Exercise sheet 12

Exercise 1. Recall that we say that a group is *divisible* if the equation $x^m = g$ has a solution x for all $g \in G$ and $m \geqslant 1$. Let G be a countable group. Show that there exists a divisible group G^* which contains G as a subgroup.

Hint: Embed G into a countable group in which elements of every order exist, and then use Exercise 4 of Sheet 10.

Exercise 2. Let $\Gamma = (V, E)$ be a graph and $|\Gamma|$ be the geometric realisation of Γ. Show the following assertions:

1. $|\Gamma|$ is Hausdorff.
2. $|\Gamma|$ is compact if and only if the sets V and E are finite.
3. If $\Gamma' \subseteq \Gamma$ is a subgraph, then $|\Gamma'|$ is closed in $|\Gamma|$.

Exercise 3. Let X be a topological space.

1. Let $A \subseteq X$ be a subspace and $r : X \to A$ be a retraction. Show that if X is contractible then so is A.
2. Show that X is contractible if and only if for every topological space Y, any continuous map $f : X \to Y$ is homotopic to a constant mapping.

Definition 1

> A connected graph $\Gamma = (V, E)$ is called a *tree* if it contains no cycle, i.e. there is no reduced edge path $(e_1, \dots, e_n)$ in Γ such that $(e_1)_0 = (e_n)_1$.

Exercise 4. Let Γ be a tree. Let $n \in \mathbb{N}$ such that $n \geqslant 2$ and consider $T_1, \dots, T_n$ subtrees of Γ. Show that if $T_i \cap T_j \neq \emptyset$ for all $i, j \in \{1, \dots, n\}$ then $\cap_{i=1}^n T_i \neq \emptyset$.

Please hand in your solutions on the morning of January, 20th before the lecture (letterbox 162 or electronically in the Learnweb).