Westfälische Wilhelms-Universität Münster

Workshop on Numerical Schemes for Surface Partial Differential Equations

Westfälische Wilhelms-Universität Münster

Applied Mathematics Münster

Funded by DFG

February 22-24, 2016
Münster, Germany

Second Applied Mathematics Symposium Münster
Workshop: Numerical Schemes for Surface Partial Differential Equations

Invited Speakers

Charles M. Elliott
Mats G. Larson
Andreas Rätz
Arnold Reusken
Axel Voigt

The goal of this workshop is to foster the intensive exchange of recent results and ideas by bringing together experts on surface PDEs and associated numerical schemes. Particularly challenging topics include complex-shaped surfaces, surfaces which evolve in time, incorporation of mass conservation into the schemes and coupling to bulk processes.

http://www.wwu.de/math/surf2016

Organizers
Christian Engwer, Sebastian Westerheide

General information:

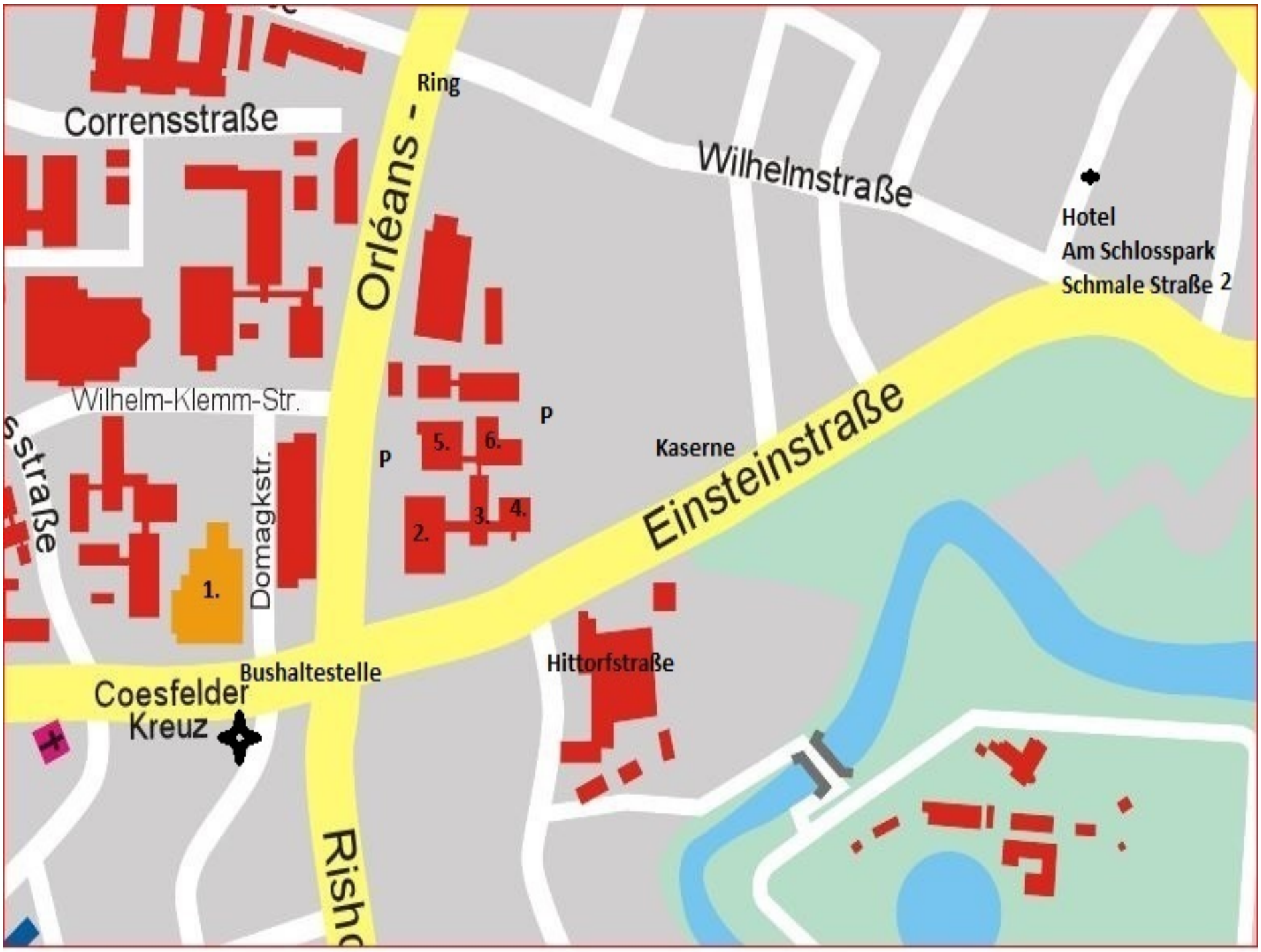
Welcome to the Workshop on Numerical Schemes for Surface Partial Differential Equations 2016!

In the following, we will give you some information on the activities and the schedule of the workshop.

- In your conference folder you will find vouchers for lunch in the canteen (Mensa, Map, No 1) . Please drop the voucher at the cashpoint after picking up your meal. We reserved tables on the first floor.
  Please return unused vouchers to Mrs. Gietz after the workshop.
- The conference dinner will take place on Monday, February 22 in the entry-hall of the lecture building. (Map, No 5)
- Wifi access is available. Your password is in your conference folder.
- Alternatively, if you are part of the eduroam community, you may connect to the network "eduroam" as usual.
- Information on the invited talks of the workshop are provided in the attached book of abstracts.
- All talks will take place at SRZ 19 (Map, No 5)
- Please registrer your spontaneous talk at the registration desk by 09:45 using the form contained in your conference folder.

Information for speakers:
You may use your own notebook and presenter for your presentation, or the ones provided in the lecture room.
Please make sure that everything works before your actual talk (e.g. in one of the coffee/lunch breaks before your session)

In case of questions, we are happy to assist you. You find contact persons at the registration desk and during the breaks.



1. Mensa
2. Hörsaalgebäude / Lecture Hall — Einsteinstr. 64
3. Hochhaus Mathematik/Informatik / highrise Mathematics/ Informatics — Einsteinstr. 62
4. ZIV
5. Seminargebäude / seminar building — Orléansring 12
6. Angewandte Mathematik / Applied Mathematics — Orléansring 10

P- Parkplatz / parking

* Bushaltestelle Coesfelder Kreuz / bus stop Coesfelder Kreuz

Monday, Feb 22 Time	Speaker / Program	Title
09:15	Registration and welcome coffee	
09:45	Welcome address by Christian Engwer	
10:00	Charlie Elliott	PDEs on surfaces and envolving domains
11:00-11:30	Coffee break	
11:30	Arnold Reusken	Space-time trace FEM for PDEs on evolving surfaces
12:30-13:30	Lunch	
13:30	Spontaneous talks session 1	
15:30-16:00	Coffee break	
16:00	Spontaneous talks session 2	
18:00	Drinks in the lobby	
19:00	Workshop dinner (open end)	

Tuesday, Feb 23 Time	Speaker / Program	Title
09:55	Short welcome address	
10:00	Axel Voigt	Positionally ordered liquid crystals on curved manifolds
11:00-11:30	Coffee break	
11:30	Spontaneous talks session 3	
12:30-13:30	Lunch	
13:30	Andreas Rätz	Diffuse-interface methods for coupled bulk - - surface PDEs and an application to modelling of lipid raft formation in cell membranes
14:30	Spontaneous talks session 4	
15:30-16:00	Coffee break	
16:00	Spontaneous talks session 5	
17:00	End of day	

Wednesday, Feb 24 Time	Speaker / Program	Title
09:25	Short welcome address	
09:30	Mats. G. Larson	Cut Finite Element Methods for Convection-Diffusion Problems on Evolving Domains
10:30-11:00	Coffee break	
11:00	Spontaneous talks session 6	
12:00	Closing remarks and optional lunch	

Book of Abstracts (invited talks)

Charlie Elliott - PDEs on surface and evolving domains

Many physical models give rise to the need to solve partial differential equations in time dependent regions. The complex morphology of biological membranes and cells coupled with biophysical mathematical models present significant computational challenges as evidenced within the Newton Institute programme "Coupling Geometric PDEs with Physics for Cell Morphology, Motility and Pattern Formation". In this talk we discuss the mathematical issues associated with the formulation of PDEs in time dependent domains in both flat and curved space. Here we are thinking of problems posed on time dependent d-dimensional hypersurfaces Gamma(t) in R^{d+1}. The surface Gamma(t) may be the boundary of the bounded open bulk region Omega(t). In this setting we may also view Omega(t) as (d+1)-dimensional sub-manifold in R^{d+2}. Using this observation we may develop a theory applicable to both surface and bulk equations. We will present an abstract framework for treating the theory of well-posedness of solutions to abstract parabolic partial differential equations on evolving Hilbert spaces using generalised Bochner spaces. This theory is applicable to variational formulations of PDEs on evolving spatial domains including moving hyper-surfaces. Our setting is abstract and not restricted to evolving domains or surfaces. Then we show well-posedness to a certain class of parabolic PDEs under some assumptions on the parabolic operator and the data. Specifically, we study in turn a surface heat equation, an equation posed on a bulk domain, a novel coupled bulk-surface system and an equation with a dynamic boundary condition. We give some background to applications in cell biology. We describe how the theory may be used in the development and numerical analysis of evolving surface finite element spaces which unifies the discrtetisation methodology for evolving surface and bulk equations. We give some computational examples from cell biology involving the coupling of surface evolution to processes on the surface.

Arnold Reusken - Space-time trace FEM for PDEs on evolving surfaces

We present a particular class of finite element methods for the solution of partial differential equations on evolving surfaces. The evolving hypersurface in R^d defines a d-dimensional space-time manifold in the space-time continuum R^{d+1}. We derive and analyze a variational formulation for a class of diffusion problems on the space-time manifold. For this variational formulation new well-posedness and stability results are derived. The analysis is based on an inf-sup condition and involves some natural, but non-standard, (anisotropic) function spaces. Based on this formulation a discrete in time variational formulation is introduced that is very suitable as a starting point for a discontinuous Galerkin (DG) space-time finite element discretization. This FEM employs discontinuous piecewise linear in time -- continuous piecewise linear in space finite elements. Trial and test surface finite element spaces consist of traces of standard volumetric elements on the space-time manifold. This DG space-time method is explained and results of numerical experiments are presented that illustrate its properties. Results of a discretization error analysis are discussed.

Axel Voigt - Positionally ordered liquid crystals on curved manifolds

We discuss positional and orientational ordering on curved surfaces. While the first can be described by scalar valued surface PDEs, for which various numerical methods exist, the second requires vector valued surface PDEs. This leads to new mathematical issues: What is the correct surface Laplacian? How to discretize it efficiently? Are there additional extrinsic curvature effects? We introduce various numerical methods to solve vector valued surface PDEs, including parametric finite elements, diffuse interface approximations and discrete exterior calculus to study the interplay of order, geometry and topology.

Andreas Rätz - Diffuse-interface methods for coupled bulk--surface PDE's and an application to modelling of lipid raft formation in cell membranes

In this talk, we investigate a model for lipid raft formation and dynamics in biological membranes. The model describes the lipid composition of the membrane and an interaction with cholesterol. To account for cholesterol exchange between cytosol and cell membrane we couple a bulk-diffusion to a surface PDE on the membrane. The latter describes a relaxation dynamics for an energy taking lipid-phase separation and lipid-cholesterol interaction energy into account. It takes the form of an (extended) Cahn–Hilliard equation. For numerical simulations and investigation of the long-time behavior of the model we choose a diffuse-interface method for the coupled bulk--surface system.

Mats G. Larson - Cut Finite Element Methods for Convection-Diffusion Problems on Evolving Domains

We give an introduction to cut finite element (CutFEM) approximation of problems on evolving domains. CutFEM is based on representation of geometry and solutions on a fixed background mesh. The geometry is allowed to cut through the background mesh in arbitrary way resulting in so called cut elements. To deal with the cut elements certain stabilization terms are added to the variational form of the problem leading to stable formulations and stable linear systems of equations. We present basic theoretical results for both bulk and surface PDEs. Finally, we present several illustrating applications including convection-diffusion on evolving bulk-surface domains, fluid structure interaction, and CutFEM for curves embedded in R^3.

List of Participants

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For further Information and reimbursement

Information for all who get some reimbursement:

Please contact Carolin Gietz in some way, she has forms which have to be signed.

You will find her at Orléansring 10, 2nd floor, office 120.001 and during the coffee breaks.

Carolin Gietz
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