The Steinberg formula for orbit groups

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Abstract

The orbit space of unimodular rows under elementary action, over certain rings R, was originally studied by topologists. Under suitable restrictions, they could put group structures on this orbit space. Motivated by these ideas, algebraists too obtained similar results for the orbit space, for any commutative, Noetherian ring with unity.

A variety of interesting formulae in the orbit groups are known due to the works of A. A. Suslin, L. N. Vaserstein and W. van der Kallen. We show that the following relation, which we call the Steinberg formula, holds in the orbit groups:

$$[(x, a_2, \ldots, a_n)] * [(1 - x, a_2, \ldots, a_n)] = [(x(1 - x), a_2, \ldots, a_n)].$$