{ay}.M) \\ L(l_{am}) \xrightarrow{a;l_{am};m} L(l_{am}) \end{array}\right\} \quad S \xrightarrow{\bullet;l_{am};\bullet} \overline{a}mb.a(l_{(xy)}).(L(l_{xy}) \,|\, \overline{a}bc.p') \,|\, m(y).(l_{ay})(\overline{m}al_{ay}.M) \,|\, q \,|\, L(l_{am}) \,|\, L(l'_{ma})$$

$$M = m(x).m(y).(l_{xy})(\overline{m}xl_{xy}.M)$$
$$p = \overline{a}ma.\overline{a}mb.a(l_{(xy)}).(L(l_{xy}) \,|\, \overline{a}bc.p')$$
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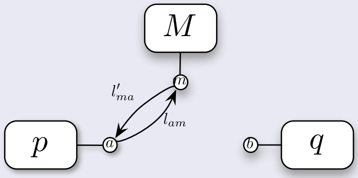

$$S = p \,|\, M \,|\, q \,|\, L(l_{am}) \,|\, L(l'_{ma})$$

Interleaving behavior

$$\cdots \xrightarrow{\bullet;l_{am};\bullet} a(l_{(xy)}).(L(l_{xy}) \,|\, \overline{a}bc.p') \,|\, (l_{ab})(\overline{m}al_{ab}.M) \,|\, q \,|\, L(l_{am}) \,|\, L(l'_{ma})$$

$$\xrightarrow{\bullet;l'_{ma};\bullet} (l_{ab})(L(l_{ab}) \,|\, \overline{a}bc.p' \,|\, M) \,|\, q \,|\, L(l_{am}) \,|\, L(l'_{ma})$$

$$\xrightarrow{\bullet;\bullet} (l_{ab})(L(l_{ab}) \,|\, p' \,|\, M) \,|\, q'[{}^{c}/_{x}] \,|\, L(l_{am}) \,|\, L(l'_{ma})$$

$$M = m(x).m(y).(l_{xy})(\overline{m}x l_{xy}.M)$$
$$p = \overline{a}ma.\overline{a}mb.a(l_{(xy)}).(L(l_{xy}) \mid \overline{a}bc.p')$$
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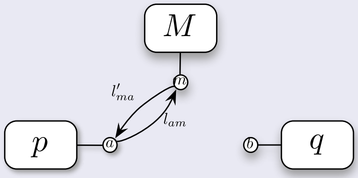

$$S = p \mid M \mid q \mid L(l_{am}) \mid L(l'_{ma})$$

Concurrent behavior

$$S \xrightarrow{\bullet;\overline{a}ma \mid a;l_{am};m \mid mma;\bullet} \overline{a}mb.a(l_{(xy)}).(L(l_{xy}) \mid \overline{a}bc.p') \mid m(y).(l_{ay})(\overline{m}al_{ay}.M) \mid q \mid L(l_{am}) \mid L(l'_{ma})$$

Syntax

Processes

$$p ::= \mathbf{0} \mid \pi.p \mid p+p \mid p|p \mid (r)p \mid A(r_1, r_2, \ldots, r_n)$$

$$\pi ::= \overline{a}br \mid a(s) \mid l_{ab} \mid \tau$$

$$r ::= a \mid l_{ab} \qquad s ::= a \mid l_{(ab)}$$

$$A(s_1, s_2, \ldots, s_n) \stackrel{\text{def}}{=} p \qquad i \neq j \Rightarrow \mathrm{n}(s_i) \cap \mathrm{n}(s_j) = \varnothing$$

Renamings

Pair of functions $\langle \sigma_{\mathcal{S}} : \mathcal{S} \to \mathcal{S}, \sigma_{\mathcal{L}} : \mathcal{L} \to \mathcal{L} \rangle$ s.t.

$$\sigma_{\mathcal{L}}(l_{ab}) = l'_{a'b'} \;\Rightarrow\; \sigma_{\mathcal{S}}(a) = a' \wedge \sigma_{\mathcal{S}}(b) = b' \wedge (ab \neq a'b' \Rightarrow l \neq l')$$

Well-formed processes

- Restricted sites are never endpoints of free links in their scope

$$(a) l_{ab}.p$$

- Links with different endpoints have different labels

$$l_{ab}.p \,|\, l'_{a'b'}.q \qquad ab \neq a'b' \Rightarrow l \neq l'$$

- Structural congruence (α-conversion) $\equiv$ only for such processes

Paths

$$
\begin{array}{lll}
\alpha ::= a; W; b & & \text{(Service Path)} \\
\quad \mid \; \bullet; W; \bullet & & \text{(Complete Path)} \\
\quad \mid \; \bullet; W; \overline{a}br & & \text{(Output Path)} \\
\quad \mid \; abr; W; \bullet & & \text{(Input Path)} \\
\quad \mid \; ab(s); W; \bullet & \mathrm{n}(s) \cap (\mathrm{n}(W) \cup \{a, b\}) = \varnothing & \text{(Bound Input Path)} \\
\quad \mid \; (r)\alpha & & \text{(Extrusion Path)}
\end{array}
$$

$$
W ::= l_{ab} \mid W; W \mid \varepsilon \qquad r ::= a \mid l_{ab} \qquad s ::= a \mid l_{(ab)}
$$

Structural congruence $\equiv_{\alpha}$ given by monoidality of ; and reordering of restrictions

Interleaving transition system

Smallest relation $p \xrightarrow{\alpha} p'$ s.t.

- α is up to structural congruence
- $p \equiv_I p'$, $q \equiv_I q'$ and $p \xrightarrow{\alpha} q$ implies $p' \xrightarrow{\alpha} q'$, where $\equiv_I$ is $\equiv$ with commutative monoidality of $|$

(RES) $$\frac{p \xrightarrow{\alpha} q}{(r)p \xrightarrow{\alpha/r} (r)q}$$

(COM) $$\frac{p_1 \xrightarrow{\bullet;W;\bar{a}br} q_1 \quad p_2 \xrightarrow{abr;W';\bullet} q_2}{p_1 \,|\, p_2 \xrightarrow{\bullet;W;W';\bullet} q_1 \,|\, q_2}$$

(SRV-OUT) $$\frac{p_1 \xrightarrow{(R)\ \bullet;W;\bar{a}br} q_1 \quad p_2 \xrightarrow{a;W';c} q_2}{p_1 \,|\, p_2 \xrightarrow{(R)\ \bullet;W;W';\bar{c}br} q_1 \,|\, q_2}$$

Interleaving network conscious bisimilarity

A binary, symmetric and reflexive relation $\mathcal{R}$ is an *interleaving network conscious bisimulation* if $(p, q) \in \mathcal{R}$ and $p \xrightarrow{\alpha} p'$, with:

1. $\mathrm{bn}(\alpha) \cap \mathrm{fn}(q) = \varnothing$;
2. $l_{ab} \in \mathrm{bn}(\alpha) \cup \mathrm{obj}_{\mathrm{in}}(\alpha) \Rightarrow \forall a'b' \neq ab : l_{a'b'} \notin \mathrm{fn}(q)$

implies that there is q' such that $q \xrightarrow{\alpha} q'$ and $(p', q') \in \mathcal{R}$. The bisimilarity is the largest such relation and is denoted by $\sim_I^{NC}$.

Preserves all the operators except the input prefix

π-calculus encoding

Linkless NCPi

Subcalculus of NCPi such that:

- No links appear in processes
- The output prefix is of the form $\overline{a}ab$

There is a one-to-one correspondence between processes and transitions of π-calculus and linkless NCPi

Concurrent paths

$$
\begin{aligned}
\Lambda ::= & \mathbf{1} && \text{(Empty)} \\
| \; & \beta && \text{(Singleton)} \\
| \; & \Lambda_1 | \Lambda_2 && \text{(Union)} \\
| \; & (r)\Lambda && \text{(Extrusion)}
\end{aligned}
$$

where β is a path without extrusion restrictions.

Structural congruence $\equiv_\Lambda$ given by

- Commutative monoidality of $|$
- Scope extension
- $\equiv_\alpha$ for singletons

Concurrent transition system

Smallest relation $p \stackrel{\Lambda}{\Rightarrow} q$ s.t.

- Λ is up to $\equiv_\Lambda$
- closed under composition with $\equiv$

$$(\text{PAR}) \qquad \frac{p_1 \stackrel{\Lambda_1}{\Longrightarrow} q_1 \quad p_2 \stackrel{\Lambda_2}{\Longrightarrow} q_2}{p_1 \mid p_2 \xrightarrow{\Lambda_1 \mid \Lambda_2} q_1 \mid q_2}$$

$$(\text{COM}) \qquad \frac{p \xRightarrow{(R)\ (\bullet; W; \bar{a}br \mid ab'x; W'; \bullet \mid \Theta)} q}{p \xRightarrow{(R')\ (\bullet; W; W'; \bullet \mid \Theta)} (R'')\ q\sigma}$$

Properties

Concurrent bisimilarity

$\sim^{NC}$ is an obvious extension of $\sim_I^{NC}$

Conservativity of the extension

For all singletons α

$$p \stackrel{\alpha}{\Rightarrow} q \iff p \stackrel{\alpha}{\to} q$$

Congruence property

$\sim^{NC}$ is a congruence with respect to all NCPi operators.

Concurrent transition system for the π-calculus

The concurrent rules generate a concurrent semantics for the π-calculus, via the linkless NCPi encoding, whose bisimilarity is a congruence.

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   - Saturated semantics in $\mathbf{Set}^{\mathbf{G}}$

Consider the algebraic specification of multigraphs as the category

$$\rightrightarrows \quad := \quad e \underset{t}{\overset{s}{\rightrightarrows}} v$$

The category $\mathbf{FinSet}^{\rightrightarrows}$ has finite multigraphs as objects and graph homomorphisms as morphisms.

Definition

The category $\mathbf{G}$ is the skeletal category of $\mathbf{FinSet}^{\rightrightarrows}$. We denote by $\mathbf{G}_{\mathrm{I}}$ the subcategory of $\mathbf{G}$ with only monos.

- $\mathbf{G}$ and $\mathbf{G}_{\mathrm{I}}$ are small and have all limits
- $\mathbf{G}$ has also all colimits, those of $\mathbf{G}_{\mathrm{I}}$ can be computed in $\mathbf{G}$

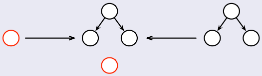
Allocation operators on **G**

Let

- $[n]$ be the discrete graph with n vertices
- k_n be the graph with one edge for each ordered pair of vertices

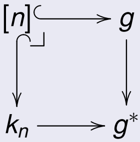
Allocation of vertices

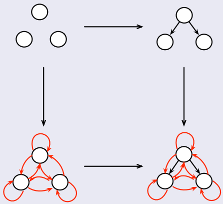
$$\delta_v(g) := g + [1]$$



Allocation of edges

$\delta_e(g) := g^*$ where





Actions on morphisms defined through universal properties

Operators on $\mathbf{Set}^{\mathbf{G}}$

- Countable powerset $\mathcal{P}_c : \mathbf{Set}^{\mathbf{G}} \to \mathbf{Set}^{\mathbf{G}}$

$$\mathcal{P}_c(P)(g) := \{S \subseteq P(g) \mid S \text{ is countable}\}$$

- Names functors $\mathcal{S}, \mathcal{L}, \mathcal{N} : \mathbf{G} \to \mathbf{Set}$

$$\mathcal{S}(g) := \mathbf{G}[\bullet, g] \qquad \mathcal{L} := \mathbf{G}[\bullet \to \bullet, g] \qquad \mathcal{N} := \mathcal{S} + \mathcal{L}$$

- Allocation functors $\Delta_e, \Delta_v : \mathbf{Set}^{\mathbf{G}} \to \mathbf{Set}^{\mathbf{G}}$

$$\Delta_v := (-) \circ \delta_v \qquad \Delta_e := (-) \circ \delta_e$$

Behavioral functor in $\mathbf{Set}^{\mathbf{G_I}}$

$$
\begin{aligned}
B(P) = & \mathcal{P}_c(\mathcal{S} \times \mathcal{L}^\star \times \mathcal{S} \times P && \text{(Service Path)} \\
+ & \ \mathcal{L}^\star \times P && \text{(Complete Path)} \\
+ & \ \mathcal{S} \times \mathcal{N} \times P && \text{(Input Path)} \\
+ & \ \mathcal{L}^\star \times \mathcal{S} \times \mathcal{N} \times P && \text{(Free Output Path)} \\
+ & \ \mathcal{L}^\star \times \mathcal{S} \times \Delta_v P && \text{(Bound Site Output Path)} \\
+ & \ \mathcal{L}^\star \times \mathcal{S} \times \mathcal{S} \times \mathcal{S} \times \Delta_e P) && \text{(Bound Link Output Path)}
\end{aligned}
$$

where $\mathcal{L}^\star(g) := \{\text{strings on } \mathcal{L}(g)\}$

Theorem

The category B-Coalg has a final coalgebra

B-coalgebras as indexed transition systems

B-coalgebras on $P : \mathbf{G}_{\mathrm{I}} \to \mathbf{Set}$ can be characterized as transition systems such that

1. States are pairs $g \vdash p$, meaning $p \in P(g)$
2. Transitions are of the form

$$g \vdash p \xrightarrow{\alpha} g' \vdash p'$$

   where α, g' and p' depends on B (e.g. if α extrudes a site then $g' = \delta_v(g)$ and $p' \in \Delta_v(P)$)
3. Transitions are preserved and reflected by morphisms of $\mathbf{G}_{\mathrm{I}}$, namely injective graph homomorphisms

NCPi coalgebra

- Presheaf $N : \mathbf{G} \to \mathbf{Set}$ of processes

$$N(g) := \{p \mid \mathrm{fn}(p) \subseteq \mathcal{N}(g)\}/\equiv \qquad N_{\mathrm{I}} := \lfloor N \rfloor$$

$$N(\sigma) := \text{extension of } \mathcal{N}(\sigma) \text{ to processes}$$

where $\lfloor - \rfloor := (-) \circ (\mathbf{G}_{\mathrm{I}} \hookrightarrow \mathbf{G})$

- Indexed transition relation from the ordinary one, e.g.

$$\frac{p \xrightarrow{\bullet;W;\overline{a}(b)} p'}{g \vdash p \xrightarrow{\bullet;W;\overline{a}(\star)} \delta_v(g) \vdash p'[\star/b, g \hookrightarrow \delta_v(g)]} \quad \text{(extrusion of a site)}$$

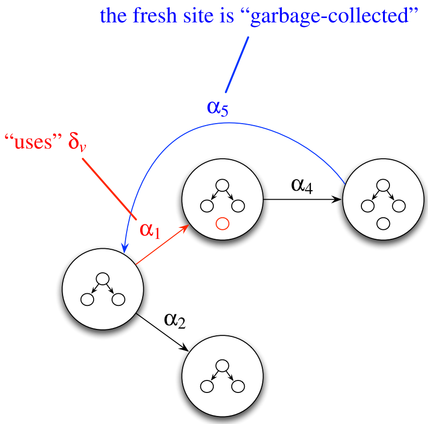
"normalizes" p'

- This induces the NCPi coalgebra (N_{I}, ν)

Theorem

Coalgebraic bisimulations on (N_I, ν) are the ordinary NCPi bisimulations closed under injective renamings

HD-automata



G$_{\text{I}}$ and N_{I} have all the nice properties allowing the existence of a HD-automaton

Extending the semantics to $\mathbf{Set}^{\mathbf{G}}$

Problem: the operational semantics does not immediately correspond to a natural transformation in $\mathbf{Set}^{\mathbf{G}}$

$$\bar{a}bc \mid l_{ab} \mid b'(x) \xmapsto{\nu_g} \{\bar{a}bc \mid l_{ab} \mid b'(x) \xrightarrow{\bullet;\bar{a}bc} l_{ab} \mid b'(x),\ \bar{a}bc \mid l_{ab} \mid b'(x) \xrightarrow{\bullet;l_{ab};\bar{b}bc} b'(x),\ \bar{a}bc \mid l_{ab} \mid b'(x) \xrightarrow{b'b'c;\bullet} \bar{a}bc \mid l_{ab}, \dots\}$$

$$\downarrow b' \mapsto b$$

$$\bar{a}bc \mid l_{ab} \mid b(x) \xmapsto{\nu_{g'}} \{\bar{a}bc \mid l_{ab} \mid b(x) \xrightarrow{\bullet;\bar{a}bc} l_{ab} \mid b(x),\ \bar{a}bc \mid l_{ab} \mid b(x) \xrightarrow{\bullet;l_{ab};\bar{b}bc} b(x),\ \bar{a}bc \mid l_{ab} \mid b(x) \xrightarrow{bbc;\bullet} \bar{a}bc \mid l_{ab},\ \color{red}{\bar{a}bc \mid l_{ab} \mid b(x) \xrightarrow{\bullet;l_{ab};\bullet} \mathbf{0}}, \dots\}$$

Solution: right Kan extensions

- Canonical way of extending the domain of a functor
- The following adjunction is exploited

$$\mathbf{Set}^{\mathbf{G}} \underset{\lceil - \rceil}{\overset{\lfloor - \rfloor}{\rightleftarrows}} \mathbf{Set}^{\mathbf{G}_{\mathrm{I}}} \qquad \lfloor - \rfloor \dashv \lceil - \rceil$$

where $\lceil P \rceil$ is the right Kan extension of $P \in |\mathbf{Set}^{\mathbf{G}_{\mathrm{I}}}|$

$$B : \mathbf{Set}^{\mathbf{G}_{\mathrm{I}}} \to \mathbf{Set}^{\mathbf{G}_{\mathrm{I}}} \quad \rightsquigarrow \quad \widehat{B} := \lceil B(\lfloor P \rfloor) \rceil : \mathbf{Set}^{\mathbf{G}} \to \mathbf{Set}^{\mathbf{G}}$$

$$(N_{\mathrm{I}}, \nu) \quad \rightsquigarrow \quad (N, \ \widehat{\nu} : N \xrightarrow{\eta_N} \lceil \lfloor N \rfloor \rceil \xrightarrow{\lceil \alpha \rceil} \lceil B(\lfloor N \rfloor) \rceil \ )$$

Theorem

Coalgebraic bisimulations on $(N, \widehat{\nu})$ are ordinary NCPi bisimulations closed under all renamings, thus are congruences

$$
\begin{array}{ccccc}
\bar{a}bc \mid l_{ab} \mid b'(x) & \xmapsto{(\eta_N)_g} & \langle \ldots, (\bar{a}bc \mid l_{ab} \mid b(x))_{b' \mapsto b}, \ldots \rangle & \xmapsto{(\lceil \nu \rceil)_g} & \langle \ldots, \{\bar{a}bc \mid l_{ab} \mid b(x) \xrightarrow{\bullet; l_{ab}; \bullet} \mathbf{0}, \ldots\}_{b' \mapsto b}, \ldots \rangle \\
\Big\downarrow {\scriptstyle b' \mapsto b} & & & & \Big\downarrow \\
\bar{a}bc \mid l_{ab} \mid b(x) & \xmapsto[(\eta_N)_{g'}]{} & \langle \ldots, (\bar{a}bc \mid l_{ab} \mid b(x))_{id}, \ldots \rangle & \xmapsto[(\lceil \nu \rceil)_{g'}]{} & \langle \ldots, \{\bar{a}bc \mid l_{ab} \mid b(x) \xrightarrow{\bullet; l_{ab}; \bullet} \mathbf{0}, \ldots\}_{id}, \ldots \rangle
\end{array}
$$

This corresponds to the saturated transition system

$$
p \xrightarrow{\sigma, \alpha} p' \iff p\sigma \xrightarrow{\alpha} p'
$$