The growth of the infinite long-range percolation cluster

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Abstract

I will consider long-range percolation on $\mathbb{Z}^d$, where the probability that two vertices at distance r are connected by an edge is given by $p(r) = 1 - \exp[-\lambda(r)] \in (0, 1)$ and the presence or absence of different edges are independent. Here $\lambda(r)$ is a strictly positive, non-increasing regularly varying function.

I will discuss the asymptotic growth of the size of the k-ball around the origin, $|\mathcal{B}_k|$, i.e. the number of vertices that are within graph-distance k of the origin, for different $\lambda(r)$.

I will show that conditioned on the origin being in the (unique) infinite cluster, non-empty classes of non-increasing regularly varying $\lambda(r)$ exist for which respectively

1) $|\mathcal{B}_k|^{1/k} \to \infty$ almost surely, 2) there exist $1 < a_1 < a_2 < \infty$ such that $\lim_{k\to\infty} \mathbb{P}(a_1 < |\mathcal{B}_k|^{1/k} < a_2) = 1$, 3) $|\mathcal{B}_k|^{1/k} \to 1$ almost surely.

This result can be applied to spatial epidemics. In particular, regimes are identified for which the basic reproduction number, R_0, which is an important quantity for epidemics in unstructured populations, has a useful counterpart in spatial epidemics.