Oberseminar Mathematische Stochastik

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Permutation Invariant Functionals of Lévy Processes and applications

(joint work with F. Baumgartner, Innsbruck)

Abstract:
Itô's chaos expansion [2] describes L_2-random variables, defined on an abstract Wiener-Lévy space, uniquely by deterministic chaos kernels. The description of randomness by deterministic kernels has its price: the chaos kernels might have a complicated structure so that they cannot be used in certain applications. Itô itself used the chaos expansion to study the structure of translation invariant functionals on the time interval $\mathbb{R}$.

In [1] we continue this investigation by studying natural invariance properties of functionals under permutations defined on Lévy processes $X = (X_t)_{t\in[0,1]}$ on the finite time interval $[0,1]$ in contrast to under translations on the infinite time interval $\mathbb{R}$ (as used by Itô). We show that our permutation properties, that appear naturally in various applications, can be described by a simplified structure of the deterministic chaos kernels in Itô's chaos expansion. These structural properties of the kernels relate intrinsically to a measurability with respect to invariant σ-algebras. This makes it possible to apply deterministic functions to invariant functionals on Lévy processes while keeping the simplified structure of the kernels. This stability is crucial for applications. In particular, the invariance properties yield to the notion of a *locally ergodic* subset of the time domain $[0,1]$, the Lévy process X is defined on. One of the main results is that the chaos kernels can be chosen to be constant on locally ergodic subsets. Applications are given for backward stochastic differential equations (BSDEs) driven by Lévy processes and for the chaos expansion of Nualart and Schoutens based on the normalized Teugels martingales.

[1] F. Baumgartner and S. Geiss: Permutation Invariant Functionals of Lévy Processes. arXiv:1407.3645. In revision for *Trans. AMS*.

[2] K. Itô: Spectral type of the shift transformation of differential processes with stationary increments. *Trans. AMS*, 81(2), 253–263, 1956.