Optimisation 2

Homework 1

Exercise 1: Predator-Prey Relationship - Optimal Control

Consider two different bug species, where one of them (the predator - species 2) feeds on the other (the prey - species 1), hence the population densities depend on each other. Let

- y_i^1 be the number of individuals of species 1 in year i
- y_i^2 be the number of individuals of species 2 in year i

Additionally, we can influence the reproduction by providing an amount of food u_i in year i. Assume that both populations mate once a year. The number of individuals in year $i+1$ can be computed by the following equations:

$$y_{i+1}^1 = y_i^1 \exp[ru_i(1 - \tfrac{y_i^1}{K}) - y_i^2 \tfrac{A}{y_i^1+B}] \tag{1}$$

$$y_{i+1}^2 = y_i^2 \exp[l(1 - h\tfrac{y_i^2}{y_i^1})] \tag{2}$$

Here, we have

- r, l reproduction rates
- K population saturation for species 1
- h, A, B saturation constants

In the beginning (year 0), the number of individuals is given by $(y_0^1, y_0^2) = (1000, 100)$.

1. Explain the different terms in equations (1) and (2).
2. After N years, the ratio between both species should achieve a constant value α, i.e. $\frac{y_N^1}{y_N^2} \approx \alpha$. Set up a corresponding quadratic objective functional $J(y,u)$ including the desired final ratio of the species depending on the amount of provided food. Also include feeding costs with parameter λ.
3. Set up the optimal control problem including all the constraints on y and u.
4. For a given nonlinear problem $\min\limits_{u,y} \; J(y,u)$ s.t. $F(y,u) = 0$, derive the adjoint equation
$$D_yF(y,u)^T p = \partial_y J(y,u)$$
and the reduced gradient
$$\nabla f(u) = -\nabla_u F(y,u)p + \nabla_u J(y,u)$$

5. Compute the adjoint equation and the reduced gradient for the above problem.
6. Implement a reduced gradient flow in Matlab. You can make use of the Matlab ODE solvers.
7. Solve the problem numerically via the gradient flow. Use the following parameters: $r = l = 1$, $h = 10$, $A = 1$, $B = 100$, $K = 1000$, $N = 10$, $\alpha = \frac{1}{2}$.
8. Change the parameter λ and investigate how the final ratio $\frac{y_N^1}{y_N^2}$ changes.