

## 6. Übung zur Vorlesung Gebäude

Please hand in your solutions on the morning of Friday 18 May before the lecture.

We denote the set of all chambers in a building  $\Delta$  by  $Cham(\Delta)$ .

### Aufgabe 6.1 (1. Diameters)

(2 marks) Suppose that  $\Delta$  is a building of type  $\circ \xrightarrow{m} \circ$ , with  $m \in \{2, 3, \dots, \infty\}$ . We put  $diam(\Delta) = \sup\{\ell(\delta(a, b)) \mid a, b \in Cham(\Delta)\}$

Show that  $diam(\Delta) = m$ .

### Aufgabe 6.2 (2. The building encodes the Coxeter diagram)

(4 marks) Suppose that  $\Delta$  is a building of type  $(W, I)$ . If  $a$  is a simplex of type  $I \setminus \{i, j\}$ , with  $i \neq j$ , show that the entry  $m_{i,j}$  of the Coxeter matrix can be read off from  $lk(a)$ .

Show that the simplicial complex  $\Delta$  determines both  $(W, I)$  and the type function uniquely, up to isomorphism. (What kind of isomorphism?).

### Aufgabe 6.3 (3. Properties of the $W$ -valued distance)

Let  $\Delta$  be a building of type  $(W, I)$  and let  $\delta : Cham(\Delta) \times Cham(\Delta) \longrightarrow W$  denote the  $W$ -valued distance.

(2 marks) Show that  $\delta(a, b) = \delta(b, a)^{-1}$  and show that  $\delta(a, b) = 1$  holds if and only if  $a = b$ .

(2 marks) Suppose that  $\delta(b, c) = w$  and that  $\delta(a, b) = i \in I$ . Show that  $\delta(a, c) \in \{iw, w\}$ . If  $\ell(iw) = 1 + \ell(w)$ , show that  $\delta(a, c) = iw$ .

(2 marks) Suppose that  $\delta(b, c) = w$  and that  $i \in I$ . Show that there is a chamber  $a$  such that  $\delta(a, b) = i$  and  $\delta(a, c) = iw$ .