

## EXERCISE SHEET 7

**Exercise 1.** Prove that the category **abgrp** of abelian groups is complete and cocomplete.

**Exercise 2.** Denote by **grp** the category of groups. Show that the abelianization functor  $G \rightarrow G_{\text{ab}}$  is left adjoint to the inclusion  $\mathbf{abgrp} \hookrightarrow \mathbf{grp}$ .

**Exercise 3.** Show that the free abelian group functor  $\text{FA} : \mathbf{set} \rightarrow \mathbf{abgrp}$  is not continuous.

**Exercise 4.** Prove or disprove that the category of compact Hausdorff spaces is complete.

**Bonus exercise.** Give a category-theoretic proof that

$$\left( \prod_{i \in I} G_i \right)_{\text{ab}} = \bigoplus_{i \in I} (G_i)_{\text{ab}}.$$

*Please hand in your solutions on the morning of December, 2nd before the lecture (letterbox 162 or electronically in the Learnweb).*