

4. Übung zur Vorlesung Gebäude

Please hand in your solutions on the morning of Friday 4 May before the lecture.

Aufgabe 4.1 (1. Stabilisers)

Suppose that (W, I) is a Coxeter system and $\Sigma := \Sigma(W, I)$.

- (a) (2 marks) Find the stabiliser W_a of a simplex $a = wW_J \in \Sigma$ under the action of W .
- (b) (2 marks) Prove that W_a acts transitively on the chambers containing a .

Aufgabe 4.2 (2. Flag complexes)

The flag complex $\Delta(P)$ of a partially ordered set (P, \leq) is the set of all finite subsets $\{p_1, p_2, \dots, p_n\} \subseteq P$ such that $p_1 \leq p_2 \leq \dots \leq p_n$. We say that $\{p_1, p_2, \dots, p_n\}$ is a flag. The flag complex is a partially ordered set with respect to the inclusion relation between subsets of P . Prove that:

- (a) (4 marks) For all partially ordered sets P the flag complex $\Delta(P)$ is a simplicial complex.
- (b) (2 marks) The flag complex $\Delta(K, n)$ (which is defined in the lecture notes) is a chamber complex.

Aufgabe 4.3 (3. Coxeter groups and Coxeter complexes)

Do the following tasks for these three Coxeter groups:

- (a) $D_\infty = \langle i, j \mid i^2, j^2 \rangle$ with Coxeter system $(D_\infty, \{i, j\})$,
- (b) $W = \langle i, j, k \mid i^2, j^2, k^2, (ij)^3, (jk)^3, (ki)^2 \rangle$ with Coxeter system $(W, \{i, j, k\})$ and
- (*) $W = \langle i, j, k \mid i^2, j^2, k^2, (ij)^3, (jk)^3, (ki)^3 \rangle$ with Coxeter system $(W, \{i, j, k\})$

- (i) (1 mark for each Coxeter group) Write down the Coxeter matrix and Coxeter diagram.
- (ii) (1 mark for each Coxeter group) Try to draw the Coxeter complexes. Label which simplices correspond to which cosets of W_J for which subsets $J \subseteq I$.
- (iii) (1 mark for each Coxeter group) Draw the chamber graphs of the Coxeter complexes.