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Winter term 2024/25
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Locally Compact Groups

Sheet 11

Hand in: Friday January 17, 2025 after class in letterbox No 4.

We put $\mathbb{T} = \mathrm{U}(1) = \{z \in \mathbb{C} \mid |z| = 1\}$. This is a compact abelian group, the 1-torus.

Problem 1.
Let C be a finite cyclic group. Show that $A = \mathrm{Hom}(C, \mathbb{T})$ is a cyclic group isomorphic to C.
Hint. Let c be a generator of C. A homomorphism $f : C \to \mathbb{T}$ is uniquely determined by $f(c)$, and the order of $f(c)$ divides the order of c.

Problem 2.
Show that the only subgroup of $\mathbb{T}$ which is contained in the open set $U = \{z \in \mathbb{T} \mid \mathrm{Re}(z) > 0\}$ is the trivial group $\{1\}$.

Problem 3.
Let A be a discrete abelian group. Show that $\mathrm{Hom}(A, \mathbb{T})$, viewed as a subset of $\mathbb{T}^A$, is a compact group.

Problem 4.
Let B be a compact abelian group. We endow the group $\mathrm{Mor}(B, \mathbb{T})$ of morphisms $f : B \to \mathbb{T}$ with the metric $d(f, h) = \max\{|f(b) - h(b)| \mid b \in B\}$. Show that the topology determined by d on $\mathrm{Mor}(B, \mathbb{T})$ is discrete.
Hint. Use Problem 11.2.

Bonus Problem 1.
Let $m \in \mathbb{N}$. Show that every profinite subgroup of $\mathbb{T}^m$ is finite.