

# The next 700 cryptosystems

Gilles Barthe

IMDEA Software Institute, Madrid, Spain

Based on joint work with:

Benjamin Grégoire (INRIA Sophia Antipolis, France)

César Kunz (U. Politecnica Madrid/IMDEA, Spain)

Yassine Lakhnech (U. de Grenoble/CNRS, France)

Santiago Zanella Béguelin (MSR Cambridge, UK)

# Historical perspective

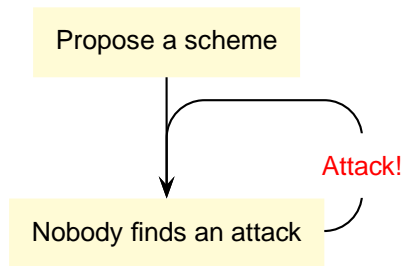
Propose a scheme

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graph TD; A[Propose a scheme] --> B[Nobody finds an attack]
```

Nobody finds an attack

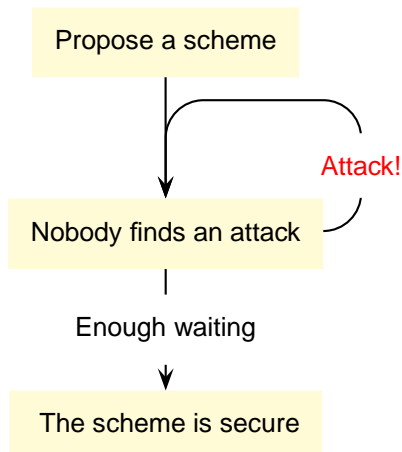
Before 1980

# Historical perspective



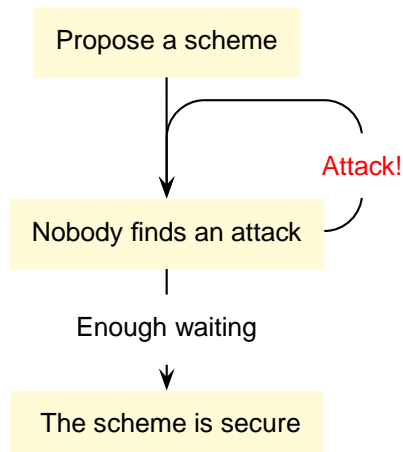
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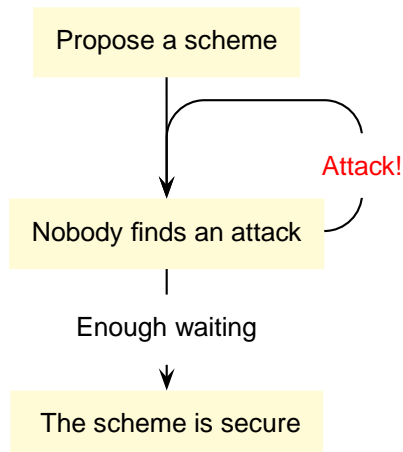
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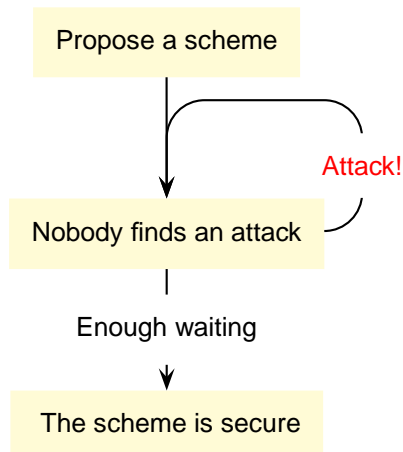
How much time is *enough*?

# Historical perspective



1 year? 5 years? 10 years?

# Historical perspective

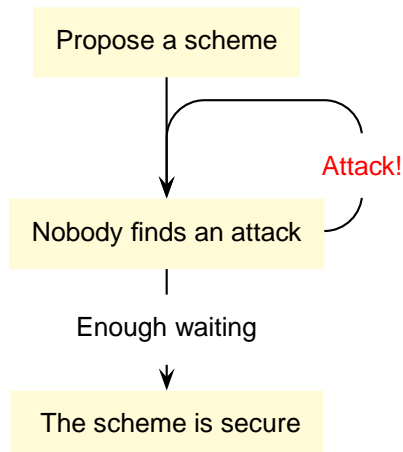


Before 1980

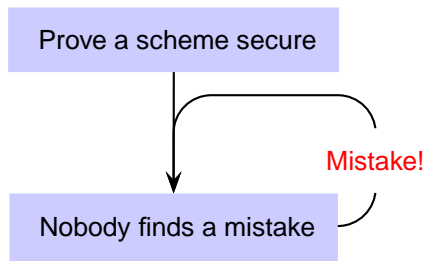
Prove a scheme secure

After 1980

# Historical perspective



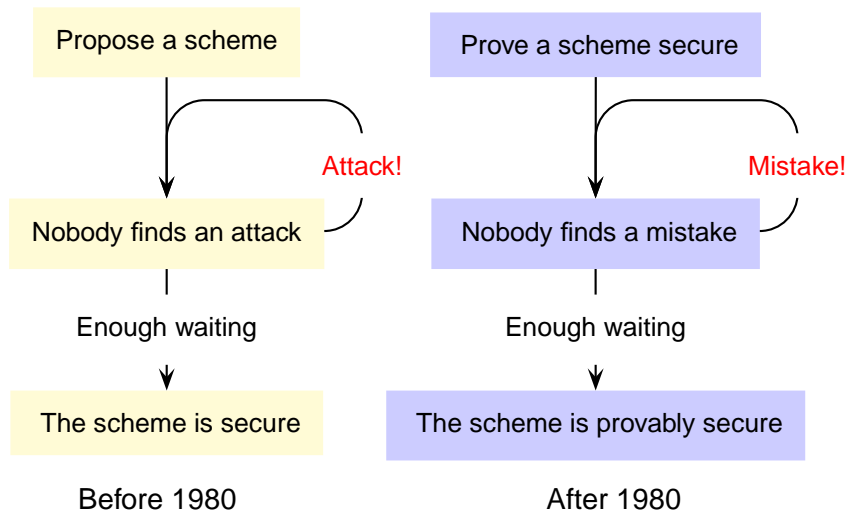
Before 1980



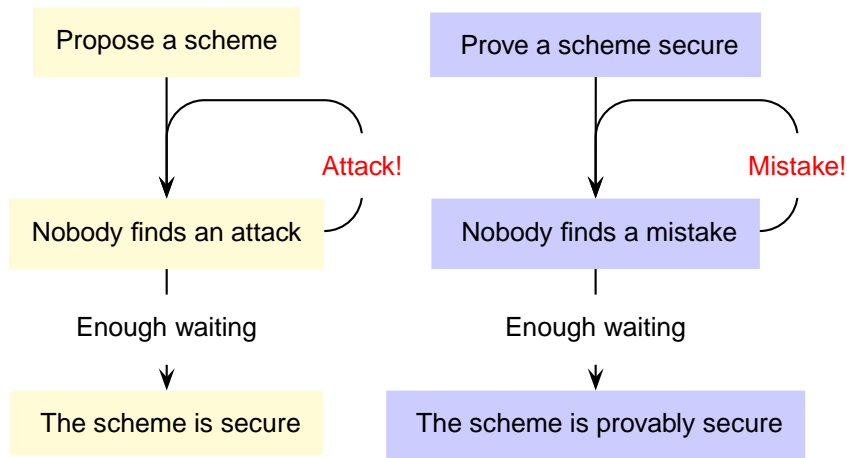
After 1980



# Historical perspective

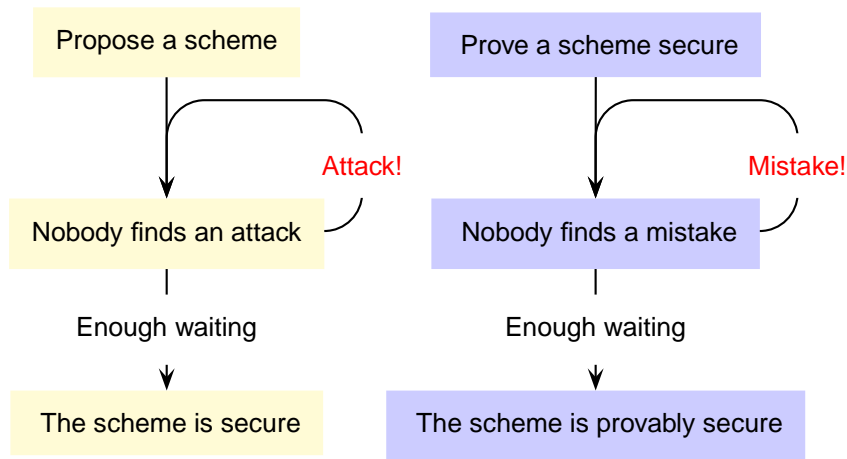


# Historical perspective



How much time is *enough*?

# Historical perspective



1 week? 1 year? 5 years? 10 years? 20 years?

# A famous example: RSA-OAEP

**Oracle**  $\text{Enc}_{pk}(m)$  :

$r \xleftarrow{\$} \{0, 1\}^{k_0}$ ;

$s \leftarrow G(r) \oplus (m \parallel 0^{k_1})$ ;

$t \leftarrow H(s) \oplus r$ ;

return  $f_{pk}(s \parallel t)$

**Oracle**  $\text{Dec}_{sk}(c)$  :

$(s, t) \leftarrow f_{sk}^{-1}(c)$ ;

$r \leftarrow t \oplus H(s)$ ;

if  $[s \oplus G(r)]_{k_1} = 0^{k_1}$  then return  $[s \oplus G(r)]^n$   
else return  $\perp$

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Game IND-CCA2 :

$(sk, pk) \leftarrow \mathcal{KG}()$ ;  
 $(m_0, m_1, \sigma) \leftarrow \mathcal{A}_1(pk)$ ;  
 $b \xleftarrow{\$} \{0, 1\}$ ;  
 $c^* \leftarrow \text{Enc}(pk, m_b)$ ;  
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**Oracle**  $G(x)$  :

if  $x \notin \text{dom}(L_G)$  then  $L_G[x] \xleftarrow{\$} \{0, 1\}^{n+k_1}$ ;  
return  $L_G[x]$

**Oracle**  $H(x)$  :

if  $x \notin \text{dom}(L_H)$  then  $L_H[x] \xleftarrow{\$} \{0, 1\}^{k_0}$ ;  
return  $L_H[x]$

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Game POW :

$(sk, pk) \leftarrow \mathcal{KG}()$ ;  
 $y \xleftarrow{\$} \{0, 1\}^{n+k_1}$ ;  
 $z \xleftarrow{\$} \{0, 1\}^{k_0}$ ;  
 $y' \leftarrow \mathcal{I}(f_{pk}(y \parallel z))$ ;  
return  $y = y'$

## A famous example: RSA-OAEP

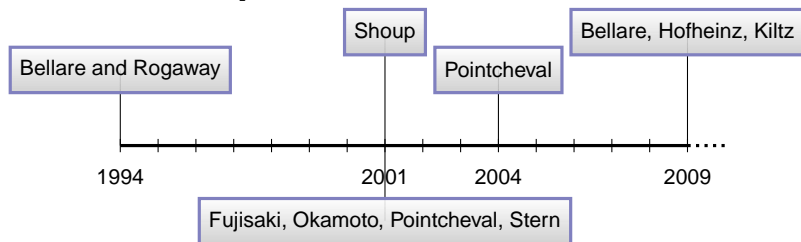
For every IND-CCA2 adversary  $\mathcal{A}$  there exists an inverter  $\mathcal{I}$  s.t.

$$\begin{aligned} \text{Adv}_{\text{IND-CCA2}}(\mathcal{A}) &= \left| \Pr_{\text{IND-CCA2}}[b = b'] - \frac{1}{2} \right| \\ &\leq \mathbf{Succ}_f^{\text{POW}}(\mathcal{I}) + \frac{3q_D q_G + q_D^2 + 4q_D + q_G}{2^{k_0}} + \frac{2q_D}{2^{k_1}} \end{aligned}$$

where

$$\mathbf{Succ}_f^{\text{POW}} = \Pr_{\text{POW}}[y = y']$$

# A famous example: RSA-OAEP



**1994** Purported proof of chosen-ciphertext security

**2001** Proof establishes a weaker security notion, but desired security can be achieved

- 1 ...for a modified scheme, or
- 2 ...under stronger assumptions

**2004** Filled gaps in Fujisaki et al. 2001 proof

**2009** Security definition needs to be clarified

**2010** Filled gaps and improved bounds from 2004 proof

**2012** Improved bound from 2010 proof



# Provable security as program verification!

CertiCrypt/EasyCrypt project, 2006-

High-assurance cryptographic proofs

- ▶ based on rigorous methods from programming languages
  - program verification
  - compiler verification
- ▶ with machine support building upon off-the-shelf tools

# Code-based game-playing proofs

(Bellare & Rogaway 2004, Halevi 2005)

Games as probabilistic programs

|                   |   |                 |
|-------------------|---|-----------------|
| $\mathcal{C} ::=$ | $\mathcal{V} \leftarrow \mathcal{E}$                                  | assignment      |
|                   | $\mathcal{V} \xleftarrow{\$} \mathcal{D}$                             | random sampling |
|                   | $\mathcal{C}; \mathcal{C}$  | sequence        |
|                   | if $\mathcal{E}$ then $\mathcal{C}$ else $\mathcal{C}$                | conditional     |
|                   | while $\mathcal{E}$ do $\mathcal{C}$                                  | while loop      |
|                   | $\mathcal{V} \leftarrow \mathcal{P}(\mathcal{E}, \dots, \mathcal{E})$ | procedure call  |

- ▶ For cryptographers: rigorous notation for games
- ▶ In our work: rigorous justification of game-based proofs

|                   |                                     |               |
|-------------------|-------------------------------------|---------------|
| $\mathcal{E} ::=$ | $\mathcal{E} \oplus \mathcal{E}$    | xor           |
|                   | $\mathcal{E} \parallel \mathcal{E}$ | concatenation |
|                   | ...                                 |               |

# The game-playing approach

(Shoup 2004, Bellare & Rogaway 2004, Halevi, 2005)

**For every** feasible adversary  $\mathcal{A}$  against scheme  $\mathbf{S}$  (wrt goal  $\mathbf{G}$ )  
**there exists** a feasible adversary  $\mathcal{B}$  against assumption  $\mathbf{H}$  st

$$\Pr_{G_a}[\mathcal{A} \text{ breaks } \mathbf{S}] \leq h(\Pr_{G_h}[\mathcal{B} \text{ breaks } \mathbf{H}])$$

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**Game  $G_a$  :**

...  
...  $\leftarrow \mathcal{A}(\dots)$ ;  
...

**Game  $G_1$  :**

...  
...  
...

...

**Game  $G_h$  :**

...  
...  $\leftarrow \mathcal{B}(\dots)$ ;  
...

$$\Pr_{G_a}[\mathcal{A} \text{ breaks } \mathbf{S}] \leq h_1(\Pr_{G_1}[E_1]) \leq \dots \leq h(\Pr_{G_h}[\mathcal{B} \text{ breaks } \mathbf{H}])$$

# pRHL: a Relational Hoare Logic for pWHILE

(after Benton 2004)

- ▶ Judgment:

$$c_1 \sim c_2 : P \Rightarrow Q$$

where  $P$  and  $Q$  are relations on memories

- ▶ Validity:

$$\models c_1 \sim c_2 : P \Rightarrow Q$$

iff for all memories  $m_1$  and  $m_2$

$$(m_1, m_2) \models P \rightarrow (\llbracket c_1 \rrbracket_{m_1}, \llbracket c_2 \rrbracket_{m_2}) \models Q^\sharp$$

- ▶ Lifting  $Q^\sharp$  asserts *existence* of maximal flow in flow network (beware of existential quantification)

pRHL captures common patterns of reasoning in crypto proofs

## Conditionals

$$\frac{\vDash c_1 \sim c : P \wedge e\langle 1 \rangle \Rightarrow Q \quad \vDash c_2 \sim c : P \wedge \neg e\langle 1 \rangle \Rightarrow Q}{\vDash \text{if } e \text{ then } c_1 \text{ else } c_2 \sim c : P \Rightarrow Q}$$

## Assignment

$$\frac{}{\vDash x \leftarrow e \sim \text{nil} : Q\{x\langle 1 \rangle := e\langle 1 \rangle\} \Rightarrow Q}$$

## Random assignment

$$\frac{f \text{ is 1-1 and } Q' \stackrel{\text{def}}{=} \forall v, Q\{x\langle 1 \rangle := f v, x\langle 2 \rangle := v\}}{\vDash x \xleftarrow{\$} A \sim x \xleftarrow{\$} A : Q' \Rightarrow Q}$$

## Adversary calls

$$\frac{\forall \mathcal{O}. \vDash z \leftarrow \mathcal{O}(\vec{w}) \sim z \leftarrow \mathcal{O}(\vec{w}) : Q \wedge =_W \Rightarrow Q \wedge =_{\{z\}}}{\vDash x \leftarrow \mathcal{A}(\vec{y}) \sim x \leftarrow \mathcal{A}(\vec{y}) : Q \wedge =_Y \Rightarrow Q \wedge =_{\{x\}}}$$

# Tool support and examples

## **CertiCrypt: formally verified Coq libraries**

- ▶ Optimizations and probabilistic relational Hoare logic
- ▶ Verified against operational semantics based on ALEA

## **EasyCrypt: SMT-based verification tool**

- ▶ Probabilistic relational Hoare logic
- ▶ Verification condition generation + why3 back-end
- ▶ Accessible to cryptographers

## **Examples**

- ▶ Crypto: public-key encryption, block ciphers, signatures, hash designs, zero-knowledge proofs of knowledge, authenticated key exchange protocols
- ▶ Differential privacy: continuous statistics, approximation algorithms, synthetic databases, 2-party computation

# A simple example: BR93 encryption

Game IND-CPA :

$(sk, pk) \leftarrow \mathcal{KG}()$ ;  
 $(m_0, m_1, \sigma) \leftarrow \mathcal{A}_1(pk)$ ;  
 $b \xleftarrow{\$} \{0, 1\}$ ;  
 $c^* \leftarrow \text{Enc}(pk, m_b)$ ;  
 $b' \leftarrow \mathcal{A}_2(pk, c^*, \sigma)$ ;  
return  $b = b'$

$\text{Enc}_{pk}(m)$  :

$r \xleftarrow{\$} \{0, 1\}^\ell$ ;  
 $s \leftarrow G(r) \oplus m$ ;  
 $y \leftarrow f_{pk}(r) \parallel s$ ;  
return  $y$

Game OW :

$(sk, pk) \leftarrow \mathcal{KG}()$ ;  
 $y \xleftarrow{\$} \{0, 1\}^\ell$ ;  
 $y' \leftarrow \mathcal{I}(f_{pk}(y))$ ;  
return  $y = y'$

$G(x)$  :

if  $x \notin \text{dom}(L_G)$  then  $L_G[x] \xleftarrow{\$} \{0, 1\}^k$ ;  
return  $L_G[x]$

For every IND-CPA adversary  $\mathcal{A}$  making at most  $q_G$  queries to  $G$ , there exists an inverter  $\mathcal{I}$  against OW such that

$$\left| \Pr_{\text{IND-CPA}} [b = b'] - \frac{1}{2} \right| \leq q_G \text{ Succ}_f^{\text{OW}}(\mathcal{I})$$



# Step 1: failure event

## Game $G_0$ :

$L_G \leftarrow \emptyset; Q_G \leftarrow [];$   
 $(sk, pk) \leftarrow \mathcal{KG}();$   
 $(m_0, m_1, \sigma) \leftarrow \mathcal{A}_1(pk);$   
 $b \xleftarrow{\$} \{0, 1\};$   
 $r \xleftarrow{\$} \{0, 1\}^\ell;$   
 $g \leftarrow G(r);$   
 $s \leftarrow g \oplus m_b;$   
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## Game $G_1$ :

$L_G \leftarrow \emptyset; Q_G \leftarrow [];$   
 $(sk, pk) \leftarrow \mathcal{KG}();$   
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 $c^* \leftarrow f_{pk}(r) \parallel s;$   
 $b' \leftarrow \mathcal{A}_2(pk, c^*, \sigma);$

The games are equivalent until the adversary queries  $G$  with  $r$

$$|\Pr_{\text{IND-CPA}}[b = b'] - \Pr_{G_1}[b = b']| \leq \Pr_{G_1}[r \in Q_G]$$

## Step 2: optimistic sampling

### Game $G_1$ :

$L_G \leftarrow \emptyset; Q_G \leftarrow [];$   
 $(sk, pk) \leftarrow \mathcal{KG}();$   
 $(m_0, m_1, \sigma) \leftarrow \mathcal{A}_1(pk);$   
 $b \xleftarrow{\$} \{0, 1\};$   
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### Game $G_2$ :

$L_G \leftarrow \emptyset; Q_G \leftarrow [];$   
 $(sk, pk) \leftarrow \mathcal{KG}();$   
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 $c^* \leftarrow f_{pk}(r) \parallel s;$   
 $b' \leftarrow \mathcal{A}_2(pk, c^*, \sigma);$

Games are equivalent and  $c^*$  is independent from  $b$ , hence

$$\left| \Pr_{\text{IND-CPA}}[b = b'] - \frac{1}{2} \right| \leq \Pr_{G_2}[r \in Q_G]$$

## Step 3: reduction

### Game $G_2$ :

$L_G \leftarrow \emptyset; Q_G \leftarrow [];$   
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### Game OW :

$(sk, pk) \leftarrow \mathcal{KG}();$   
 $y \xleftarrow{\$} \{0, 1\}^\ell;$   
 $y' \leftarrow \mathcal{I}(f_{pk}(y));$   
return  $y = y'$

### Adversary $\mathcal{I}(x)$ :

$L_G \leftarrow \emptyset; Q_G \leftarrow [];$   
 $(m_0, m_1, \sigma) \leftarrow \mathcal{A}_1(pk);$   
 $s \xleftarrow{\$} \{0, 1\}^k; y \leftarrow x \parallel s;$   
 $b' \leftarrow \mathcal{A}_2(pk, y, \sigma);$   
 $i \xleftarrow{\$} [1, |Q_G|];$   
return  $Q_G[i];$

## Step 3: reduction

### Game $G_2$ :

$L_G \leftarrow \emptyset; Q_G \leftarrow [];$   
 $(sk, pk) \leftarrow \mathcal{KG}();$   
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 $b' \leftarrow \mathcal{A}_2(pk, y, \sigma);$   
 $i \xleftarrow{\$} [1, |Q_G|];$   
return  $Q_G[i];$

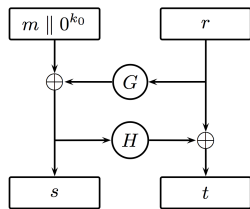
Inverter wins with probability  $\frac{1}{q_G}$  if  $r \in Q_G$ , and 0 otherwise

$$\left| \Pr_{\text{IND-CPA}} [b = b'] - \frac{1}{2} \right| \leq q_G \text{Succ}_f^{\text{OW}}(\mathcal{I})$$

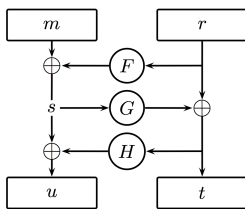
# Beyond OAEP

Over 100 variants of OAEP in the literature

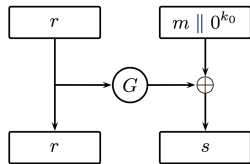
- ▶ Are they all secure?
- ▶ Are there common patterns in proofs?
- ▶ Can proofs be automated?



(a) OAEP



(b) 3-round OAEP



(c) SAEP

# The next 700 cryptosystems

(After Landin, 1966)

The question arises, do the idiosyncracies reflect basic logical properties of the situations that are being catered for? Or are they accidents of history and personal background that may be obscuring fruitful developments?

[...] We must think in terms, not of cryptosystems, but of families of cryptosystems. That is to say we must systematize their design so that a new cryptosystem is a point chosen from a well-mapped space, rather than a laboriously devised construction.



# Generation

|               |     |                                     |                          |
|---------------|-----|-------------------------------------|--------------------------|
| $\mathcal{E}$ | ::= | $m$                                 | input message            |
|               |     | $0$                                 | zero bitstring           |
|               |     | $\mathcal{R}$                       | uniform random bitstring |
|               |     | $\mathcal{E} \oplus \mathcal{E}$    | xor                      |
|               |     | $\mathcal{E} \parallel \mathcal{E}$ | concatenation            |
|               |     | $H(\mathcal{E})$                    | hash                     |
|               |     | $f(\mathcal{E})$                    | trapdoor permutation     |

# Filtering

Eliminate schemes that are not :

- ▶ invertible  $f(r)$
- ▶ IND-CPA
  - is decryption possible without a key?  $m \parallel f(r)$
  - is encryption randomized?  $f(m)$
  - is randomness extractable without a key?  $r \parallel f(m \oplus r)$
- ▶ IND-CCA2
  - is encryption malleable?  $f(r) \parallel m \oplus G(r)$

## Deducibility relation

$$\frac{e \vdash e_1 \quad e \vdash e_2}{e \vdash e_1 \parallel e_2} [\text{Conc}] \quad \frac{e \vdash e_1 \quad e \vdash e_2}{e \vdash (e_1 \oplus e_2)} \downarrow [\text{Xor}] \quad \frac{e \vdash e'}{e \vdash H(e')} [\text{H}]$$
$$\frac{e \vdash e_1 \parallel e_2}{e \vdash e_j} [\text{Proj}_j] \quad \frac{e \vdash e'}{e \vdash f(e')} [f] \quad \boxed{\frac{e \vdash f(e')}{e \vdash e'} [\text{finv}]}$$

# Chosen-plaintext security

## Step 1: proof finding

**Optimistic sampling** Replace  $e \oplus r$ , where  $r$  is fresh, by  $r$

**Permutation** Replace  $f(r)$ , where  $r$  is fresh, by  $r$

**Failure event** Replace  $H(e)$  by fresh  $r$

**Probability** Compute probability of  $b = b'$  or  $e \in L$

## Step 2: proof generation and proof checking

**Generation** Output EasyCrypt file

**Checking** Independent verification of EasyCrypt file (< 120 s)

# Chosen-ciphertext security

If decryption oracle is of the form

```

$$u_0 \dots u_n, t_n \leftarrow \text{Extract}(c);$$

$$\text{for } i \leftarrow n \dots 1 \text{ do } t_{i-1} \leftarrow u_i \oplus H_i(t_i);$$

$$\text{if Test}(\vec{t}, \vec{u}, c) \text{ then GetMsg}(\vec{t}) \text{ else return } \perp$$

```

and IND-CPA and IND-CPA-like properties then IND-CCA2

## Proof intuition

- ▶ Plaintext-awareness: infeasible to get valid ciphertext otherwise than by encrypting a known plaintext

IND-CPA + plaintext-awareness  $\implies$  IND-CCA2

- ▶ Successively modify decryption oracle to reject ciphertexts for which corresponding hash queries have not been made
- ▶ Yields plaintext extractor and reduction to IND-CPA
- ▶ IND-CPA-like properties provide bounds for failure events

# Experiments

- ▶ Generated over 100,000 schemes
- ▶ Filters leave 4,500 schemes
- ▶ Proved IND-CPA security of 3,000 schemes
- ▶ Proved IND-CCA2 security of 2,000 schemes

## Assessment

- ▶ Systematic exploration of design space
- ▶ Tens of papers automated
- ▶ IND-CCA2 checker fails on redundant-free schemes

# ZAEP

(with David Pointcheval)

Two minimal schemes

$$\text{BR93} : f(r) \parallel (G(r) \oplus m) \quad \text{ZAEP} : f(r \parallel G(r) \oplus m)$$

**ZAEP is redundant-free**

$$\text{Dec}(c) : r \parallel t \leftarrow f_{sk}^{-1}(c); g \leftarrow G(r); \text{return } t \oplus g$$

**INDCCA Security of ZAEP for RSA exponent 2 and 3**

$$\left| \Pr_{\text{IND-CCA2}}[b = b'] - \frac{1}{2} \right| \leq \mathbf{Succ}_f^{\text{OW}}(\mathcal{I}) + \frac{q_D}{2^n}$$

Based on existence of two efficient algorithms:

- ▶ CIE: given  $f(r, s_1), f(r, s_2)$  with  $s_1 \neq s_2$ , returns  $s_1, s_2$  and  $r$
- ▶ SIE: given  $f(r, s)$  and  $r$  returns  $s$

# Conclusion

## High-assurance cryptographic proofs

- ▶ Rigorous proofs using PL techniques (pRHL)
- ▶ Independent verification and (in principle) certified proofs

## Automated generation of schemes and proofs

- ▶ public-key encryption
- ▶ zero-knowledge compilers

## Directions for future work

- ▶ Program logics, decision procedures, relational invariant inference. . .
- ▶ Probabilistic encapsulation, modularity
- ▶ Towards implementations
- ▶ Synthesis: revisit classical cryptography