A method for pricing American options using semi-infinite linear programming

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Abstract: A new approach for the numerical pricing of American options is introduced. The main idea is to choose a finite number of suitable excessive functions (randomly) and to find the smallest majorant of the gain function in the span of these functions. The resulting problem is a linear semi-infinite programming problem, that can be solved using standard algorithms. This leads to good upper bounds for the original problem. For our algorithms no discretization of space and time and no simulation is necessary. Furthermore it is applicable even for high-dimensional problems. The algorithm provides an approximation of the value not only for one starting point, but for the complete value function on the continuation set, so that the optimal exercise region and e.g. the Greeks can be calculated. Furthermore one furthermore easily obtains good lower bounds. We apply the algorithm to (one- and) multidimensional diffusions and to Lévy processes, and show it to be fast and accurate.

Keywords: optimal stopping, excessive functions, upper bounds, semi-infinite linear programming