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SS 2012

8.Übung zur Vorlesung Gebäude

Please hand in your solutions on the morning of Friday 8 June before the lecture.

Aufgabe 8.1 (1. Normal subgroups and BN-Pairs)

Let G be a group, let N and B be subgroups of G such that $B \cap N \triangleleft N$. Let S be a subset of $W := N/(B \cap N)$. Suppose that Z is a normal subgroup of G, contained in B. Let B' and N' be the images of B and N in $G' := G/Z$.
(a) Show that the canonical projection $p : N \to N'$ induces an isomorphism ι from W to $W' := N'/(B' \cap N')$.
(b) Let S' be the image of S unter ι, where ι is defined as in (a).
Show that (G, B, N, S) is a Tits sytem exactly when (G', B', N', S') is.

Aufgabe 8.2 (2. Projective planes)

Recall that an incidence structure (P, L, I) is a set of points P, a set of lines L and an incidence relation $I \subset P \times L$. Two elements $p \in P$ and $l \in L$ are said to be incident if $(p, l) \in I$. An incidence structure is a projective plane, if and only if:

(i) Given two distinct points, there is exactly one line incident to both of them.
(ii) Given two distinct lines, there is exactly one point incident to both of them.
(iii) There are at least four points, such that no line is incident to any three of them. (non-degeneracy)

The pairs of incident points and lines (p, l) are called flags.

Suppose that G is a group that acts transitively on the incident point-line pairs of a projective plane. Let (p, l) be a flag and $A = G_p$ and $B = G_l$.

Prove that

a) $G = ABA = BAB$
b) $AB \neq BA$
c) $(AB) \cap (BA) = A \cup B$

Conversely, show that if G is a group with subgroups A, B satisfying these three conditions, then there is a projective plane on which G acts flag-transitively.