Exercise sheet 11

Exercise 1. Show that every covering map is an open map.

Exercise 2. Let $n \neq 0$.

1. Show that the map $f : z \in S^1 \to z^n \in S^1$ is a covering map from S^1 to S^1.
2. Compute the endomorphism f_* on the fundamental group $\pi_1(S^1, 1)$.

Exercise 3. Let $f : E \to X$ be a covering map.

1. If X is path connected show that for all $x_1, x_2 \in X$ we have $\#f^{-1}(x_1) = \#f^{-1}(x_2)$.
2. Show that the last assertion is still true if X is only connected.

Definition 1

We say that a functor $F : \mathcal{C} \to \mathcal{D}$ is *faithful* if F is injective on $\mathrm{Mor}_{\mathcal{C}}(a, b)$ for all objects a and b in $\mathcal{C}$.

Exercise 4. Let $f : E \to X$ be a covering map. Show that f_* is a faithful functor on the fundamental groupoids of E and X.

Definition 2

We say that a group G *has exponent* m if $g^m = e$ for all $g \in G$. We say that a group is *divisible* if the equation $x^m = g$ has a solution x for all $g \in G$ and $m \geqslant 1$.

Bonus exercise. Prove or disprove

1. A group of finite exponent $m > 0$ is not divisible.
2. A non-trivial residually finite group is not divisible.

Please hand in your solutions on the morning of January, 13th before the lecture (letterbox 162 or electronically in the Learnweb).