Discrete-time population dynamics on the space of measures

Horst R. Thieme (`hthieme@asu.edu`)
School of Mathematical and Statistical Sciences
Arizona State University, Tempe, AZ 85287-1804, USA

If a structured population is considered and the individual state space is given by a metric space S, measures μ on the σ-algebra of Borel subsets T of S offer a modeling tool with a natural interpretation: $\mu(T)$ is the number of individuals with structural characteristics in the set T. A discrete-time population model is given by a map F on the vector space of finite (signed) Borel measures that maps the structural population distribution of a given year to the one of the next year. F can be interpreted as a population turnover map. Under suitable assumptions, F has a first order approximation at the origin (the zero measure, the extinction fixed point), which is a positive linear operator or, in the case of sexual reproduction, a homogeneous order-preserving operator on the cone of nonnegative measures. This first order approximation can be interpreted as a basic population turnover operator. In the case of a semelparous population, it can be identified with the next generation operator. Even if it is only homogeneous, a spectral radius can be defined by the usual Gelfand formula and be interpreted as a basic population turnover number. We investigate in how far it serves as a threshold parameter between population extinction and population persistence. The variation norm on the space of measures is too strong to give the basic turnover operator enough compactness such that its spectral radius is an eigenvalue associated with a positive eigenmeasure or a positive eigenfunctional which can be used in standard persistence techniques. If S is compact, one can use that the space of measures is the dual space of the space of continuous functions on S. If S is not compact, the flat norm (also known as (dual) bounded Lipschitz norm) is a suitable alternative.