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Übungen zur Vorlesung

Partielle Differentialgleichungen 2

WiSe 19/20, Blatt 6

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Aufgabe 1

Consider the functional $F : \mathcal{A} \to [0, +\infty]$ defined by

$$F(u) = \int_0^1 \left((1 - (u')^2)^2 + u^2\right) \mathrm{d}x,$$

where

$$\mathcal{A} := \{u \in W^{1,4}([0,1]) : \ u(0) = u(1) = 0\}.$$

(a) Prove that $\inf_{u \in \mathcal{A}} F(u) = 0$.

(b) Does there exist a minimizer $u \in \mathcal{A}$? Justify your statement.

Aufgabe 2

Consider the functional $F : \mathcal{A} \to [0, +\infty]$ defined by

$$F(u) = \int_0^1 (|u'| + |u|) \, \mathrm{d}x,$$

where

$$\mathcal{A} := \{u \in W^{1,1}([0,1]) : \ u(0) = 0, u(1) = 1\}.$$

(a) Prove that $\inf_{u \in \mathcal{A}} F(u) = 1$.

(b) Does there exist a minimizer $u \in \mathcal{A}$? Justify your statement.

Aufgabe 3

Let $1 < p < 4/3$. Consider

$$F(u) = \int_0^1 x^{\frac{1}{2}} |u'(x)|^2 \, \mathrm{d}x$$

Prove that the minimum problem

$$\inf\{F(u) : u \in W^{1,p}((0,1)), u(0) = 0, u(1) = 1\}$$

admits a solution. Proceed as follows.

(a) Prove that F is lower-semicontinuous with respect to the weak $W^{1,p}$-convergence.

(b) Show that F is coercive with respect to the weak $W^{1,p}$-topology.