Prof. Dr. L. Kramer
Dr. Rupert McCallum
Antoine Beljean

SS 2012

2.Übung zur Vorlesung Gebäude

Please hand in your solutions on the morning of Friday 20 April before the lecture.

Aufgabe 2.1 (1. Realisations of Simplicial Complexes)

(6 marks)

Let X be a topological space and let $\{U_i\}_{i\in I}$ be an open cover of X. As discussed in the lectures, the nerve N of the open cover $\{U_i\}_{i\in I}$ is a simplicial complex.

We say that a family of continuous functions $\{f_i\}_{i\in I}$ from X to the unit interval $[0,1]$ is a partition of unity subordinate to the open cover $\{U_i\}_{i\in I}$ if the following hold:

(i) for each point $x \in X$ there is a neighbourhood of x in which all but finitely many of the f_i are zero;
(ii) $\Sigma_{i\in I} f_i(x) = 1$ for all $x \in X$;
(iii) for each $i \in I$ the support of f_i, defined to be the closure of $\{x \mid f(x) \neq 0\}$ is contained in U_i.

Show how to construct from a given partition of unity subordinate to $\{U_i\}_{i\in I}$ a continuous mapping from X to the geometric realisation of the nerve N.

Aufgabe 2.2 (2. The dihedral groups)

(4 marks)

Describe the centre of the dihedral group D_m for each possible value of m including ∞. Describe the conjugacy classes in D_m for each possible value of m including ∞.

Aufgabe 2.3 (3. Free Groups)

Suppose that X is a set, and that FX is a free group over X.

(a) (2 marks) Show that the free group FX has the universal property described in the lectures.
(b) (4 marks) Suppose that m and n are distinct positive integers, and that X is a set of cardinality m and Y is a set of cardinality n. Prove that FX is not isomorphic to FY. (Hint: Consider their abelianisations.)
(*) Look up the definition of adjoint functors, for example in the (English) Wikipedia, or in the math library. Show that the functor $F : \text{Set} \to \text{Group}$ which sends a set X to the free group over X is left adjoint to the forgetful functor $G : \text{Group} \to \text{Set}$.