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Übungen zur Vorlesung

Partielle Differentialgleichungen 2

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Aufgabe 1

Let $U \subset \mathbb{R}^3$ be an open, bounded set. Let $X := \{u \in H_0^1(U) \colon \operatorname{div} u = 0\}$. Let $A : X \to X'$ be the operator defined by

$$\langle Au, \varphi \rangle := \int_U [\nabla u]u \cdot \varphi \, dx \quad \forall \, \varphi \in X,$$

where

$$([\nabla u]u)_i = \sum_{j=1}^{3} u_j \partial_j u_i = u \cdot \nabla u_i, \;\; i = 1, 2, 3.$$

Prove that A is bounded and strongly continuous.

Aufgabe 2

Consider the nonlinear boundary-value problem

$$\begin{cases} -\Delta u + b(Du) = f & \textit{in } U \\ u = 0 & \textit{on } \partial U. \end{cases}$$

Use Banach's fixed point theorem to show that there exists a unique weak solution $u \in H^2(U) \cap H_0^1(U)$ provided $b : \mathbb{R}^n \to \mathbb{R}$ is Lipschitz continuous, with $\operatorname{Lip}(b)$ small enough.