

# Program of the Winterschool on Analysis and Applied Mathematics

22-26 February 2021

## Lecturers

- Andrea Braides: *Interfacial Energies on Networks*
- Lucia Scardia: *Multiscale Problems in Dislocation Theory*
- Ulisse Stefanelli: *De Giorgi's Energy-Dissipation Principle*

## Schedule

|             | Monday       | Tuesday      | Wednesday    | Thursday   | Friday     |
|-------------|--------------|--------------|--------------|------------|------------|
| 13:30-13:45 | Opening      |              |              |            |            |
| 13:45-14:45 | Braides      | Stefanelli   | Scardia      | Stefanelli | Braides    |
| 14:45-15:45 | Stefanelli   | Scardia      | Braides      | Braides    | Scardia    |
| 15:45-17:15 | Poster/Break | Poster/Break | Poster/Break | Discussion | Stefanelli |
| 17:15-18:15 | Scardia      | Braides      | Stefanelli   | Scardia    |            |

## Poster Session

### Monday: 22nd, February 2021

- Lukas Abel: *Epitaxy and Dislocations - On Variational Models and their Analysis*
- Omar Boussaid: *On Symmetric Polyconvexity*
- Katharina Brazda: *Modeling multiphase biomembranes with varifolds*
- Marco Bresciani: *Linearized von Kármán theory for magnetoelastic plates*
- Leon Bungert & Tim Roith: *Continuum Limit of Lipschitz Learning on Graphs*
- Stefano Ceci: *On vortex dynamics for the 2D Euler equation*
- Nicolas Clozeau: *Quantitative homogenization theory for random monotone operators*
- Matteo Fogato: *Modal analysis of some nonlinear beam equations*
- Dennis Gallenmüller: *On the Selection of Measure-Valued Solutions for the Isentropic Euler System*
- Wojciech Górny: *1-Laplacian on metric random walk spaces*
- Max Griehl: *Deriving a bending plate model for nematic liquid-crystal elastomers via Gamma-convergence*

**Tuesday: 23rd, February 2021**

- Anastasiia Hraivoronska: *Asymptotic limits of random walks via generalized gradient flows*
- Erica Ipocoana: *Non-isothermal Cahn-Hilliard models and tumor growth*
- Sascha Knüttel: *Gradientfree approximation of the Willmore energy*
- Lukas Koch: *The Jacobian equation with data in  $L^p$*
- Joshua Kortum: *Weak compactness of two-dimensional liquid crystal and magnetic flows*
- Melanie Koser: *Asymptotic self-similarity in a model of shape-memory alloys*
- Michał Łasica: *Existence of the 1-harmonic map flow*
- Lorenzo Liverani: *On the Moore-Gibson-Thompson equation – with memory with nonconvex kernels*
- Roberta Marziani: *Asymptotic analysis of singularly perturbed elliptic functionals*
- Jiri Minarcik: *Minimal surface generating flow for space curves of non-vanishing torsion*
- Antonella Nastasi: *Regularity and existence of solutions for a Neumann  $p$ -Laplacian problem in the metric setting*
- Victor Navarro Fernández: *Optimal stability estimates for advection-diffusion equations*

**Wednesday: 24th, February 2021**

- Lukas Niebel: *Kinetic maximal  $L^p$ -regularity with temporal weights and application to quasilinear kinetic diffusion equations*
- Fumihiko Onoue: *(Dis)connectedness of nonlocal minimal surfaces*
- Valerio Pagliari: *Constant rank differential operators and homogenization*
- Clara Patriarca: *Existence and uniqueness result for a fluid-structure-interaction evolution problem in an unbounded 2D channel*
- Bhagwat Ram: *A  $q$ -Sequential Quadratic Programming Method for Multi-objective Optimization Problems*
- Sanjeev Singh: *Approximate-Karush-Kuhn-Tucker Conditions and Interval Valued Vector Variational Inequalities*
- Dominik Stantejsky: *The Saturn ring effect in nematic liquid crystals with external field: effective energy and hysteresis*
- Andrea Torricelli: *Limits of Non-Local Anisotropic Perimeters*
- Jacopo Ulivelli: *Generalization of Klains Theorem to Minkowski Symmetrization of compact sets and related topics*
- Dominik Winkler: *A well-posedness result for a system of cross-diffusion equations*
- Sylvain Wolf: *Homogenization of the Poisson equation and Stokes system in a non periodically perforated domain*
- Konstantinos Zemas: *Rigidity estimates for isometric and conformal maps on the sphere*

# Abstracts

Monday, 22nd February 2021

Lukas Abel

## Epitaxy and Dislocations - On Variational Models and their Analysis

**Abstract:** Epitaxy is a special form of crystal growth. The constituents of a film mimic the structure of the substrate on which the film grows on. The principle of epitaxy is of great importance in modern technology, especially in the making of semi-conducting devices.

If there is a lattice misfit between the film and the substrate dislocations may appear. Dislocations are topological defects of the crystallographic lattice that can act as a stress relief mechanism.

Based on the results of [1], we compare variational models for epitaxially strained films based on linear elasticity, which account for the presence of dislocations. This is based on joint work with J. Ginster and B. Zwicknagl.

## References

- [1] Fonseca, I., Fusco, N., Leoni, G., and Morini, M. A model for dislocations in epitaxially strained elastic films. *Journal de Mathématiques Pures et Appliquées* 111 (05 2016).

Omar Boussaid

## On Symmetric Polyconvexity

**Abstract:** Symmetric quasiconvexity plays a key role for energy minimization in geometrically linear elasticity theory. Due to the complexity of this notion, a common approach is to retreat to necessary and sufficient conditions that are easier to handle. This poster focuses on symmetric polyconvexity, which is a sufficient condition. We prove a new characterization of symmetric polyconvex functions in the two- and three-dimensional setting, and use it to investigate relevant subclasses like symmetric polyaffine functions and symmetric polyconvex quadratic forms. In particular, we provide an example of a symmetric rank-one convex quadratic form in  $3d$  that is not symmetric polyconvex. The construction takes the famous work by Serre from 1983 on the classical situation without symmetry as inspiration. Beyond their theoretical interest, these findings may turn out useful for computational relaxation and homogenization.

Katharina Brazda

## Modeling multiphase biomembranes with varifolds

**Abstract:** Biological membranes are thin layers of lipid molecules and can be described as fluid surfaces with bending rigidity. A variational model for their equilibrium configurations consists in minimizing the Canham-Helfrich energy under area and volume constraints. We consider multiphase membranes with sharp phase interfaces and present an existence result obtained in the geometric measure theory framework of oriented curvature varifolds. This is joint work with Luca Lussardi and Ulisse Stefanelli.

Marco Bresciani

### Linearized von Kármán theory for magnetoelastic plates

**Abstract:** We study the problem of dimension reduction for magnetoelastic plates in the linearized von Kármán regime. The model features a mixed Eulerian-Lagrangian formulation, as magnetizations are defined on the deformed configuration. We do not include any higher-order term in the energy, but we impose a lower bound on the scaling.

Leon Bungert & Tim Roith

### Continuum Limit of Lipschitz Learning on Graphs

**Abstract:** In semi-supervised learning one is confronted with a large set of data points, only very few of which are labelled. The task is to find a labelling function which extends these labels to the whole data set. In order to find useful labelling functions, in graph-based semi-supervised learning one represents the data set as weighted graph and poses a smoothness constraint on the labelling function. In this context  $p$ -Laplacian learning has become very popular and consists in finding a  $p$ -harmonic function which coincides with labels on the labelled set. However, this method is asymptotically ill-posed if  $p$  is smaller than the dimension of the data space, and is not feasible for most applications. In this poster, we will therefore present Lipschitz-learning which aims to find a Lipschitz-extension of the labels and is well-posed in arbitrary dimension. Our main result is a discrete-to-continuum limit of Lipschitz-extensions as the data set grows to a continuum. Our theory uses Gamma-convergence and Hausdorff convergence of the data set. As a by-product we obtain a continuum limit for a nonlinear eigenvalue problem related to geodesic distance functions.

Stefano Ceci

### On vortex dynamics for the 2D Euler equation .

**Abstract:** We investigate the motion of vortices in an incompressible, inviscid fluid moving in a two-dimensional domain according to the Euler equation in vorticity form. The vortices move as a result of the interactions with each other and with the boundary of the domain. It is well-known that this motion approximates the so-called Kirchhoff-Routh system, in which vortices are represented by points. We extend previous results to the case of a possibly unbounded vorticity, which is required to be in  $L^p$ , for some  $p > 2$ , and investigate how well the solution to Euler approximates the Kirchhoff-Routh system. More precisely, we derive an estimate for the 2-Wasserstein distance between the vorticity and the measure associated to the point-vortex system. This is joint work with Christian Seis.

Nicolas Clozeau

### Quantitative homogenization theory for random monotone operators.

**Abstract:** We consider a model problem of diffusion or conductivity in a random medium (for instance, a heterogeneous domain obtained by mixing randomly two different phases, one being the matrix and the other the inclusions). The model that we use takes the form of a quasi-linear elliptic equation with high oscillatory (at scale  $S \ll 1$ ) and random nonlinear monotone operator. We are interested with the asymptotic behaviour of the solution when  $S$  tends to zero.

The qualitative theory is by now understood, since the works of Dal Maso, Braides, Defranceschi, Jikov, Kozlov, Oleĭnik, and Pankov. It is well known that the solution, provided the law of the monotone operator is stationary and ergodic, can be approximated by its two-scale expansion: that is a first-order expansion in  $S$  taking account of the oscillation at scale  $S$ . This two-scale expansion involves the so-called first-order corrector which allows to reconstruct the oscillation of the gradient.

This work aims to turn this theory into a quantitative theory. We show the optimal growth the correctors and linearized correctors for stretched exponential moments and that the error in the two-scale expansion is optimally of order  $O(S)$  for  $d = 3$  also with stretched exponential moments. This is joint work with Antoine Gloria.

Matteo Fogato

### Modal analysis of some nonlinear beam equations

**Abstract:** We consider the

$$u_{tt} + \delta u_t + A^2 u + \|A^{\theta/2} u\|^2 A^\theta u = g$$

where  $A^2$  is a diagonal, self-adjoint and positive-definite operator and  $\theta \in [0, 1]$  and we study some finite-dimensional approximations of the problem. First, we analyze the dynamics in the case when the forcing term  $g$  is a combination of a finite number of modes. Next, we estimate the error we commit by neglecting the modes larger than a given  $N$ . We then prove, for a particular class of forcing terms, a theoretical result allowing to study the distribution of the energy among the modes and, with this background, we refine the results. Some generalizations and applications to the study of the stability of suspension bridges are given.

Dennis Gallenmüller

### On the Selection of Measure-Valued Solutions for the Isentropic Euler System

**Abstract:** We show that there exist infinitely many generalized measure-valued solutions to the two-dimensional isentropic Euler system with quadratic pressure law, which behave deterministically up to a certain time and, although they are energy admissible, can be discarded as unphysical, as they do not arise as vanishing viscosity limits. In fact, these measure-valued solutions also do not arise from a sequence of weak solutions of the Euler equations, in contrast to the incompressible case. The proof uses an  $\mathcal{A}$ -free rigidity argument for non-constant states and combines this with a compression wave solution evolving into infinitely many weak solutions, constructed by Chiodaroli, De Lellis, and Kreml.

Wojciech Górny (joint work with J.M. Mazn)

### 1-Laplacian on metric random walk spaces

**Abstract:** The nonlocal 1-Laplacian operator is closely related to the median value property. We study this operator in a quite general framework of metric random walk spaces, which include as special cases locally finite graphs and Euclidean spaces or Carnot groups with nonlocality given by a radial kernel. We provide some geometric conditions on the space and the random walk under which the problem admits a solution and study its properties.

Max Griebel

### Deriving a bending plate model for nematic liquid-crystal elastomers via Gamma-convergence

**Abstract:** Liquid-crystal elastomers (LCEs) are a class of materials, whose shape can be controlled via external stimulation. Here, we see a three-dimensional model describing the deformations. Its terms include the elastomer's hyperelastic energy (coupled to the liquid-crystal structure) and the liquid-crystal's Oseen-Frank energy. Using Gamma-convergence, we derive a dimension-reduced model, effectively describing the bending behaviour for thin LCE-plates.

**Tuesday, 23 February 2021**

Anastasiia Hraivoronska

**Asymptotic limits of random walks via generalized gradient flows**

**Abstract:** We consider the law of continuous-time random walks on graphs as an approximation for solutions of diffusion equations. The motivation comes from developing structure-preserving numerical schemes. Our approach is based on exploiting the gradient structure of both evolution types and passing to the discrete-to-continuous limit in the corresponding variational formulation. We postulate that the limit may depend wildly on the graph geometry and can be surprisingly nontrivial.

Erica Ipocoana

**Non-isothermal Cahn-Hilliard models and tumor growth**

**Abstract:** We first want to establish new regularity results for a non-isothermal Cahn-Hilliard system in the two dimensional setting. In particular the model consists of a non-linear PDE system corresponding to the Cahn-Hilliard system for phase separation coupled with the internal energy balance, which describes the evolution of temperature. The main source of difficulty is directly related to the thermodynamic consistency of the model. Our model system is part of a more general model [1], taking into account also the Navier-Stokes equation. On the other hand, the results on uniqueness of the solution were proved with great effort. The further regularity that we gain in this work allows us to show a simplest uniqueness proof. Moreover, since here we assume null velocity vector field, we infer a crucial estimate for the temperature, obtained by a suitable Moser iteration scheme. The said model, studied in [3], can be considered as a starting point to describe the growth of a tumor surrounded by healthy tissues in the presence of a nutrient [2].

## References

- [1] M. Eleuteri, E. Rocca and G. Schimperna, Existence of solutions to a two-dimensional model for nonisothermal two-phase flows of incompressible fluids, *Ann. Inst. H. Poincaré Anal. Non Linéaire* **33** (2016), 14311454.
- [2] E. Ipocoana, A non-isothermal Cahn-Hilliard model for tumor growth, (2021) in preparation.
- [3] E. Ipocoana and A. Zafferi, Further regularity and uniqueness results for a non-isothermal Cahn-Hilliard equation, *Comm. Pure Appl. Anal.*, DOI: 10.3934/cpaa.2020289 (2020).

Sascha Knüttel

**Gradientfree approximation of the Willmore energy**

**Abstract:** On the poster I present a new phase field approximation of the Willmore energy. I start from a diffuse perimeter approximation considered by Amstutz-van Goethem and motivate from this an approximation of the Willmore energy. I show a  $\Gamma$ -lim sup estimate for the approximation and justify by a formal asymptotic expansion that a corresponding  $L^2$ -Gradient Flow converges to the Willmore Flow. Joint work with M. Röger, TU Dortmund.

Lukas Koch

**The Jacobian equation with data in  $L^p$**

**Abstract:** I present a number of results concerning the Jacobian equation  $\det Du = f$  with  $L^p$ -data. In particular, generically no solutions in  $W^{1,2p}$  exist and for spherically symmetric  $L^2$ -data minimisers of the Dirichlet energy need not be spherically symmetric. The results are based on joint work with A. Guerra (Oxford) and S. Lindberg (Aalto).

Joshua Kortum

**Weak compactness of two-dimensional liquid crystal and magnetic flows**

**Abstract:** We investigate the existence and weak stability of global weak solutions for certain complex fluid models. These comprise a suitably adapted version of the Navier-Stokes equations and a harmonic map heat flow-like equation. The Ericksen-Leslie model for liquid crystals is the simplest non-trivial system of such a type. A second, more general example represents the flow of ferromagnetic thin films and involves a variant of the Landau-Lifshitz-Gilbert equation. The construction of weak solutions relies on variants of the Ginzburg-Landau approximation where the main problem consists of the limit passage in the Navier-Stokes equation. This issue is handled by invoking partial regularity techniques and the method of concentration-cancellation originally introduced for the incompressible Euler equations.

Melanie Koser

**Asymptotic self-similarity in a model of shape-memory alloys**

(based on a joint work with Sergio Conti, Johannes Diermeier and Barbara Zwicknagl)

**Abstract:** We study energy-driven pattern formation in shape memory alloys. No matter how shape memory alloys are deformed at a low temperature, they return to their original shape by heating them above a critical temperature. The cause of this phenomenon is a first order solid-to-solid phase transformation. Microstructures close to phase boundaries of martensitic nuclei can be modeled variationally. We consider a model by Kohn and Müller (1992 & '94), and prove asymptotic self-similarity of minimizers. This generalizes results by Conti (2000) to various physically relevant boundary conditions, more general domains, and arbitrary volume fractions, including low-hysteresis shape memory alloys. The proof relies on pointwise estimates and local energy scaling laws of a minimizer.

Michał Łasica

**Existence of the 1-harmonic map flow**

**Abstract:** Similarly as in the real-valued case, the total variation of maps taking values in a Riemannian manifold extends to a lower semicontinuous functional on  $L^2$ . However, in general this functional is not geodesically semiconvex, so the existence of its gradient flow is not provided by general variational theory. Alternatively, one can try to apply the theory of parabolic PDE systems, mimicking the approach used for  $p$ -harmonic map flows,  $p > 1$ . This poses some difficulties, because the PDE system corresponding to the flow is strongly nonlinear, singular and degenerate. However, in some cases, this approach was successful. I will present known results on the existence of the flow, focusing on my work with Lorenzo Giacomelli and Salvador Moll.

Lorenzo Liverani  
**On the Moore-Gibson-Thompson equation  
with memory with nonconvex kernels**

**Abstract:** We consider the MGT equation with memory

$$\partial_{ttt}u + \alpha\partial_{tt}u - \beta\Delta\partial_tu - \gamma\Delta u + \int_0^t g(s)\Delta u(t-s)ds = 0.$$

We prove an existence and uniqueness result removing the convexity assumption on the convolution kernel  $g$ , usually adopted in the literature. In the subcritical case  $\alpha\beta > \gamma$ , we establish the exponential decay of the energy, without leaning on the classical differential inequality involving  $g$  and its derivative  $g'$ , namely,

$$g' + \delta g \leq 0, \quad \delta > 0,$$

but only asking that  $g$  vanishes exponentially fast. This is a joint work with my PhD supervisor prof. Vittorino Pata and prof. Monica Conti.

Roberta Marziani  
**Asymptotic analysis of singularly perturbed elliptic functionals**

**Abstract:** We study the  $\Gamma$ -convergence of general singularly perturbed elliptic functionals of the form

$$\mathcal{F}_\varepsilon(u, v, A) = \int_A \psi(v)f_\varepsilon(x, \nabla u)dx + \frac{1}{\varepsilon} \int_A g_\varepsilon(x, v, \varepsilon\nabla v)dx \quad (0.1)$$

where  $A \subset \mathbb{R}^m$  is open and bounded,  $(u, v) \in W^{1,p}(A; \mathbb{R}^m) \times W^{1,p}(A)$  with  $0 \leq v \leq 1$ . The function  $\psi: [0, 1] \rightarrow [0, 1]$  is increasing, continuous and such that  $\psi(0) = 0$ ,  $\psi(1) = 1$ , and  $\psi(s) > 0$  for  $s > 0$ ; whereas, for every  $\varepsilon > 0$ , the integrands  $f_\varepsilon: \mathbb{R}^n \times \mathbb{R}^{m \times n} \rightarrow [0, +\infty)$  and  $g_\varepsilon: \mathbb{R}^n \times [0, 1] \times \mathbb{R}^n \rightarrow [0, +\infty)$  belong to suitable classes of functions denoted, respectively, by  $\mathcal{F}$  and  $\mathcal{G}$ . The choice of  $\mathcal{F}$  and  $\mathcal{G}$  is such that the functional (0.1) is generalisation of functionals of Ambrosio-Tortorelli type; i.e.

$$AT_\varepsilon(u, v) = \int_A \psi(v) |\nabla u|^p dx + \int_A \left( \frac{(1-v)^p}{\varepsilon} + \varepsilon^{p-1} |\nabla v|^p \right) dx. \quad (0.2)$$

It is well known that the latter  $\Gamma$ -converges to the free discontinuity functional

$$\int_A |\nabla u|^p dx + c_p \mathcal{H}^{n-1}(S_u). \quad (0.3)$$

where now the variable  $u$  belongs to the space of generalised special functions of bounded variation  $GSBV^p(A; \mathbb{R}^m)$ . Accordingly to this, the asymptotic analysis of  $\mathcal{F}_\varepsilon$  gives rise to a decoupling volume-surface effect, more precisely we show that (0.1)  $\Gamma$ -converges to the functional

$$\mathcal{F}(u, 1, A) = \int_A f_\infty(x, \nabla u)dx + \int_{S_u \cap A} g_\infty(x, \nu) d\mathcal{H}^{n-1} \quad (0.4)$$

where  $f_\infty$  is characterised as double limit of scaled minimisation problems involving only the bulk energy (i.e. the first term on the right hand-side of (0.1)) while  $g_\infty$  is obtained by the double limit of scaled minimisation problems only involving the surface energy (i.e. the second term on the right hand-side of (0.1)) The characterisation result for  $g_\infty$  implies, in particular, that the surface term in (0.4) does not depend on the jump-opening  $[u]$ . This result was obtained in collaboration with Annika Bach and Caterina Ida Zeppieri.



Jiri Minarcik

**Minimal surface generating flow for space curves of non-vanishing torsion**

**Abstract:** This contribution introduces geometric flow of space curves during which the curve traces out a zero mean curvature surface. Results concerning the long term behaviour of this motion are presented along with an estimate of the generated area and the terminal time. The flow is illustrated on a simple analytical example involving the helix curve.

Antonella Nastasi

**Regularity and existence of solutions for a Neumann  $p$ -Laplacian problem in the metric setting.**

**Abstract:** We use a variational approach to study existence and regularity of solutions for a Neumann  $p$ -Laplacian problem with a reaction term on metric spaces equipped with a doubling measure and supporting a Poincaré inequality. Trace theorems for functions with bounded variation are applied in the definition of the variational functional and minimizers are shown to satisfy De Giorgi type conditions.

Victor Navarro Fernández

**Optimal stability estimates for advection-diffusion equations.**

**Abstract:** The advection-diffusion equation is of remarkable importance on many different physical contexts. It describes the evolution in time and space of a scalar quantity when it is subjected to an advection field and when diffusion plays a role due to the molecular motion of the involved particles. On this poster we focus on advection-diffusion equations with rather rough velocity fields. First, we derive optimal stability estimates for advection-diffusion equations in a setting in which the velocity field is Sobolev regular in the spatial variable. This estimate is formulated with the help of Kantorovich-Rubinstein distances with logarithmic cost functions. Second, we extend the stability estimates to advection-diffusion equations with velocity fields whose gradients are singular integrals of  $L^1$  functions, entailing a new well-posedness result.

Wednesday, 24th February 2021

Lukas Niebel

**Kinetic maximal  $L^p$ -regularity with temporal weights and application to quasilinear kinetic diffusion equations**

**Abstract:** The poster gives a short overview over two recent preprints by Rico Zacher and Lukas Niebel [1, 2]. In these preprints we introduce the concept of kinetic maximal  $L^p$ -regularity with temporal weights and prove that this property is satisfied for the (fractional) Kolmogorov equation. The initial value problem is characterized in terms of anisotropic Besov spaces and continuity of solutions with values in the trace space is shown. These results are then applied to obtain local existence of solutions to a class of quasilinear kinetic equations.

## References

- [1] L. Niebel and R. Zacher, *Kinetic maximal  $L^2$ -regularity for the (fractional) Kolmogorov equation*, ArXiv e-prints arXiv:2006.11531 (2020)
- [2] L. Niebel and R. Zacher, *Kinetic maximal  $L^p$ -regularity with temporal weights and application to quasilinear kinetic diffusion equations*, ArXiv e-prints arXiv:2006.11531 (2020)

Fumihiko Onoue

**(Dis)connectedness of nonlocal minimal surfaces**

**Abstract:** We investigate the shape of nonlocal minimal surfaces in a cylinder with a prescribed datum given by the complement of slabs. We show the disconnectedness and connectedness of nonlocal minimal surfaces when the width of the slabs is sufficiently large and small, respectively. Moreover, we reveal the property that nonlocal minimal surfaces in the case that the width is small coincide with the cylinder itself. This is joint work with Serena Dipierro and Enrico Valdinoci from The University of Western Australia.

Valerio Pagliari

**Constant rank differential operators and homogenization**

**Abstract:** We present some recent results about the existence of potentials and extensions for maps lying in the kernel of linear,  $k$ -th order, homogeneous differential operators with constant coefficients and constant rank. We also suggest possible applications to homogenization problems.

Clara Patriarca

**Existence and uniqueness result for a fluid-structure-interaction evolution problem in an unbounded 2D channel**

**Abstract:** In an unbounded 2D channel, we consider the vertical displacement of a rectangular obstacle in a regime of small flux for the incoming flow field, modelling the interaction between the cross-section of the deck of a suspension bridge and the wind (see Figure 1). I will briefly illustrate the strategy to prove an existence and uniqueness result for a fluid-structure-interaction evolution problem

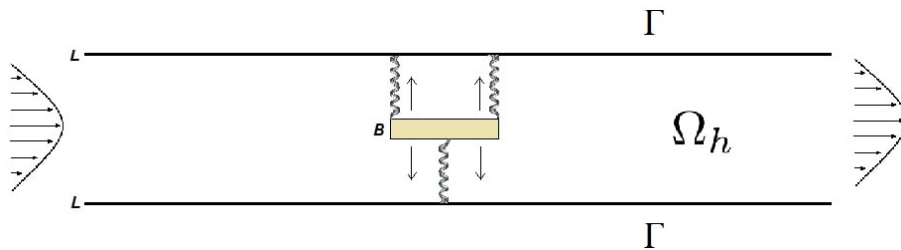


Figure 1: The channel with the vertically moving obstacle

set in this channel, where at infinity the velocity field of the fluid has a *Poiseuille flow* profile. We introduce a suitable definition of weak solutions and we make use of a penalty method. In order to prevent collisions of the obstacle with the boundary of the channel, we introduce a *strong force* in the differential equation governing the motion of the rigid body and we find a unique global-in-time solution.

## References

- [1] C. Patriarca. Existence and uniqueness result for a fluid-structure-interaction evolution problem in an unbounded 2D channel, preprint. 2021
- [2] C. Conca, H.J. San Martin, and M. Tucsnak. Existence of solutions for the equations modelling the motion of a rigid body in a viscous fluid. *Communications in Partial Differential Equations*, 25, 2000.
- [3] O. Glass and F. Sueur. Uniqueness results for weak solutions of two-dimensional fluid-solid systems. *Arch. Rational Mech. Anal.*, 218, 2015.
- [4] H. Fujita and N. Sauer. On existence of weak solutions of the Navier-Stokes equations in regions with moving boundaries. *Journal of the Faculty of Science. Section I A*, 17, 1970.

Bhagwat Ram

### A q-Sequential Quadratic Programming Method for Multi-objective Optimization Problems

By

S. K. Mishra<sup>1</sup>, M.E. Samie<sup>2</sup>, B. Ram<sup>3</sup>

<sup>1</sup>Department of Mathematics, Institute of Science, Banaras Hindu University, Varanasi, India;

<sup>2</sup>Department of Mathematics, Bu-Ali Sina University, Hamedan, Iran,

<sup>3</sup>DST-Centre for Interdisciplinary Mathematical Sciences, Institute of Science, Banaras Hindu University, Varanasi, 221005, India

**Abstract:** In this poster, a globally convergent q-sequential quadratic programming (q-SQP) method is developed for multi-objective optimization problems with inequality type constraints. A feasible q-descent direction is obtained using a linear approximation of all objective functions and constraint functions. The sub-problem of the sequence generates the feasible solution. The method is compared

with the existing method on a suitable set of test problems and shown that proposed method is more efficient.

Sanjeev Singh

### **Approximate-Karush-Kuhn-Tucker Conditions and Interval Valued Vector Variational Inequalities**

**Abstract:** This Article deals with the Approximate Karush-Kuhn-Tucker (AKKT) optimality conditions for interval valued multiobjective function as a generalization of Karush-Kuhn-Tucker optimality conditions. Further, we establish relationship between vector variational inequality problems and multiobjective interval valued optimization problems under the assumption of LU□convex smooth and nonsmooth objective functions.

Dominik Stantejsky

### **The Saturn ring effect in nematic liquid crystals with external field: effective energy and hysteresis**

**Abstract:**We consider the Landau-de Gennes model for liquid crystals with an external magnetic field to model the occurrence of the Saturn ring effect under the assumption of rotational equivariance. After a rescaling of the energy, a variational limit is derived. Our analysis relies on precise estimates around the singularities and the study of a radial auxiliary problem in regions, where a continuous director field exists. Studying the limit problem, we explain the transition between the dipole and Saturn ring configuration and the occurrence of a hysteresis phenomenon, giving a rigorous explanation of what was derived and simulated previously by [H. Stark, Eur. Phys. J. B 10, 311321 (1999)].

Andrea Torricelli

### **Limits of Non-Local Anisotropic Perimeters**

**Abstract:** We compute the pointwise limit of a family of functionals which penalize a non-local and anisotropic interaction between a finite perimeter set and its complement. We completely characterize the limit which turns out to be an anisotropic perimeter and we discuss some particular cases.

Jacopo Ulivelli

### **Generalization of Klains Theorem to Minkowski Symmetrization of compact sets and related topics**

**Abstract:** Symmetrizations and rearrangements serve as very powerful tools in geometry and analysis, and in such tools lie the motivations and interest of this work. We first focus on generalizing to compact sets of  $\mathbb{R}^n$  a convergence result for sequences of symmetrizations. This result, originally due to Klain, was later extended to a more general frame by Bianchi, Gardner and Grochi. Such convergence follows mainly from the structural properties of Minkowski Symmetrization and the classic result from Shapley, Folkman and Starr. We use similar techniques to investigate the behaviour of the Minkowski sum of a specific class of compact sets and its idempotence. Amongst the applications we find the extension of the validity of an estimate method in PDE developed by Salani as well as an approximation result for the ball through means of isometries from Klartag.

Dominik Winkler

### A well-posedness result for a system of cross-diffusion equations

**Abstract:** We investigate the well-posedness of certain cross-diffusion equations in the class of bounded functions. More precisely, we show existence, uniqueness and stability of bounded weak solutions under the assumption that the system has a dominant linear diffusion. As an application, we provide a new well-posedness theory for a cross-diffusion system that originates from a hopping model with size exclusions. Our approach is based on a fixed point argument in a function space that is induced by suitable Carleson-type measures.

Sylvain Wolf

### Homogenization of the Poisson equation and Stokes system in a non periodically perforated domain

**Abstract:** We consider the homogenization of the Poisson Equation and the Stokes system in non periodically perforated domains. In this setting, the size of the perforations is proportional to the distance between neighbouring cells and scales like  $\varepsilon \ll 1$ . The behaviour of the solutions of these PDEs is well known when the holes are periodically distributed in the macroscopic domain. We propose to generalize these results when the perforations are locally non periodic but tend to be periodic far from the origin. We study classical objects of the homogenization theory such as two-scale expansions, existence of correctors and we derive convergence rates of the exact solutions towards their homogenized approximations. In this poster session, we will emphasize on the geometric properties that we impose on the non-periodic perforations and we will summarize the results obtained in [1, 2].

## References

- [1] Xavier Blanc and Sylvain Wolf. Homogenization of the Poisson equation in a non-periodically perforated domain. *Asymptotic Analysis*, pages 1–27, 2021.
- [2] Sylvain Wolf. Homogenization of the Stokes system in a non periodically perforated domain. *Submitted*, 2021.

Konstantinos Zemas

### Rigidity estimates for isometric and conformal maps on the sphere

**Abstract:** This poster is meant to present results obtained in collaboration with S. Luckhaus on both linear and nonlinear stability aspects of the class of rigid motions (resp. Möbius transformations) of the standard round sphere among maps from the sphere into the ambient Euclidean space.

Unlike similar in flavour results for maps defined on domains, not only an isometric (resp. conformal) deficit is necessary in this more flexible setting, but also a deficit measuring the distortion of the sphere under the maps in consideration. The latter is defined as an associated isoperimetric type of deficit.

The focus is mostly on the case when the ambient dimension is 3 and we also explain why, in both cases, the estimates are optimal in their corresponding settings. The adaptations needed in higher dimensions will also be addressed. We also obtain linear stability estimates for both cases in all dimensions. These can be regarded as Korn-type inequalities for the combination of the quadratic form associated with the isometric (resp. conformal) deficit on the sphere and the isoperimetric one.