# Exercise sheet 9

**Exercise 1.** Let $F : \mathcal{C} \to \mathcal{D}$ be a functor between two categories.
Prove or disprove the following assertion: "the image of $F$ in $\mathcal{D}$ is a subcategory of $\mathcal{D}$".

**Exercise 2.** Write the following groups as amalgamated products:

1. $G = \langle x, y \mid x^3y^{-3},\ y^6 \rangle$;
2. $H = \langle x, y \mid x^{30},\ y^{70},\ x^3y^{-5} \rangle$.

**Exercise 3.** Show that there exist uncountably many groups that are generated by two elements and *not* finitely presented.

**Exercise 4.** Let $G$ be a group and $A, B \leqslant G$ two subgroups. Denote $C := A \cap B$.
Show that $G \simeq A*_C B$ if and only if: all $g \in G \backslash C$ can be written as a product $g = g_1 \cdots g_n$ with $g_i \in G_i \backslash C$ where

$$G_i \in \{A, B\} \quad \text{and} \quad G_i \neq G_{i+1}$$

and all such products are different from $1$.

**Bonus exercise.** Let $G, H$ be two groups and $\alpha : G \to H$ be an epimorphism.
Assume that $H := H_1 *_{H_3} H_2$ and let $G_i := \alpha^{-1}(H_i)$ for $i = 1, 2, 3$. Show that $G := G_1 *_{G_3} G_2$.
*Hint: Use the previous exercise.*

*Please hand in your solutions on the morning of December, 16th before the lecture (letterbox 162 or electronically in the Learnweb).*