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Universität Münster

Locally Compact Groups

Sheet 9

Hand in: Friday December 13, 2024 after class in letterbox No 4.

Problem 1.
Determine the Haar integral on the locally compact group $(\mathbb{R}^*, \cdot)$.

Problem 2.
Put

$$G = \mathrm{GL}_m(\mathbb{R}) = \{g \in \mathbb{R}^{m \times m} \mid \det(g) \neq 0\}.$$

Show that G is unimodular.

Problem 3.
Put

$$H = \{\left(\begin{smallmatrix} s & t \\ 0 & 1 \end{smallmatrix}\right) \mid s, t \in \mathbb{R},\ s > 0\}.$$

Show that H is a locally compact group. Determine the Haar integral on H. Show that H is not unimodular.

Problem 4.
Let G be a locally compact group. Show that G is a finite discrete group if and only if $C_c(G, \mathbb{R})$ has finite dimension.

Bonus Problem 1.
An element g in a locally compact group G is called *compact* if g is contained in some compact subgroup. Show that for such an element $\mathrm{mod}(g) = 1$.