$\aleph_0$- saturated o-minimal expansions of the reals.

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Abstract.

Let R be a (non-archimedean) real closed field. For uncountable cardinal $\kappa = \aleph_\alpha$, it follows by o-minimality that R is κ - saturated if and only if the linear ordering $(R, <)$ is an η_α - set in the sense of Hausdorff.

We consider the case $\kappa = \aleph_0$. We give necessary and sufficient conditions on the valued field (R, v) (where v is natural valuation on R) for R to be $\aleph_0$ saturated. The conditions are threefold: conditions on the (divisible ordered abelian) value group, on the residue field, and on pseudo-convergence of certain pseudo Cauchy sequences.

Thus to complete the characterization, we also give necessary and sufficient conditions for a divisible ordered abelian group to be $\aleph_0$-saturated.

We discuss a generalization of this theorem to arbitrary o-minimal expansions of ordered fields, focusing on exponential expansions.