9. Übungszettel zur Vorlesung „Räume nichtpositiver Krümmung“

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WWU Münster

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Each question is worth 4 points.

Aufgabe 9.1 (Semisimple isometries of products)
Let X and Y be CAT(0) spaces and let g and h be isometries of X and Y respectively. Show that the isometry of $X \times Y$ given by $(x, y) \mapsto (g(x), h(y))$ is semisimple if and only if both g and h are semisimple.

Aufgabe 9.2 (Isometries of $\mathbb{H}^2$)
Using the Poincaré upper half space model of the hyperbolic space, we may identify $\mathbb{H}^2$ with the upper half complex plane $\{z \in \mathbb{C} \mid \operatorname{Im} z > 0\}$. Consider the action of $\mathrm{SL}(2, \mathbb{R})$ on $\mathbb{H}^2$ given by

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} z = \frac{az + b}{cz + d}.$$

a) Show that $\mathrm{SL}(2, \mathbb{R})$ acts transitively on $\mathbb{H}^2$.

b) Show that the stabilizer of i is $O(2)$.

Aufgabe 9.3 (Semisimple isometries of $\mathbb{H}^2$)
Determine all the semisimple elements in $\mathrm{SL}(2, \mathbb{R})$ in its action on $\mathbb{H}^2$.

Aufgabe 9.4 (Adjoint)
Let $M(n, \mathbb{R})$ denote the algebra of $n \times n$ matrices with real entries. For $A \in M(n, \mathbb{R})$, A^t will denote the transpose of A. For $A \in M(n, \mathbb{R})$, define an endomorphism ad_A on $M(n, \mathbb{R})$ by $\mathrm{ad}_A(B) = AB - BA$.

Verify that the formula $(A|B) = \mathrm{Tr}(AB^t)$ defines a scalar product on $M(n, \mathbb{R})$. Show that if X is a symmetric matrix in $M(n, \mathbb{R})$, then ad_X is a self-adjoint operator on $M(n, \mathbb{R})$, i.e., for any $A, B \in M(n, \mathbb{R})$, we have $(\mathrm{ad}_X A|B) = (A|\mathrm{ad}_X B)$.

9.*-Aufgabe (Trace, det, and exp)
Show that for $A \in M(n, \mathbb{R})$, $\exp(\mathrm{Tr}(A)) = \det(\exp A)$.

Abgabe bis: Donnerstag, den 21.1.2021, 8 Uhr online im Learnwebkurs.