Towards a Theory of Architectural Contracts: Schemes and Patterns of Assumption/Promise Based System Specification

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What is a (discrete) system?

A system

- has a scope (a boundary)
- a behaviour
 - black box view: interface
 - syntactic interface: defines the discrete events at the system boundary by input and output via ports, channels, messages (events, signals)
 - dynamic interface, interface behaviour: the processes of interaction in terms of discrete events at the system boundary
 - glass/white box view: an internal structure (state and/or distribution into sub-systems)
 - architecture in terms of sets of sub-systems and their relationships (communication connections)
 - state space

and a behaviour

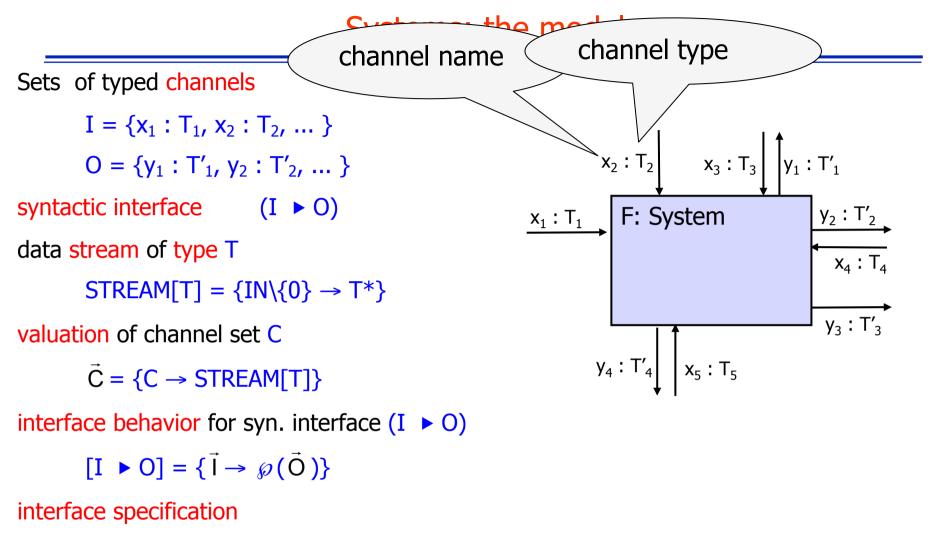
- state transition relation with input and output
- interactions between components

properties

♦ quality profile (performance, ...)

◇ ...





 $p: I \cup O \quad \rightarrow IB$

represented as interface assertion S - logical formulae with channel names as attributes of type stream



Definition. Causal Behavior

For a mapping

$$\mathsf{F}: \vec{\mathsf{I}} \to \wp(\vec{\mathsf{O}})$$

the set

 $\mathsf{dom}(\mathsf{F}) = \{ x : \mathsf{F}(x) \neq \varnothing \}$

is called the *domain* of F.

F is called *total*, if dom(F) = \vec{I} , otherwise F is called *partial*.

F is called *causal*, if for all $t \in IN$ and all input histories $x, z \in \vec{I}$: $x, z \in dom(F) \land x \downarrow t = z \downarrow t \Rightarrow \{y \downarrow t: y \in F(x)\} = \{y \downarrow t: y \in F(z)\}$ F is called *strongly causal*, if (for all $t \in IN$ and all input histories $x, z \in \vec{I}$): $x, z \in dom(F) \land x \downarrow t = z \downarrow t \Rightarrow \{y \downarrow t+1: y \in F(x)\} = \{y \downarrow t+1: y \in F(z)\}$



Realizability

For an interface behaviour F: $\vec{I} \rightarrow \wp(\vec{O})$ a strongly causal total function f: $\vec{I} \rightarrow \vec{O}$ such that $\forall x \in \vec{I} : f(x) \in F(x)$

is called realisation.

F is called *realizable* if there exists a realisation for F.

```
Realisation f: \vec{I} \rightarrow \vec{O} provides a deterministic strategy to calculate
for every input history x a particular output history f(x) with
f(x) \in F(x)
```

f essentially defines a deterministic Mealy machine.



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Example: Nonrealizable strongly causal behaviour

Consider

 $F(x) = \{y: x \neq y\}$

Note that F is strongly causal but not realizable.



Let [F] denote the set of realisations for F.

Definition: Full Realizability

An interface behaviour **F** is called fully realizable if it is realizable and for all input histories **x**:

 $\mathsf{F}(\mathsf{x}) = \{\mathsf{f}(\mathsf{x}): \mathsf{f} \in [\mathsf{F}]\}$

holds.



Theorem:

A strongly causal behaviour F is realizable iff there exists a deterministic Moore machine with f as its interface abstraction that is a realisation of F.

A strongly causal behaviour F is fully realizable iff there exists a Moore machine with F as its interface abstraction.



Designing Architectures: Composing and Decomposing Systems



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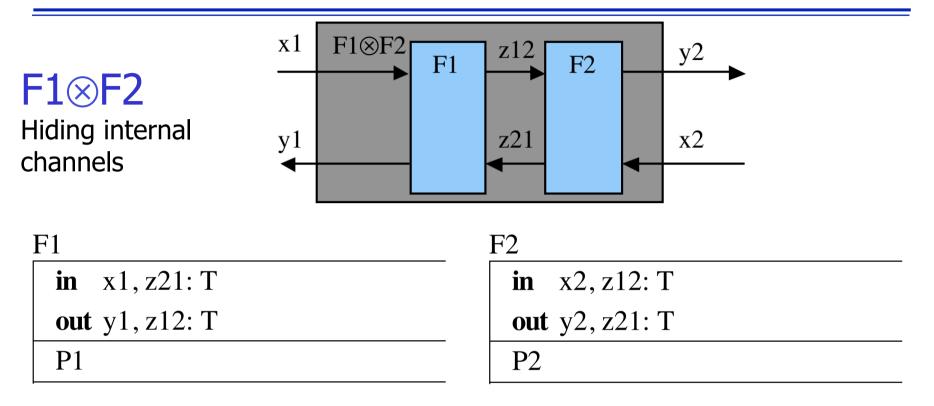


Composition and Decomposition of Systems

 $F_k \in IF[I_k \triangleright O_k]$ for k = 1, 2 where $O_1 \cap O_2 = \emptyset$, $I_k \cap O_k = \emptyset$ shared channels: $I_1 \ C_2$ $O_2 C_2$ F_1 F_2 $C_1 = O_1 \cap I_2$ $C_2 = O_2 \cap I_1$ $O_1 \setminus C_1$ C_2 $I_2 \ C_1$ $I = I_1 \setminus C_2 \cup I_2 \setminus C_1$ $O = O_1 \setminus C_1 \cup O_2 \setminus C_2$ $F_1 \times F_2 \in IF[I \triangleright C]$, where $C = (O_1 \cup O_2 \cup C_1 \cup C_2)$ $z \in H[I_1 \cup I_2 \cup O_1 \cup O_2]$ $(F_1 \times F_2).(x) = \{z | C: x = z | I \land z | O_1 \in F_1(z | I_1) \land z | O_2 \in F_2(z | I_2)\}$ $F_1 \otimes F_2 \in IF[I \triangleright O]$ $(F_1 \otimes F_2).(x) = \{y | O: y \in (F_1 \times F_2).(x)\}$ "closed view"

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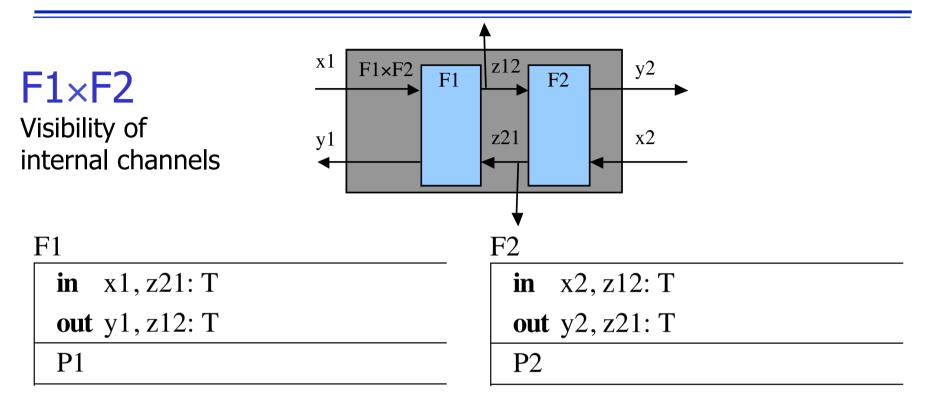
Interface specification composition rule (closed form: Hiding Internal Channels)



F1⊗F2	
in x1, x2: T	
out y1, y2: T	
∃ z12, z21: P1 ∧ P2	



Interface specification composition rule (open form)



F1×F2	
in x1, x2: T	_
out y1, y2, z12, z21: T	
P1 ^ P2	



• Given a set of components K with $F_k \in \ IF$ we write $\bigotimes \{F_k : k \in K \}$

for the interface behaviour of the architecture formed by

 $\mathsf{F_1} \otimes \ \mathsf{F_2} \otimes \ \mathsf{F_3} \otimes \ \mathsf{F_4} \ldots$

- Operator \otimes is parallel composition including feedback
- Operator
 is logically represented by logical conjunction for interface assertions and existential quantification for channel hiding
- Strong causality
 - reflects the flow of time
 - guarantees unique fixpoints of feedback loops in the case of deterministic systems



Contracts Assumption/Promise



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Contracts

An interface specification can be seen as a contract between

- the user of a system
 - ♦ interacting with the system
 - vising the system as sub-system ("component") in a larger system
- the implementer of a system

Key idea:

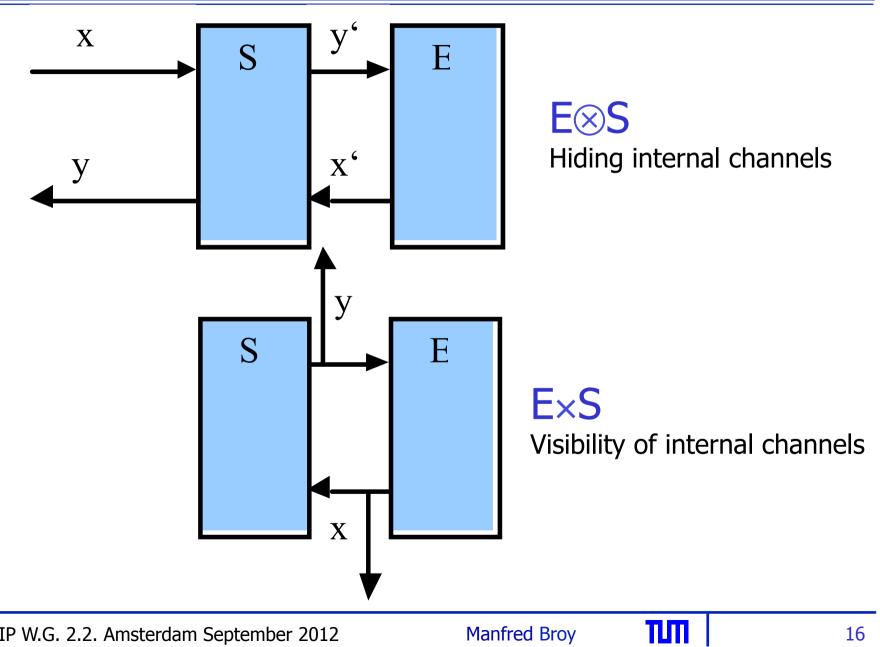
The contract includes all information needed

- for proper use
- for correct implementation

of the SuC (system under consideration)



Composition





A/P-Pattern

- Let System be the set of all systems.
- Composing system S ∈ System with environment E ∈ Env(S) ⊆ System results in

 $E \times S \in System$

 Based on composition operator × we formulate contracts by assumptions and promises:

```
Con(S) \equiv \forall E \in Env(S): Asu(E) \Rightarrow Pro(E \times S)
```

where

- Con(S) is a system specification called contract,
- ♦ Asu(E) is an environment specification called assumption and
- ♦ Pro(E×S) is a specification about the system E×S called a promise.
- The predicates specify system properties

Con, Asu, Pro: System \rightarrow IB



Semantics of Assumption/Promise: Interface Assertions

Given a syntactic interface $(I \triangleright O)$ an interface assertion is a Boolean expression p(x, y) where p is a predicate

p:
$$\vec{I} \times \vec{O} \rightarrow IB$$
 .

and $\mathbf{x} \in \vec{\mathbf{I}}$ and $\mathbf{y} \in \vec{\mathbf{O}}$ are input and output histories



Semantics of Contracts by Logical Implication

• Interface assertions structured into following pattern:

```
assumption: asu(x, y)
promise: pro(x, y)
with the meaning: if the environment fulfils the assumption
asu(x, y)
then the system fulfils the promise
```

```
pro(x, y)
```

• We require of environment E the assumption specified by $Asu(E) \equiv [\forall x, y: x \in E(y) \Rightarrow asu(x, y)]$

and of the system S and its environment ${\sf E}$ the promise is specified by

 $Pro(E, S) = [\forall x, y: y \in (E \times S)(x) \Rightarrow pro(x, y)]$

The combination of these predicates then specifies a contract

 $Con(S) \equiv [\forall E: Asu(E) \Rightarrow Pro(E, S)]$

This defines the meaning of a functional contract.



Deriving Implicative Assertions from Contracts

• Consider

Asu(E) = $[\forall x, y: x \in E(y) \Rightarrow asu(x, y)]$ Pro(E, S) = $[\forall x, y: y \in (E \times S)(x) \Rightarrow pro(x, y)]$

$$Con(S) \equiv [\forall E: Asu(E) \Rightarrow Pro(E, S)]$$

which unfolds into

 $Con(S) \equiv$

 $[\forall E: [\forall x, y: x \in E(y) \Rightarrow asu(x, y)] \Rightarrow$

 $[\forall x, y: y \in (E \times S)(x) \Rightarrow pro(x, y)]]$

 The restriction of causality and realizability for environment E and S allows us to derive further contract properties.

- In case assertion asu(x, y) is causal and fully realizable there exists a most general environment E_{gen} such the following property holds :
 ∀ x, y: x ∈ E_{gen}(y) ⇔ asu(x, y)
- If a most general environment exists, then

 $Con(S) = [\forall x, y: y \in (E_{gen} \times S)(x) \Rightarrow pro(x, y)]$

This semantic interpretation of the A/P pattern is equivalent to $Con(S) \equiv [\forall x, y: y \in S(x) \land x \in E_{gen}(y) \Rightarrow pro(x, y)]$ which leads by the specification of E_{gen} to the following contract: $Con(S) \equiv \forall x, y: asu(x, y) \Rightarrow (y \in S(x) \Rightarrow pro(x, y))$

and to interface assertion con(x, y) for contract Con(S)

 $con(x, y) \equiv [asu(x, y) \Rightarrow pro(x, y)]$



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Why Assumptions have to Speak about System Output

- Consider a system with input channel x and output channel y which numbers as messages specified by
 asu(x, y) ≡ ∀ t: ∀ n ∈ IN: n#(x↓t) ≤ (n#y↓t)+1
 pro(x, y) ≡ ∀ n ∈ IN: n#x = n#y
- We get the specification in terms of an interface assertion con(x, y) = [asu(x, y) ⇒ pro(x, y)]
- The promise is only guaranteed if a next copy of a number n is never sent to the system before the copy previously sent has been forwarded.



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Tab. 1 Cases of Validity of con(x, y), asu(x, y), and pro(x, y)

con(x, y)	asu(x, y)	pro(x, y)	Interpretation
true	true	true	for system S history output y is a correct output for legal input history x
false	true	false	for system S history y is an incorrect output for legal input history x
true	false	true	for system S and output history y
true	false	false	input history x is illegal, environments that produce y in reaction to x are illegal, y is irregular output



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Architectural Contracts & Implicative Assertions

• Given an A/P specification

```
assumption: asu(x, y)
promise: pro(x, y)
```

one interpretation is that the system S is only used in environments E for which assumption asu(x, y) holds. Then we get

 $asu(x, y) \land pro(x, y)$

This interpretation is called *architectural contract*.

• The implicative assertion

 $con(x, y) \equiv [asu(x, y) \Rightarrow pro(x, y)]$

specifies the properties implied for system S by the A/P specification.

• If system S is used in environments E with specifying assertion env(x,

y) we get by composition for the composite system $E \times S$

 $env(x, y) \land (asu(x, y) \Rightarrow pro(x, y))$

which is different to the architectural contract interpretation.

Example: General Implicative Assertions

- Let n be a given natural number.
- Given system contract with input channel x and output channel y, both carrying natural numbers

 $con(x, y) \equiv [n\#y = 0 \Rightarrow n\#x = 0]$

- The premise is not a meaningful assumption, since
 - \diamond there does not exist an environment that guarantees assertion n#y=0
 - it does not speak about input x but only about output y.
- Assertion n#y = 0 is not causal in history y, since

 $y \downarrow t = y' \downarrow t \Rightarrow \forall x: (n \# y = 0) \equiv (n \# y' = 0)$

which does not hold.

 Assertion n#y = 0 is not a healthy assumption, since it is not realizable by any environment.

Assertion

 $\operatorname{con}'(x, y) \equiv [n\#x > 0 \Rightarrow n\#y > 0]$

is equivalent to assertion con(x, y) by contraposition

- Assertion n#x > 0 is causal in history y since the formula $y \downarrow t = y' \downarrow t \Rightarrow \forall x: (n#x \downarrow t > 0) \equiv (n#x \downarrow t > 0)$ holds.
- This assertion may be interpreted as an A/P-format
 assumption: n#x > 0
 promise: n#y > 0

which is a meaningful (but rather simple) contract.



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 We call assumption Asu(E) about environment E nonsatisfiable if there does not exist some environment E such that Asu(E) holds.

Then contract Con(S) is trivial.

- Let Asu be specified based on asu as defined above.
 - \diamond If asu(x, y) is false, then Asu is non-satisfiable.
 - Even in cases where asu(x, y) is not identical to false, predicate Asu may be non-satisfiable.



Theorem:

- If every environment E can be represented by a total Mealy machine, then Asu(E) is satisfiable if and only if asu(x, y) is realizable (for the environment with input y and output x).
- Proof:

For a specification asu(x, y) there exists a Mealy machine that satisfies asu(x, y) if and only if asu(x, y) is realizable.



Safety and Liveness of Interface Assertions



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Notation

- s \downarrow t prefix of length t \in IN of the stream s (which is a sequence of length t),
- $z\!\downarrow\! t$ history of streams prefix of length $t\in{\rm I\!N}$ of the history $z\in\vec{C}$

 $(z\downarrow t)(c) = (z(c))\downarrow t$

Notation: Given a predicate p: $\vec{C} \rightarrow \mathbb{I}B$ we specify for every time $t \in \mathbb{I}N$ $p(z \downarrow t) \equiv \exists z' \in \vec{C} : z \downarrow t = z' \downarrow t \land p(z')$



```
A predicate R is a pure safety property if the following equivalence holds for all histories x and y:
```

 $R(x, y) \equiv \forall t: R(x \downarrow t, y \downarrow t)$

R is a pure liveness property if \forall t: R(x \ t, y \ t)



Canonical Decomposition Assertions into Safety and Liveness Parts

- The safety part R* of an interface assertion R(x, y) is given by the following equation
 R*(x, y) ≡ ∀ t: R(x↓t, y↓t)
- R is called safety realizable if:
 ∀ x: ∃ y: R*(x, y)
- For predicate R we get liveness property R^{∞} included in property R by $R^{\infty}(x, y) \equiv (\neg R^{*}(x, y) \lor R(x, y))$
- To show that R[∞] is a liveness property we have to prove ∀ t: R[∞](x↓t, y↓t)
- Theorem: $R(x, y) \equiv R^*(x, y) \land R^{\infty}(x, y)$



Assumption/Promises as Safety and Liveness Properties

- We consider the interface assertion con(x, y) with con(x, y) = [asu(x, y) ⇒ pro(x, y)]
- The liveness conditions in assertion asu(x, y) for input history x may depend on safety properties of y.
- A typical example would be
 - If y(t) is a query, then
 - \diamond there exists a time t' > t such that x(t') is a reply to this query.



Assumption asu and promise pro are both safety property

• Then the A/P-scheme is equivalent to the following assertion:

 $con(x, y) \equiv \forall t: [asu(x \downarrow t, y \downarrow t) \Rightarrow pro(x \downarrow t, y \downarrow t)]$

This is the consequence of the required causality of con(x, y).



Assumption asu is safety, promise pro is liveness property

 Then the A/P-specification con(x, y) is equivalent to the following assertion:

 $con(x, y) \equiv [\forall t: asu(x \downarrow t, y \downarrow t)] \Rightarrow pro(x, y)$ This is the consequence of the required causality

of con(x, y).



Assumption asu is liveness, promise pro is safety property:

- In this case we can strengthen the specification according to the realizability on con(x, y) con(x, y) = pro(x, y)
- Since the violation of assumption asu(x, y) cannot be observed in finite time, but promise pro can only be violated in finite time, a computation strategy has to observe promise pro in any case.



An example

asu(x, y) ≡ (true#x = ∞)pro(x, y) ≡ (true#y = 0) Assume a realization f that fulfils true#f(x) > 0 for some x with true#x = n ∈ IN. This leads to a contradiction since by

true#f(x) > 0there exists some t withtrue#f(x) \downarrow t > 0and thus for history x' with

 $x \downarrow t = x' \downarrow t$ and true# $x' > \infty$ we get true#f(x') > 0 which violates the specification true# $x = \infty \Rightarrow$ true#f(x) = 0

Assumption asu and promise pro as liveness properties

• In this case the condition

 $asu(x, y) \Rightarrow pro(x, y)$

can be fulfilled by fulfilling promise pro(x, y) in any case.

• Otherwise, the liveness condition have to fit together.



An example

asu(x, y) \equiv (true#x = ∞) pro(x, y) \equiv (true#y < ∞)



Decomposing A/P Specification Into Safety and Liveness

 We decompose assumption asu and promise pro into pure safety properties asu_S, pro_S and pure liveness properties asu_L and pro_L such that

 $con(x, y) \equiv$

 $[asu_{S}(x, y) \land asu_{L}(x, y) \Rightarrow pro_{S}(x, y) \land pro_{L}(x, y)]$

 For a strongly causal and realizable specification con(x, y) we can derive specific assertions

> $asu_{S}(x, y) \Rightarrow pro_{S}(x, y)$ $asu_{S}(x, y) \land asu_{L}(x, y) \Rightarrow pro_{L}(x, y)$



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- Analysing the assumption/promise pattern additional consequences are derived by
 - Causality and realizability requirements
 - ♦ Safety and liveness considerations



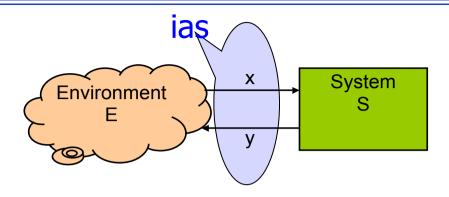
Contracts and Architectures



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From interaction assertions ias to contracts



- Let ias(x, y) be an assertion that characterizes the interaction between system S and its environment E, called interaction assertion.
- ias provides an observation/specification of the traffic between E and S
- Can we derive of a contract for S from assertion ias(x, y) that captures the obligations of system S w.r.t. ias?



- It is clear that we cannot expect to get a reasonable contract from ias(x, y) in every case.
- A simple example would be ias(x, y) = false.



• Given the healthiness condition for the interface assertion

∃ x, y: ias(x, y)

we can do a separation of R into an assumption and a promise (for the safety properties in R) as follows.

 We specify the responsibilities of the system S that accepts the input history x and issues and output history y such that assertion

ias(x, y)

holds.

 We are looking for assertions asu(x, y) and pro(x, y) such that asu(x, y) ∧ pro(x, y) ⇒ ias(x, y)

and

assumption	asu(x, y)
promise	pro(x, y)

is a healthy assumption promise specification.

 If ias(x, y) is strongly causal in x and fully realizable, then asu(x, y) = true is a valid choice.

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• If

```
\forall x: \exists y: ias(x, y)
```

does not hold, then we need to construct an assumption asu(x, y) and a promise pro(x, y) such that

(1) asu(x, y) is causal in y and realizable (2) $asu(x, y) \Rightarrow pro(x, y)$ is strongly causal in x and realizable.

and asu(x, y) \land pro(x, y) \Rightarrow R(x, y)



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- we derive an A/P-specification for system S with the weakest assumption by the following steps:
 - 1. Separate ias into a safety and a liveness part
 - 2. Do the canonical separation of the safety part of ias into an assumption and a promise
 - **3.** Separate the liveness part of ias in an assumption and a promise
 - **4.** Construct the A/P-specification of S from the liveness and safety parts of the assumption and the promise.



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```
asu(x, y) =
[ias(x, y \downarrow 0) \land (\forall t: ias(x \downarrow t, y \downarrow t) \Rightarrow ias(x \downarrow t+1, y \downarrow t))]
```

pro(x, y) = (∀ t: ias(x↓t+1, y↓t) ⇒ ias(x↓t+1, y↓t+1))

Theorem:

Under the condition that assertion ias(x, y) is a pure safety property:

 $(asu(x, y) \land pro(x, y)) \Leftrightarrow ias(x, y)$



- There are liveness conditions that can be seen as assumptions as well as promises.
- An example is the assertion

 ${1}#x + {0}#y = \infty$

which can either be fulfilled

- by assuming an infinite number of copies of 1 in input history x or
- by promising an infinite number of copies of 0 in y (or both).



Given an interaction assertion

ias(x, y)

that is a liveness condition we define an assumption asu_{ias} as follows

 $asu_{ias}(x) \equiv \exists y: ias(x, y)$

and a promise pro_{ias} by the equation

 $pro_{ias}(x, y) \equiv ias(x, y)$

by this definition those parts of the liveness property ias that can either be fulfilled by the environment or by the system under consideration.

- ♦ In the later case they are made part of the promise.
- This way we get a weakest assumption and the strongest promise.

Example. From interaction to interface assertions

• Given the specification

 $ias(x, y) \equiv (x \approx y \land \forall t: (\#y \downarrow t)+b \ge \#x \downarrow t \land \#x \downarrow t \ge \#y \downarrow t)$ where x and y are streams of data and b is a given number and $x \approx y$ specifies that x and y carry the same stream of messages (eliminating empty slots "-")

• We choose the assumption

 $asu(x, y) \equiv \forall t: (\#y \downarrow t) + b \ge \#x \downarrow t$

- asu(x, y) is causal in y and realizable.
- We choose the promise

 $pro(x, y) \equiv (x \approx y \land \forall t: \#x \downarrow t \geq \#y \downarrow t)$

- Assertion pro(x, y) is strongly causal and fully realizable.
- We get

```
pro(x, y) \land asu(x, y) \Rightarrow ias(x, y)
```



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Example. Non-realizable Specification

 Consider a system with input channel x and output channel y both carrying boolean messages:

 $R(x, y) = [(true # x < \infty \Rightarrow true # y = \infty)]$

∧ (true#x = ∞ ⇒ true#y < ∞)]

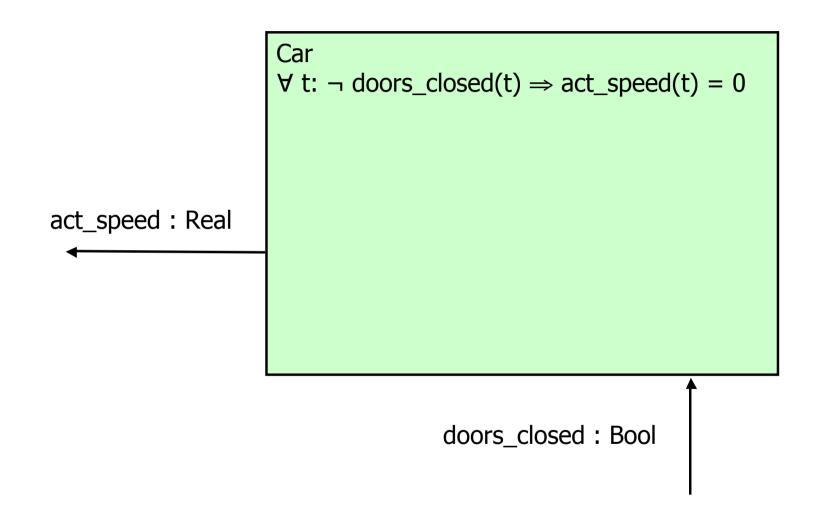
- All the involved assertions are liveness properties. We get
 ∀ x: ∃ y: R(x, y)
- However, there does not exist a causal function f with
 ∀ x: R(x, f(x))
- Otherwise, there would exist a strongly causal function f with a fixpoint y = f(y) such that R(x, y) holds which delivers a contradiction.
- The example suggests that non-realizable specifications include liveness properties that cannot be realized.

Assumptions in Architectural Modelling

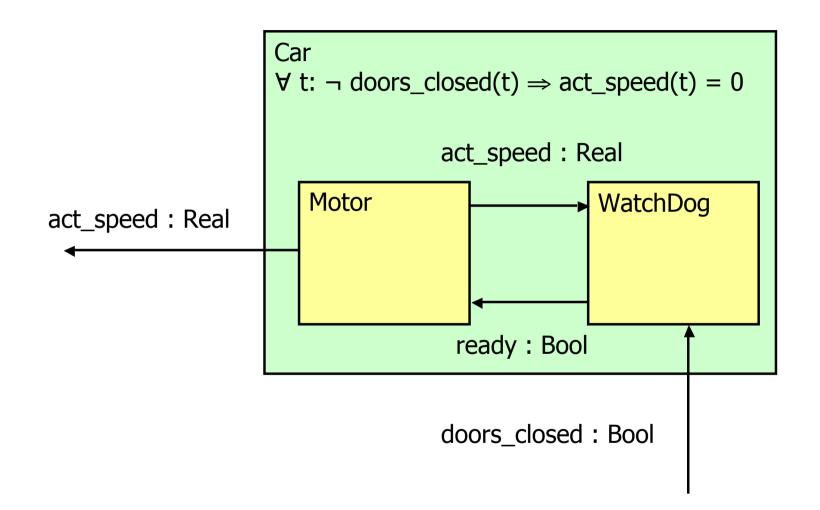


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Example: how A/P-specifications can be formulated

• The specification

 \forall t: \neg doors_closed(t) \Rightarrow act_speed(t) = 0

can only be guaranteed if the two inner components work together. This requires

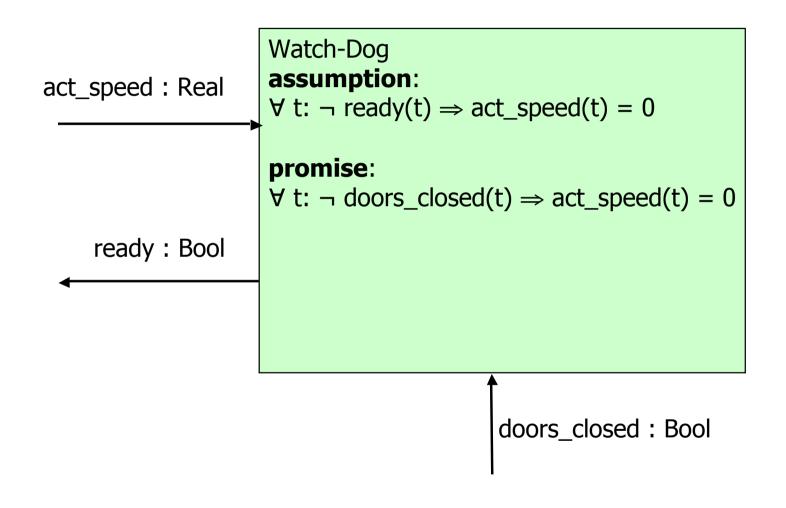
 \forall t: \neg ready(t) \Rightarrow act_speed(t) = 0

• Then the system specification holds if

 \forall t: \neg doors_closed(t) $\Rightarrow \neg$ ready(t)

- This is logically equivalent to the A/P-specification for the WatchDog assumption: ∀ t: ¬ ready(t) ⇒ act_speed(t) = 0 promise: ∀ t: ¬ doors_closed(t) ⇒ act_speed(t) = 0
- In other words,
 - the overall system specification can be guaranteed by the watchdog
 - only if the assumption about the behaviour of the component motor holds.







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Assumption/Promise to define Architectural Design Patterns

• A/P-specification

assumption: \forall t: \neg ready(t) \Rightarrow act_speed(t) = 0 **promise**: \forall t: \neg doors_closed(t) \Rightarrow act_speed(t) = 0 is logically guaranteed by the simple specification \forall t: \neg doors_closed(t) $\Rightarrow \neg$ ready(t)

- This assertion no longer speaks about the specification of the environment, but is a pure interface specification.
- The example shows the simplification of an A/Pspecification to a plain interface assertion.



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Conclusion

- The meaning of A/P specs
 - Simple conditionals/implication under what condition
- A/P specs normalized by
 - Assumption of realizability
 - ♦ Analysis of liveness and safety transformation of A/P specs
- A/P specs for architecture design
 - From interaction assertions to A/P specs
 - From A/P specs to plain specs

