Statistical Filtering of Indirect Illumination for Computer Graphics

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Outline

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  - PCA
- Motion detection
  - Global
  - Local
- Denoising
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  - Hybrid Method
  - Regression
- Artifact detection
Background

Direct Illumination (D)  Global Illumination (G)
Indirect Illumination

Global Illumination = Direct + Indirect

=> G = D+I

Global Illumination (G)

Indirect Illumination (I)

Direct Illumination (D)
Photon Mapping

- Used for indirect illumination
- Method that Pixar wishes to use
- Algorithm is computationally expensive
- Lower sampling rates generates noisy images
Our Problem

- Remove noise generated by low sample photon mapping
- Detect and replace problematic pixels
- Re-render as few pixels as possible

=>
PCA: Basis

- PCA creates a variance-ordered basis
- Basis vectors point in direction of highest successive variance
- Noise-free pixels and noise are represented by different directions in the basis

Transformation of Basis using PCA
PCA: Choosing the Basis

- Noisy animations are represented using PCA:

\[ I(\vec{x}, t) = \sum_{i=1}^{N} w_i(t) B_i(\vec{x}) \]

- The first vectors of the basis capture the noise-free indirect illumination
- The last vectors of the basis describe the noise
- Noise-free \( I(x, t) \) is determined with a truncated PCA basis:

\[ I(\vec{x}, t) \approx I_k(\vec{x}, t) = \sum_{i=1}^{k} w_i(t) B_i(\vec{x}) \]
Grand Scheme

Noisy Animation

Motion Detection
- Global
- Local

Denoising
- Error Threshold
  - Hybrid
    - Artifact Detection
      - Re-render
  - Moving Basis
- Anisotropic Diffusion
Global vs. Local Motion

Original Animation

Global motion

Local motion
Global Motion Detection

\[ I(\bar{x}, t) = \sum_{i=1}^{F} w_i(t) B_i(\bar{x}) \]

- PCA separates the animation sequence \( I(\bar{x}, t) \) into spatial and temporal bases.
- The spatial basis vectors can be used for detecting global motion.

![Imagery](image_url)
Global Motion Detection

- The first basis vectors contain information of area of motion
- The last basis vectors contain only noise
- Adding up the absolute values of $B_1$ to $B_5$ results in

\[ \alpha(x) = \sum_{i=1}^{5} |B_i(x)| \]
Local Motion Detection using z-scores

\[ z(\bar{x}, t) = \frac{I(\bar{x}, t) - \mu(\bar{x})}{\sigma(\bar{x})} \]

Original animation

Threshold z-scores
Denoising using PCA

- It represents data with a variance ordered basis

\[ I(\bar{x}, t) = \sum_{i=1}^{F} w_i(t) B_i(\bar{x}) \]

- First basis vectors contain the noise-free indirect illumination

- Last basis vectors contain noise and motion

- Noise in animation sequence can be filtered using truncated temporal PCA basis
PCA: Motivation

- Most methods denoise individual frames
- Our approach uses temporal correlation of pixel values
- PCA finds a new basis which separates meaningful pixels from noisy ones
- PCA is fast and inexpensive
PCA: Basis

- PCA creates a variance-ordered basis.
- Basis vectors point in direction of highest successive variance.
- Noise-free pixels and noise are represented by different directions in the basis.

Transformation of Basis using PCA
PCA: Choosing the Basis

- Noisy animations are represented using PCA:
  \[ I(\vec{x}, t) = \sum_{i=1}^{N} w_i(t) B_i(\vec{x}) \]
  
  - \( I(\vec{x}, t) = \) Image sequence
  - \( N = \) Number of frames
  - \( B_i(x) = \) Basis Vectors
  - \( w_i(t) = \) Observation Coefficients

- The first vectors of the basis capture the noise-free indirect illumination
- The last vectors of the basis describe the noise

- Noise-free \( I(x, t) \) is determined with a truncated PCA basis:
  \[ I(\vec{x}, t) \approx I_k(\vec{x}, t) = \sum_{i=1}^{k} w_i(t) B_i(\vec{x}) \]
Image Sequence Reconstruction

Noisy animation sequence

Reconstruction with $k = 2$

Reconstruction with $k = 10$
Error Threshold Method*

- Compute the PCA basis
- Create reconstructions for each truncation $T_k$
  \[ I_k(\tilde{x}, t) = \sum_{i=1}^{k} w_i(t) B_i(\tilde{x}) \]
- Calculate the error (between the reconstruction and the noisy image)
  \[ error(k, \tilde{x}) = [I(\tilde{x}, t) - I_k(\tilde{x}, t)]^2 \]
- Calculate the difference of the error
  \[ \Delta(error(k, \tilde{x})) = error(k, \tilde{x}) - error(k + 1, \tilde{x}) \]

* MARK MEYER, JOHN ANDERSON, 2007, Statistical Acceleration for Animated Global Illumination
Error Threshold Method

Noisy Animation

Error Threshold Method

\[ I_{PCA}(\bar{x}, t) \]
Error Threshold Method

Noisy Animation

Error Threshold Method
Moving Basis

- Pick a single frame
- Select former and following $P$ frames and apply Gaussian weights
- Compute PCA for selected frames
- Pick out the middle frame of the reconstruction
- Move on to next frame and repeat procedure
Moving Basis Reconstruction

Noisy animation sequence

Moving Basis

\[ I_{MB}(\vec{x}, t) \]
Moving Basis

Noisy animation sequence

Reconstruction using Moving Basis
Single Frame Denoising*

- Each frame of the image sequence is denoised using anisotropic diffusion:

\[ u_t = \nabla \cdot \left( \frac{1}{\sqrt{1 + |\nabla u|^2}} \nabla u \right) \]

- Motion is preserved
- Little noise is removed

Denoising Methods

- Error Threshold
  - Noise free Animation
  - Time Consuming Artifacts on moving parts

- Moving Basis
  - Reduced noise Motion partially recovered
  - Remaining Noise and blur

- Single Frame Denoising
  - Motion recovered
  - Texture lost on stationary parts

Hybrid Reconstruction
Hybrid Method

- Combination of:
  - Denoising methods
  - Motion detection
- Formulation

Global Motion:

\[ \alpha(\bar{x}) = \begin{cases} 
0, \text{ motion} \\
0 < \sigma < 1, \text{ motion at some point} \\
1, \text{ stationary} 
\end{cases} \]

Local Motion:

\[ \beta(\bar{x}, t) = \begin{cases} 
0, \text{ currently no motion} \\
0 < \sigma < 1, \text{ motion at some time} \\
1, \text{ motion} 
\end{cases} \]

\[
I_{Hybrid}(\bar{x}, t) = I_{PCA}(\bar{x}, t)\alpha(\bar{x}) + I_{MB}(\bar{x}, t)(1 - \alpha(\bar{x}))\beta(\bar{x}, t) + I_{SF}(\bar{x}, t)(1 - \alpha(\bar{x}))(1 - \beta(\bar{x}, t))
\]
Hybrid Method

- Noisy Animation
- Moving Basis
- Error Threshold
- Anisotropic Diffusion
- Hybrid Method
Hybrid Method

Noisy animation → Anisotropic Diffusion → Moving Basis → First basis reconstruction → Hybrid Method
Artifact Detection using Regression

- Artifact – any pixel that looks bad 😞
- Regression methods used to detect pixels that have to be re-rendered
- Polynomial regression on all pixels over time
- Degree is determined by a threshold

\[ n = \arg \min_i \left( \frac{1}{F} \left| I(\tilde{x}, t) - R_{i,\tilde{x}}(t) \right| < \delta \right) \]
Artifact Detection Using Regression

Noisy – noisy regression

Pixel over time

- Noisy sequence
- Noisy sequence regression
- High-sample render
- High-sample render regression
Regression Results

Noise-free Animation

Polynomial Regression
Summary

- Detect motion using spatial PCA basis vectors and z-scores
- Denoise image sequence with hybrid method
- Detection artifacts using regression
- Re-render artifacts
Future work

- Artifacts detection
  - Improvement of regression
  - Difference between hybrid and regression denoising
- Motion detection
  - Localization of motion using VARIMAX
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