SINGULARITIES IN THIN FILM FLOW FROM A DYNAMICAL SYSTEMS PERSPECTIVE

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We are considering the free boundary problem for the thin film equation $h_t+(h^n h_{yyy})_y = 0$ in $\{h > 0\}$ where $n \in (0, 3)$ is the mobility exponent. This equation can be derived for surface-tension driven fluid flow in a lubrication approximation under the assumption of small film heights and large viscosities. The choice of n corresponds to various nonlinear slip conditions with $n = 3$ leading to no slip at the substrate and a contact point (i.e., the triple junction where air, liquid, and solid meet) that does not move unless the dissipation is infinite (no-slip paradox). In order to understand the singular behavior of solutions at the free boundary, naturally the question of regularity of solutions at the contact point comes up.

In the first part of the talk, we are considering the regularity of special solutions to the thin-film equation, known as source-type self-similar solutions. This is based on publications [1, 2] in which dynamical systems theory is used to investigate the singularity at the free boundary. In the second part of the talk, we are going to address the full parabolic setting in which PDE methods are employed to derive regularity of the solution at the contact point. This is based on the works [3, 4], where a quadratic mobility $n = 2$ (Navier slip) and complete-wetting boundary conditions are considered. If time permits, the generalization to 3D fluid films and general mobilities for the full parabolic problem will be discussed.

References

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