Oberseminar Topologie: 25.05.2020

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„Hurwitz spaces and Moduli spaces of Riemann surfaces.“

Abstract:
Let S_d be a symmetric group, with d>=2. For k>=0, the classical Hurwitz space hur(k,S_d) parametrises d-fold branched covers of the complex plane C with precisely k branch points. We will consider a construction that amalgamates all Hurwitz spaces for all values of k into a single space Hur(S_d).

The motivation for the construction is the following. For all g>=0 and n>=1, let M_{g,n} denote the moduli space of Riemann surfaces of genus g with n boundary components. If d is large enough (with respect to g and n), then there exists a connected component of Hur(S_d) which is homotopy equivalent to M_{g,n}.

The space Hur(S_d) carries a natural structure of topological monoid graded by natural numbers h>=0; we will briefly sketch the computation of the stable, rational cohomology of its subspaces Hur(h,S_d). The result is very explicit in degrees up to roughly d/2. Letting d go to infinity, one can in particular recover the Mumford conjecture on the stable, rational cohomology of moduli spaces of Riemann surfaces, originally proved by Madsen and Weiss.