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Übungen zur Vorlesung

Partielle Differentialgleichungen 2

WiSe 19/20, Blatt 11

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Aufgabe 1

Let H be a Hilbert space and let $I : H \to (-\infty, +\infty]$ be convex, proper and lower semicontinuous. For given $w \in H$ and $\lambda > 0$, we define the functional

$$J(u) := \frac{1}{2}\|u\|^2 + \lambda I(u) - (u, w), \quad u \in H.$$

Show that for each $u \in H$ there holds

$$\partial J(u) = u - w + \lambda \partial I(u).$$

Aufgabe 2

Let $K \subset \mathbb{R}^n$ be a closed, convex, non-empty set. Define

$$I(x) := \begin{cases} 0 & \text{if } x \in K, \\ +\infty & \text{if } x \notin K. \end{cases}$$

Explicitly determine $A = \partial I$, $J_\lambda = (I + \lambda A)^{-1}$, $A_\lambda = \frac{I - J_\lambda}{\lambda} (\lambda > 0)$ in terms of the geometry of K.

Aufgabe 3

Give a simple example showing that the flow

$$u' \in -\partial I(u) \quad (t > 0) \tag{1}$$

may be irreversible. (That is, find a Hilbert space H and a convex, proper, lower semicontinuous function $I : H \to (-\infty, +\infty]$ such that the semigroup solution of (1) satisfies

$$S(t)u = S(t)\hat{u}$$

for some $t > 0$ and $u \neq \hat{u}$.)