Locally Compact Groups and Lie Groups

Paul Klee, Bunte Gruppe

Linus Kramer

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Preface

It was a dark and stormy night.
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THANK YOU
GOOD LUCK WITH THE SECOND SENTENCE!
SCHULZ

Conventions

Our conventions in group theory and topology are mostly standard. The neutral element of a group G is usually denoted by e or, if the group is abelian and written additively, by 0. For subsets $X, Y \subseteq G$ we put

$$XY = \{xy \mid x \in X \text{ and } y \in Y\} \quad \text{and} \quad X^{-1} = \{x^{-1} \mid x \in X\}$$

and for an integer $k \geq 1$ we put

$$X^{\cdot k} = \{x_1 x_2 \cdots x_k \mid x_1, \ldots, x_k \in X\}.$$

If $XX = X^{\cdot 2} \subseteq X$ holds, we call X a *subsemigroup* of G. We call a subset $X \subseteq G$ *symmetric* if $X = X^{-1}$. The *centralizer* of X is denoted by

$$\mathrm{Cen}_G(X) = \{g \in G \mid gx = xg \text{ holds for all } x \in X\}.$$

The *center* of a group G is

$$\mathrm{Cen}(G) = \mathrm{Cen}_G(G).$$

Our convention for *commutators* in groups is that

$$[a, b] = aba^{-1}b^{-1}.$$

We will mainly consider *left actions*. Such a left action of a group G on a set Z will be written as

$$G \times Z \longrightarrow Z, \quad (g, z) \longmapsto gz.$$

The *stabilizer* of a point $z \in Z$ will be denoted by

$$G_z = \{g \in G \mid gz = z\}.$$

For a subset $A \subseteq Z$ we put $GA = \{ga \mid g \in G \text{ and } a \in A\}$.

All *rings* and ring homomorphisms are assumed to be unital, i.e. there are unit elements and homomorphisms preserve unit elements. The group of units of ring R is denoted $R^\times$. Rings are not necessarily commutative.

A subset V of a topological space X is called a *neighborhood* of a point $x \in X$ if there is an open set U with $x \in U \subseteq V$. A *neighborhood basis* of a point $x \in X$ is a collection $\mathcal{V}$ of neighborhoods of x, such that for every open set $U \subseteq X$ containing x, some member of $\mathcal{V}$ is contained in U.

It is our convention is that all compact or locally compact spaces are assumed to be Hausdorff.

We consider 0 to be a natural number. The set of natural numbers is thus

$$\mathbb{N} = \{0, 1, 2, 3, \ldots\},$$

and we denote the set of all positive natural numbers by

$$\mathbb{N}_1 = \{1, 2, 3, \ldots\}.$$

The set difference of two sets X, Y is written as

$$X - Y = \{x \in X \mid x \notin Y\}$$

and the *symmetric difference* is written as

$$X \triangle Y = (X - Y) \cup (Y - X) = (X \cup Y) - (X \cap Y).$$

If $\mathcal{S}$ is a collection of subsets of a set X and if $A \subseteq X$ is a subset, we put

$$\mathcal{S}|A = \{S \cap A \mid S \in \mathcal{S}\}.$$

The *power set* of a set X is denoted as

$$\mathcal{P}(X) = \{Y \mid Y \subseteq X\}.$$

The set of all maps from a set X to a set Y is denoted

$$Y^X.$$

If $f : X \longrightarrow Y$ is a map and if $A \subseteq X$ is a subset, then we denote the restriction of f to A by $f|_A : A \longrightarrow Y$. If $B \subseteq Y$ contains $f(A)$, we may consider the restriction-corestriction $f|_A^B : A \longrightarrow B$.

We use the *axiom of choice* without further ado.