

11. Übungszettel zur Vorlesung „Räume nichtpositiver Krümmung“

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Each question is worth 4 points.

Aufgabe 11.1 (Decompositions of $\mathrm{SL}(m, \mathbb{R})$)

Prove the polar decomposition, the Cartan decomposition and the Iwasawa decomposition for the group $\mathrm{SL}(m, \mathbb{R})$, as stated in class.

Hint: You may use the analogous results for $\mathrm{GL}(m, \mathbb{R})$ in the proof.

Aufgabe 11.2 (Eigenspace of commuting morphisms)

Let A, B be commuting endomorphisms of a vector space V , and let E be an eigenspace of A . Show that $B(E) \subseteq E$.

Aufgabe 11.3 ($P(2)_1$)

Prove that the symmetric space $P(2)_1$ is isometric to the hyperbolic space \mathbb{H}^2 .

Aufgabe 11.4 (Symmetric matrices)

Let M be a set of pairwise commuting real symmetric $m \times m$ matrices. Show that the matrices in M can be simultaneously diagonalized with an orthogonal matrix.

Hint: Use induction on m .

11.*-Aufgabe (Product of subgroups)

Let H, K be subgroups of a group G . Show that HK is a subgroup of G if and only if $HK = KH$.

Abgabe bis: Donnerstag, den 4.2.2021, 8 Uhr online im Learnwebkurs.