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Übungen zur Vorlesung

Partielle Differentialgleichungen 2

WiSe 19/20, Blatt 9

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Aufgabe 1

Let $U \subset \mathbb{R}^n$ be open, bounded with Lipschitz boundary. Assume that u is a smooth minimizer of the area integral

$$F(u) = \int_U (1 + |\nabla u|^2)^{1/2} \, \mathrm{d}x,$$

subject to given boundary conditions $w = g$ on ∂U and the constraint

$$J(u) = \int_U u \, \mathrm{d}x = 1.$$

Prove that the graph of u is a surface with constant mean curvature.

Bemerkung 1. Recall that the mean curvature of the graph of u is defined by

$$Hu = \frac{1}{n} \mathrm{div} \left(\frac{\nabla u}{(1 + |\nabla u|^2)^{1/2}} \right).$$

Aufgabe 2

Let $a : \mathbb{R}^n \to \mathbb{R}^n$ be monotone, i.e.,

$$(a(p) - a(q)) \cdot (p - q) \geq 0 \quad \forall p, q \in \mathbb{R}^n,$$

and satisfies linear growth conditions, i.e.,

$$|a(p)| \leq C(1 + |p|) \quad \forall p \in \mathbb{R}^n.$$

Let $U \subset \mathbb{R}^n$ be open, bounded, with Lipschitz boundary, and let $A : H_0^1(U) \to H_0^1(U)$ be defined by

$$\langle Au, \varphi \rangle = (Au)(\varphi) = \int_U a(\nabla u) \cdot \nabla \varphi \, \mathrm{d}x \quad \forall \varphi \in H_0^1(U).$$

Prove that

(a) A is monotone,

(b) A is hemicontinuous.

Bemerkung 2. Recall that

(a) $A : H_0^1(U) \to H_0^1(U)$ is monotone if

$$\langle Au - Av, u - v \rangle \geq 0 \quad \forall u, v \in H_0^1(U),$$

(b) $A : H_0^1(U) \to H_0^1(U)$ is hemicontinuous if

$$t \mapsto \langle A(u + tv), w \rangle$$

is continuous on $[0, 1]$ for all $u, v, w \in H_0^1(U)$.

Aufgabe 3

Let X be a reflexive Banach space and let $A : X \to X'$ be monotone, hemicontinuous, and bounded. Prove that A is demicontinuous.

Bemerkung 3. $A : X \to X'$ is called bounded if the set $\{Ax \colon \|x - y\| \leq R\} \subset X'$ is bounded for all $x \in X$ and $R > 0$. A is called demicontinuous if for all $u_k \to u$ in X, we have that $Au_k \rightharpoonup Au$ in X'.