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Recent advances in the Calculus of Variations

September 16-19, 2019
Münster

Organizers:

Manuel Friedrich

Caterina Ida Zeppieri

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General information

Venue. *Registration*: Second floor of the SRZ (Orléans-Ring 12, cf. Map A) on Monday. *Lecture room*: SRZ 217 (second floor). You can find all the information on the homepage of the conference.

Wi-Fi access. If you are part of the eduroam community, you may connect to the network "eduroam" as usual. Otherwise you can connect to the SSID "GuestOnCampus" and start any web browser. You will automatically be redirected to the login page. Confirm the terms of use and click on "log in for free". 1 GB data volume is available per device and day. Please note that the connection is not encrypted.

Lunch/Coffee break/Restaurants. We provide you with vouchers for lunch at

- Canteen - Mensa am Ring, Domagkstraße 61

In the lounge (2. floor of SRZ) you can get coffee and snacks during the coffee breaks.
There are a view restaurants close to the conference venue:

- Ristorante Milano (Italian), Wilhelmstraße 26 (closed on Monday)
- Il Gondoliere (Italian), Von-Esmarch-Straße 28 (closed on Monday)
- Buddha Palace (Indian), Von-Esmarch-Straße 18
- La Gondola D'oro (Italian), Hüfferstraße 34
- A2 am See (German), Annette-Allee 3
- Bakenhof (Hotel + Restaurant), Roxeler Str. 376
- Royals & Rice (Vietnamese), Frauenstraße 51

City tour. There will be a city tour on Tuesday at 6.30pm starting at St.-Paulus-Dom (cf. Map B). In case, you have some spare time on your own, here are some suggestions: Go see the castle, its garden and/or the included botanic garden. Visit a museum, e.g. the LWL Museum of Art and Cultural History or the Picasso-Museum. Have a walk around the Aasee lake or make yourself familiar with European history at the Historical City Hall where the peace of the Thirty Years' War was negotiated.

Conference dinner. The conference dinner takes place on Wednesday at 7.00pm at the Ratskeller (cf. Map C).

Public transportation. You can check the bus schedule on the website of Stadtwerke-Münster (in German and English), or use Google maps.

Questions. In case of further questions, please ask Manuel Friedrich:
Email: manuel.friedrich@uni-muenster.de

Acknowledgements

The conference is supported by the
Cluster of Excellence "Mathematics Münster" (MM).

Monday, September 16

08:30-09:00	*Registration*
09:00-09:10	*Welcome*
09:10-10:00	**Stephan Luckhaus**: Rigidity estimates on the sphere
10:00 - 10:50	**Matteo Novaga**: Almost minimal configurations for nonlocal energies
10:50 - 11:20	*Coffee break*
11:20-12:10	**Lucia Scardia**: Multiscale problems in dislocation theory
12:10 - 14:00	*Lunch*
14:00-14:50	**Martin Kružík**: Quasistatic evolution for dislocation-free finite plasticity
14:50-15:40	**Marco Cicalese**: Does the N-clock model approximate the XY model?
15:40-16:10	*Coffee break*
16:10-17:00	**Bernd Schmidt**: Surface energy and boundary layers for a chain of atoms at low temperature
18:30	*Reception in the lounge (2. floor of SRZ)*

Tuesday, September 17

09:00-09:50 **Alexander Mielke**: Gamma convergence of dissipation functionals and EDP convergence for gradient systems

09:50-10:40 **Martin Burger**: Pattern Formation in Vectorial Variational Problems modelling Biological Interactions

10:40-11:10 *Coffee break*

11:10-12:00 **Ulisse Stefanelli**: Two structure-preserving discretizations for gradient flows

12:00-14:00 *Lunch*

14:00-14:50 **Antonin Chambolle**: On Poincaré-Korn and Korn inequalities for functions with small jump set

14:50-15:40 **Marcello Ponsiglione**: Uniform distribution of dislocations at semi-coherent interfaces

15:40-16:10 *Coffee break*

18:00-19:30 *Guided tour of the city. Meeting point: St.-Paulus-Dom*

Wednesday, September 28

09:00-09:50 **Andrea Braides**: Limits of convolution functionals
09:50-10:40 **Mark Peletier**: Continuum limit for annihilating dislocations in one dimension
10:40-11:10 *Coffee break*

11:10-12:00 **Massimiliano Morini**: Existence and uniqueness of anisotropic and crystalline mean curvature flows

12:00-14:00 *Lunch*

14:00-14:50 **Lisa Beck**: Lipschitz bounds and non-uniform ellipticity
14:50-15:40 **Florian Theil**: Presence of order in atomistic models for solids

15:40-16:10 *Coffee break*

16:10-17:00 **Antoine Gloria**: Effective properties of a fluid with suspensions

19:00 *Social dinner at Ratskeller*

Thursday, September 19

09:00-09:50 **Giovanni Alberti**: Minimal planar N-partitions and N-clusters for large N

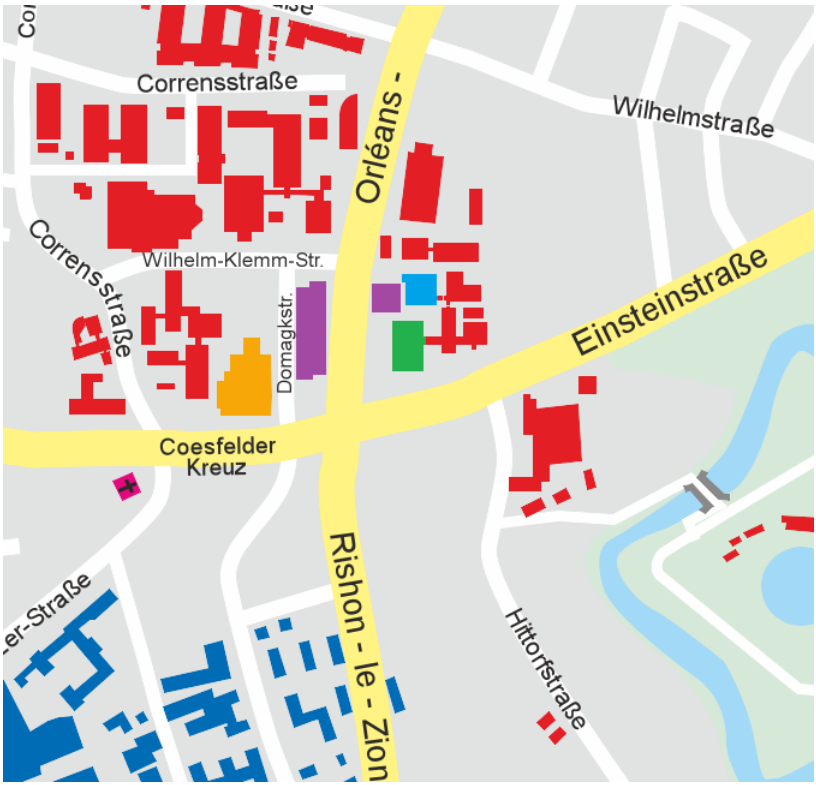
09:50-10:40 **David Bourne**: Semi-discrete optimal transport and applications

10:40-11:10 *Coffee break*

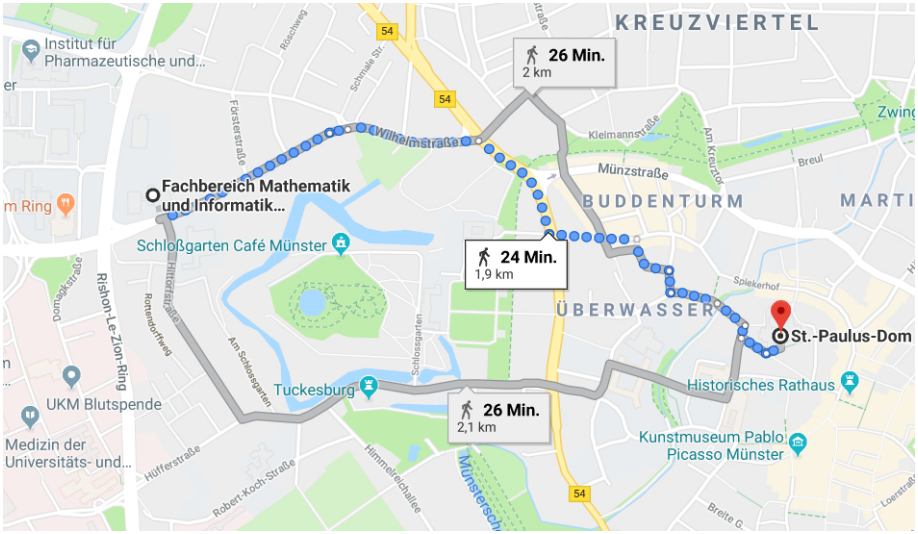
11:10-12:00 **Sergio Conti**: Dislocation microstructures and strain-gradient plasticity with a single active slip plane

12:00-14:00 *Lunch*

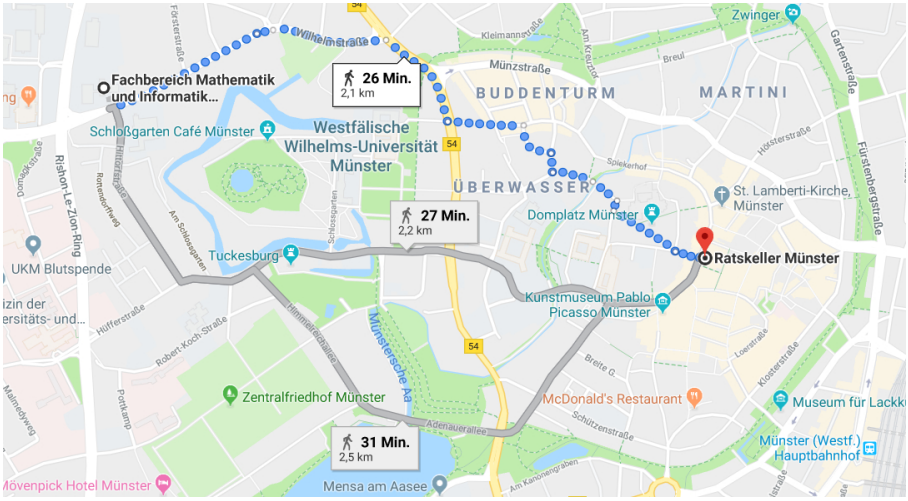
Maps and locations



Map A: Lecture building, canteen, SRZ, parking lot



Map B: Route to St.Paulus-Dom (meeting point for guided tour)



Map C: Route to Ratskeller (conference dinner)

Book of abstracts

Minimal planar N-partitions and N-clusters for large N

Giovanni Alberti

Abstract. Among all partitions of a given 2-dimensional domain E consisting of N sets (cells) with equal area, I consider those that minimize the length of the union of the boundaries of the cells. Minimal N-clusters are then obtained by further letting E vary among all sets (in the plane) with prescribed area. Among other results, T. C. Hales proved in 2001 that if E is a flat 2-dimensional torus then the regular hexagonal partition (if any exists) is the only minimal N-partition. Apart from this result, not much is known about the structure of N-partitions when N is large. In particular we expect that, away from the boundary of E, minimizing N-partitions should look ”almost” regular hexagonal. However there is evidence that when N tends to infinity non hexagonal cells (”defects”) should appear in a less than sporadic fashion also away from the boundary. Similar issues arise in the study of the asymptotic shape of minimal N-clusters. In this talk I will describe a few results obtained so far and future directions. This is a work, still in progress, with Marco Caroccia (University of Florence) and Giacomo Del Nin (University of Warwick).

Lipschitz bounds and non-uniform ellipticity

Lisa Beck

Abstract. In this talk we consider a large class of non-uniformly elliptic variational problems and discuss optimal conditions guaranteeing the local Lipschitz regularity of solutions in terms of the regularity of the data. The analysis covers the main model cases of variational integrals of anisotropic growth, but also of fast growth of exponential type investigated in recent years. The regularity criteria are established by potential theoretic arguments, involve natural limiting function spaces on the data, and reproduce, in this very general context, the classical and optimal ones known in the linear case for the Poisson equation. The results presented in this talk are part of a joined project with Giuseppe Mingione (Parma).

Semi-discrete optimal transport and applications

David Bourne

Abstract. Semi-discrete optimal transport is the important class of optimal transport problems between continuum measures (absolutely continuous measures) and discrete measures (sums of Dirac masses). In this talk I will discuss applications in optimal tiling and the steel industry.

Limits of convolution functionals

Andrea Braides

Abstract. Starting from the model case of convolution functionals approximating the Dirichlet integral, we provide a general theory of limits of convolution energies with a strong connection with discrete-to-continuum results obtained in the last twenty years. We apply such a theory to stochastic homogenization, perforated domains and thin-film limits. Works in collaboration with R. Alicandro, N. Ansini, V. Chiado' Piat, L. D'Elia, A. Piatnitski and A. Tribuzio.

Pattern Formation in Vectorial Variational Problems modelling Biological Interactions

Martin Burger

Abstract. In this talk we discuss vectorial variational problems arising as macroscopic steady states of some many particle systems in biology with different species, in local as well as nonlocal settings. We will demonstrate the the emergence of patterns such as phase separation effects, explained from relations to Cahn-Hilliard Systems, and network formation, whose theoretical understanding is still rather open. We will also comment on the difficulties of characterizing energy minimizers as stationary states of associated gradient flows, e.g. in Wasserstein spaces.

On Poincaré-Korn and Korn inequalities for functions with small jump set

Antonin Chambolle

Abstract. We discuss the validity of Poincaré-Korn and Korn inequalities for functions of "GSBD" type with a small jump set. (based on joint works with S. Conti, G. Francfort, F. Iurlano, F. Cagnetti and L. Scardia.)

Does the N-clock model approximate the XY model?

Marco Cicalese

Abstract. The N-clock model is a two-dimensional ferromagnetic spin model on the square lattice in which the spin field is constrained to take values in a set of N equi-spaced points of the unit circle. It is usually considered as an approximation of the XY model, for which instead the spin field is allowed to attain all the values of the unit circle. In the theory of superconductivity the latter models phase transitions mediated by the formation and the interaction of co-dimension 2 topological defects as in the well-known Ginzburg-Landau functional. A breakthrough result by Fröhlich and Spencer (CMP 1981) shows that the same kind of phase transitions appear in the N-clock model for N large enough. By a variational analysis we find the explicit rate of divergence of N (with respect to the number of interacting lattice points) for which the N-clock model asymptotically behaves like the XY model at zero temperature. We moreover exhaustively discuss all the other regimes of N and we show how Cartesian Currents can detect the energy concentration on sets of co-dimenion smaller or equal than 2. The results presented are contained in a recent paper in collaboration with G. Orlando (TUM) and M. Ruf (EPFL).

Dislocation microstructures and strain-gradient plasticity with a single active slip plane

Sergio Conti

Abstract. We consider a model for dislocations in a three dimensional crystal, which are restricted to move in a single plane. We derive, within the framework of Γ-convergence, a reduced model in the limit of small lattice spacing. The limiting model contains two terms. The first one is a continuous energy with linear growth, which depends on a measure which characterizes the macroscopic dislocation density; this term corresponds to the strain-gradient terms often used in mechanical applications. The second one is a nonlocal effective energy representing the far-field interaction between dislocations, it is quadratic and depends on the elastic strain. Both arise naturally as scaling limits of the nonlocal elastic interaction. Relaxation and formation of microstructures at intermediate scales are automatically incorporated in the limiting procedure. This talk is based on joint work with Adriana Garroni and Stefan Müller.

Effective properties of a fluid with suspensions

Antoine Gloria

Abstract. In this talk I'll describe and discuss some standard models of suspensions in a fluid. Depending on whether the particles have the same density as the fluid or not, the phenomena (and the analysis) are quite different. Typical questions that I will address are the notion of effective viscosity of colloidal suspensions and of the effective sedimentation speed of heavy suspensions.

Quasistatic evolution for dislocation-free finite plasticity

Martin Kružík

Abstract. We investigate quasistatic evolution in finite plasticity under the assumption that the plastic strain is compatible. This assumption is well-suited to describe the special case of dislocation-free plasticity and entails that the plastic strain is the gradient of a plastic deformation map. The total deformation can be then seen as the composition of a plastic and an elastic deformation. This opens the way to an existence theory for the quasistatic evolution problem featuring both Lagrangian and Eulerian variables. A remarkable trait of the result is that it does not require second-order gradients. This is a joint work with D. Melching and U. Stefanelli (Vienna).

Rigidity estimates on the sphere

Stephan Luckhaus

Abstract. In a joint work with K. Zemas we explore the relationship beetween geometric rigidity,the isoperimetric inequality and stability estimates for the Moebius transformations of the sphere.

Gamma convergence of dissipation functionals and EDP convergence for gradient systems

Alexander Mielke

Abstract. We consider families of gradient systems $(Q, \mathcal{E}_\varepsilon, \mathcal{R}_\varepsilon)$, where $\mathcal{E}_\varepsilon$ denotes the driving energy or entropy functional, and $\mathcal{R}_\varepsilon$ is the dissipation potential. If

the parameter ε tends to 0, it is natural to ask for the limiting, also called effective gradient system.

This can be achieved on the basis of the Energy-Dissipation Principle (EDP) where the dissipation is expressed in term of a dissipation functional, which is written as a time integral over the dual sum $\mathcal{R}_\varepsilon(q,\dot{q}) + \mathcal{R}^*_\varepsilon(q,-D\mathcal{E}_\varepsilon(q))$. The notion of EDP convergence is defined in terms of the Gamma convergence of these dissipation functionals.

As an example we treat the porous medium equation with a thin layer, where the mobility is small and suitably scaled. Using EDP convergence a new gradient structure for the effective membrane model is derived. Starting from Otto's quadratic gradient structure we obtain a transmission law (kinetic relation) that is non-quadratic.

(This is joint work with Thomas Frenzel.)

Existence and uniqueness of anisotropic and crystalline mean curvature flows

Massimiliano Morini

Abstract. An existence and uniqueness result, up to fattening, for crystalline mean curvature flows with forcing and arbitrary (convex) mobilities, is proven. This is achieved by introducing a new notion of solution to the corresponding level set formulation. Such solutions satisfy a comparison principle and stability properties with respect to the approximation by suitably regularized problems. The results are valid in any dimension and for arbitrary, possibly unbounded, initial closed sets. As a result of our analysis, we deduce the convergence of a minimizing movement scheme proposed by Almgren, Taylor and Wang (1993), to a unique (up to fattening) "flat flow".

Almost minimal configurations for nonlocal energies

Matteo Novaga

Abstract. We describe low energy configurations for nonlocal energies such as the Heitmann-Radin sticky disc functional, in the limit of diverging number of particles. We show a compactness result for configurations whose energy scales like the perimeter, after suitable renormalization. In this case, the empirical measures converge to a set of finite perimeter, while a microscopic variable, representing the orientation of the underlying lattice, converges to a locally constant function of bounded variation.

Continuum limit for annihilating dislocations in one dimension

Mark Peletier

Abstract. We prove a many-particle limit for the evolution of a system of particles in one dimension with positive and negative charges. The particles interact via a logarithmic potential, and when particles of opposite sign meet they 'annihilate' and are removed from the system.

We use the framework of viscosity solutions for this system developed by Forcadel, Imbert, and Monneau (2009), which in turn is based on Slepcev's formulation (2003) of non-local level-set evolutions. We generalize their convergence proof to the case of singular interactions and multiple signs by a careful analysis of the Hamiltonian near annihilation points.

This is joint work with Patrick van Meurs and Norbert Pozar (both Kanazawa University, Japan)

Uniform distribution of dislocations at semi-coherent interfaces

Marcello Ponsiglione

Abstract. We will introduce variational models for edge dislocations at semi-coherent interfaces between two heterogeneous crystals, and prove the optimality of uniformly distributed edge dislocations.

Specifically, we prove that, in the large interface limit, the elastic energy Gamma-converges to a limit functional comprised of two contributions: one is given by a constant gauging the minimal energy induced by dislocations at the interface, and corresponding to a uniform distribution of edge dislocations; the other one accounts for the far field elastic energy induced by the presence of further, possibly not uniformly distributed, dislocations.

After assuming periodic boundary conditions and formally considering the limit from semi-coherent to coherent interfaces, we show that the optimal configuration consists in evenly-spaced dislocations on the one dimensional circle.

These results are in collaboration with S. Fanzon and R. Scala.

Multiscale problems in dislocation theory

Lucia Scardia

Abstract. Dislocations are defects in the arrangement of atoms in metals, and they are the microscopic reason for the permanent deformation of metals at the macroscopic scale. In this talk I will present some results on the behaviour of large numbers of interacting dislocations, both from a static and from a dynamic perspective.

Surface energy and boundary layers for a chain of atoms at low temperature

Bernd Schmidt

Abstract. We report on recent results on the low-temperature behaviour for a one-dimensional chain of atoms that interact via a Lennard-Jones type potential. In particular we (1) provide a detailed analysis of the boundary layers and surface energy at temperature T = 0, (2) show that at T > 0 the Gibbs measures for infinite chains and semi-infinite chains satisfy path large deviations principles and find that the surface correction to the Gibbs free energy converges to the zero temperature surface energy, and (3) derive bounds on the decay of correlations.

Two structure-preserving discretizations for gradient flows

Ulisse Stefanelli

Abstract. The implicit Euler scheme is the reference discretization scheme for gradient flows. Despite the many appealing features, including its variational character and the optimal convergence rate, the Euler scheme fails to replicate the exact dissipation balance at the discrete level. This property is however of a crucial interest in view of the development of structure-preserving discretizations of complex systems. I will present two alternative discretization schemes, which in turn replicate exactly the dissipation balance. Their features will be compared and I will discuss existence and convergence.

This is a joint work with Ansgar Juengel and Lara Trussardi, arXiv:1811.06033.

Presence of order in atomistic models for solids

Florian Theil

Abstract. The emergence of long-range order at low temperatures in atomistic systems with continuous symmetry is a fundamental, yet poorly understood phenomenon in Physics. I a collaboration with Alessandro Giuliani (Roma Tre) we have obtained new results for a discrete microscopic model for an elastic crystal with dislocations in three dimensions, previously introduced by Ariza and Ortiz. The model is rich enough to support some realistic features of three-dimensional dislocation theory, most notably grains and the Read-Shockley law for grain boundaries. At sufficiently low temperatures the Gibbs distribution exhibits long range positional order.

Participants

- Giovanni Alberti (Pisa)
- Adolfo Arroyo Rabasa (Warwick)
- Anvarbek Atayev (Warwick)
- Annika Bach (Munich)
- Lisa Beck (Augsburg)
- Rajae Bentahar (Tétouan)
- Bibek Bhujel (Bremen)
- David Bourne (Edinburgh)
- Omar Boussaid (Chlef)
- Andrea Braides (Rome)
- Marco Bresciani (Vienna)
- Martin Burger (Erlangen)
- Marcello Carioni (Graz)
- Stefano Ceci (Münster)
- Antonin Chambolle (Paris)
- Marco Cicalese (Munich)
- Sergio Conti (Bonn)
- Vito Crismale (Paris)
- Riccardo Cristoferi (Edinburgh)
- Giacomo Del Nin (Warwick)
- Marcel Dengler (Surrey)
- Silvio Fanzon (Graz)
- Marwin Forster (Munich)
- Janusz Ginster (Berlin)
- Peter Gladbach (Leipzig)
- Antoine Gloria (Paris)
- Martin Jesenko (Freiburg)
- Leonard Kreutz (Münster)
- Martin Kružík (Prague)
- Andrea Kubin (Rome)
- Michal Lasica (Warsaw)
- Stephan Luckhaus (Leipzig)
- Alexander Mielke (Berlin)
- Marco Morandotti (Torino)
- Massimiliano Morini (Parma)
- Marilena Moruz (Leuven)
- Matteo Novaga (Pisa)
- Mario Ohlberger (Münster)
- Heiner Olbermann (Louvain)
- Ashraf H. Owis (Cairo)
- Valerio Pagliari (Pisa)
- Reza Pakzad (Pittsburgh, Bonn)
- Mark Peletier (Eindhoven)
- Matteo Perugini (Münster)
- Alessandra Pluda (Pisa)
- Marcello Ponsiglione (Rome)
- Marco Pozzetta (Pisa)
- Lucia Scardia (Edinburgh)
- Leonie Schmeller (Münster)
- Bernd Schmidt (Augsburg)
- Christoph Smoch (Bonn)
- Anna Skorobogatova (Warwick)
- Ulisse Stefanelli (Vienna)
- Angela Stevens (Münster)
- Ayk Telciyan (Coimbra)
- Florian Theil (Warwick)
- Antonio Tribuzio (Rome)
- Benedikt Wirth (Münster)