

# Absolute travel distance from optic flow

Harald Frenz \*, Markus Lappe

*Allgemeine Zoologie und Neurobiologie, Ruhr-Universität Bochum, 44780 Bochum, Germany  
Psychologisches Institut II, Westfälische Wilhelms-Universität Münster, Fliednerstrasse 21, 48149 Münster, Germany*

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## Abstract

Optic flow fields provide rich information about the observer's self-motion. Besides estimation of the direction of self-motion human observers are also able to discriminate the travel distances of two self-motion simulations. Recent studies have shown that observers estimate the simulated ego velocity of the self-motion simulation and integrate it over time. Thus, observers use a 3-D percept of the ego motion through the environment. In the present work we ask if human observers are able to use this 3-D percept of the motion simulation to build up an internal representation of travel distance and indicate it in a static scene. We visually simulated self-motion in different virtual environments and asked subjects to indicate the perceived distances in terms of static virtual intervals on the ground. The results show that human observers possess a static distance gauge, but that they undershoot the travel distances for short motion simulations. In further experiments we changed the modality of the distance indication but the undershoot in distance estimation remained. This suggests that the undershoot is linked to the perception of the optic flow field.

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## 1. Introduction

As an observer moves through the environment a wealth of information about the self-motion is registered by the sensory system. The position and orientation of the limbs are indicated by proprioception. The vestibular senses signal orientation and acceleration of the head. Self-induced visual motion signals are registered from the retinal flow field. The control of self-motion benefits from the integration of these sources of information because each sensory signal has its own strengths and weaknesses. For instance, the vestibular signal best registers fast and transient movement and is poorer for slow continuous movement. The visual system is sensitive to slow and sustained motion. To understand the

control of self-motion it is thus useful also to consider how much information can be derived from any sensory source alone. Moreover, while many sources are accessible for normal self-motion control more artificial conditions, for instance in driving simulators, restrict the amount of information available. Many simulators lack real motion in terms of moveable platforms and reduce self-motion information to the level of pure visual signals.

Visual information from the optic flow provides a lot of information about self-motion, in particular the direction of motion and the time-to-contact with obstacles. Recent studies in animals (Esch & Burns, 1995, 1996; Esch, Zhang, Srinivasan, & Tautz, 2001; Srinivasan, Zhang, Lehrer, & Collett, 1996; Sun, Carey, & Goodale, 1992) and humans (Bremmer & Lappe, 1999; Frenz, Bremmer, & Lappe, 2003; Redlick, Jenkin, & Harris, 2001; Ricke, van Veen, & Bühlhoff, 2002; Sun, Campos, & Chan, 2004, 2004) have looked at the ability to estimate also travel distances from optic flow. In

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\* Corresponding author. Address: Psychologisches Institut II, Westfälische Wilhelms-Universität Münster, Fliednerstrasse 21, 48149 Münster, Germany. Tel.: +49 2518334181; fax: +49 2518334173.  
E-mail address: [frenzh@uni-muenster.de](mailto:frenzh@uni-muenster.de) (H. Frenz).

principle the absolute travel distance cannot be judged from optic flow without involving additional scale information (e.g. landmarks or objects of known size in the scene) because optic flow fields are ambiguous with respect to scale. The speed of optic flow ( $\Phi$ ) that a moving observer experiences depends on the ego-velocity  $V$  and the distance ( $Z$ ) between objects and observer ( $\Phi \sim V/Z$ ). Hence, to judge the travel distances on the basis of optic flow fields one needs to know—or make assumptions—about  $Z$  to calculate the ego-velocity from optic flow. Bremmer and Lappe (1999) showed that it is possible in this case to discriminate the travel distances of two visually simulated self-motions. In this study human subjects had to indicate, which one of two visually simulated self-motions covered a larger distance. Subjects could correctly perform this discrimination if they made the assumption that the spatial environment was the same in the two motions, i.e. the distribution of all  $Z$  remained constant.

Observers could have used two strategies to discriminate travel distances in these experiments. The first uses directly the image motion on the retina. In this case observers would integrate the retinal image motion over time. This could be described as a 2-D hypothesis. The second possibility assumes a percept of the ego-motion in depth: the observer first extracts his or her ego-motion in depth from the retinal flow field and integrates this motion in depth over time. We call this the 3-D hypothesis. Frenz et al. (2003) performed experiments in which the optic flow field was altered by varying the translation velocity and the layout of the environment, i.e., the distribution of  $Z$ . If the change of the environment was not noticed the subjects made predictable errors in distance discrimination: they attributed the whole change of the optic flow stimulation to a change of the translation velocity, assuming that the distribution of all  $Z$  remained constant. If the subjects noticed the altered environmental structure, they could extract the amount of change in the flow field that resulted from the change of the environment from the amount that resulted from the change of translation velocity. These results support the 3-D hypothesis.

In the present work we ask whether human subjects can use their flow-derived 3-D percept of the self-motion to build up a static distance measure in the form of a ground interval or as a length estimate. In real environments length can be specified in meters or inches. The virtual scenes we use here are ambiguous, however, with respect to length and distance because no scales are available. The optic flow indicates environmental distances only up to a scale factor. Therefore, the indication of the travel distances in meters does not seem to be a reasonable measure. An unambiguous information about distance in our virtual ground-plane scene is the height above the ground of the observer's eyes (eye

height). As long as the simulated gaze direction is parallel to the ground plane, the distance of one eye height in front of the observer is always specified by the visual angle  $45^\circ$  downwards from the gaze direction. Therefore, the subjects do not have to know the depth structure of the scene or make assumptions about it to indicate the travel distance of one eye height. We use this measure to report distances of our experiments. Another advantage of using the measure of eye heights is the high reproducibility of the experiments. Only the ratio of translation velocity and distances between observers and environmental objects has to be considered to simulate identical flow fields.

We visually simulate ego-motion in different virtual environments with varying depth information. We ask human subjects to indicate the perceived travel distances in various ways. To build up a correct distance measure the subjects have to calibrate the optic flow on the basis of the environmental depth information and integrate the velocity of the ego-motion over time. But can human subjects also indicate the perceived travel distances in a stationary environment? In this case the judged distance has to be transferred to the environment in terms of a virtual distance. We instructed our subjects to indicate the perceived distances either in terms of a virtual interval on the ground, in terms of a second self-controlled visual motion, in multiple virtual eye heights, or by actively walking the same distance without visual information. In all cases we found a linear relationship of perceived to real distance but a consistent undershoot of absolute magnitude.

## 2. General methods

### 2.1. Apparatus

The stimuli were created in real-time on a Silicon Graphics Indigo2 workstation and presented on a  $120 \times 120$  cm back-projection screen (Dataframe, type CINEPLEX) using a CRT video projector (Electrohome ECP 4100). Resolution was  $1280 \times 1024$  pixel. Vertical refresh rate of the projector was locked to 72 Hz. Frame rate of the rendered images was either 36 Hz (textured stimuli) or 72 Hz (dot stimuli). The room was darkened and only illuminated by the stimuli. The subjects were positioned 0.6 m in front of the screen on a chair. The chair was adjusted in height so that the subject's physical eye height was 1.6 m above the ground. The field of view was  $90^\circ \times 90^\circ$ . Subjects were instructed to keep the head position as constant as possible. Head or eye movements were not recorded. The participants viewed the scene binocularly but without stereoscopic depth information. In a control study we found same results when the stimulus was viewed monocular or binocular with stereoscopic presentation.

2.2. Procedure

Each trial started with a visually simulated ego-motion sequence displaying forward motion over a ground plane. The velocity of the self-motion was either 0.38, 0.58, 0.96 or 1.15 eye heights/s. The duration of the simulation was 1.5, 2, 2.5 or 3 s. Accordingly, the travel distances varied between 0.58 and 3.46 eye heights. The viewing range was restricted to 11.54 eye heights in front of the observer in all scenes. Four of the travel distances (1.15, 1.44, 1.73, and 2.88 eye heights) were simulated with two different combinations of self-motion velocity and duration of the motion sequence. The virtual distances, translational velocities and simulation durations are listed in Table 1. We presented each of the 16 conditions (four velocities, four durations) 10 times in pseudorandomised order. At the end of the motion simulation two horizontal indicator lines appeared on the scene. One line (reference) was always presented 1.54 eye heights in front of the observer’s virtual position. The second line appeared 1.15 eye heights in front of the observer and was adjustable by moving a computer mouse. Both lines were positioned on the virtual ground level. The subject’s task was to indicate the virtually travel distance in terms of an interval placed on the ground plane.

We used this explicit interval adjustment task because in a pilot study which asked for egocentric distance judgements by adjusting a single line subjects used the lower edge of the projection screen as a reference point instead of their simulated position in the scene. In the data analysis an offset of one eye height occurred. Our explicit interval task avoids this confound. Additionally, we performed experiments in which we measured the perceived visual space in static virtual scenes (Frenz and Lappe, submitted for publication). In these experiments the subjects had to indicate the perceived size of

a depth interval on the ground plane with a second interval. The size of the depth interval and the distance of the reference interval from the observer was varied. In the range between 1.5 and 5 eye heights, which we used in the experiments described below, the correlation between simulated and indicated is nearly linearly. Therefore, we abstained from varying the position of the reference line in the experiments reported below. The time course of the stimulus presentation is illustrated in Fig. 1.

2.3. Data analysis

We plotted the indicated travel distances as a function of the distances of the self-motion simulation. Afterwards, we fitted linear regressions to the data points. To investigate whether the subjects possess an abstract distance gauge derived from optic flow, we calculated the correlation coefficients  $\rho$  between the simulated and indicated distances. If the subjects were able to build up an internal representation of the simulated distances, we expect high correlation coefficients. To assess the quality of distance indication we used the slope of the fitted linear regression to the data points. With accurate distance indication, the slopes of the fitted linear regressions should be 1 with an offset of 0. Slopes larger than 1 indicate overshoot of the simulated travel distance. Slopes smaller than 1 indicate undershoot of the simulated travel distances. We also calculated the 95% confidence interval for the slope of the linear regression (Draper & Smith, 1966). For this calculation

Table 1  
Parameters of the motion simulation: virtual travel distances, translation velocities, and duration of the self-motion simulations

Distance [eye heights]	Velocity [eye heights/s]	Duration [s]
0.58	0.38	1.5
0.77	0.38	2
0.96	0.38	2.5
1.15	0.38	3
0.87	0.58	1.5
1.15	0.58	2
1.44	0.58	2.5
1.73	0.58	3
1.44	0.96	1.5
1.92	0.96	2
2.40	0.96	2.5
2.88	0.96	3
1.73	1.15	1.5
2.30	1.15	2
2.88	1.15	2.5
3.46	1.15	3

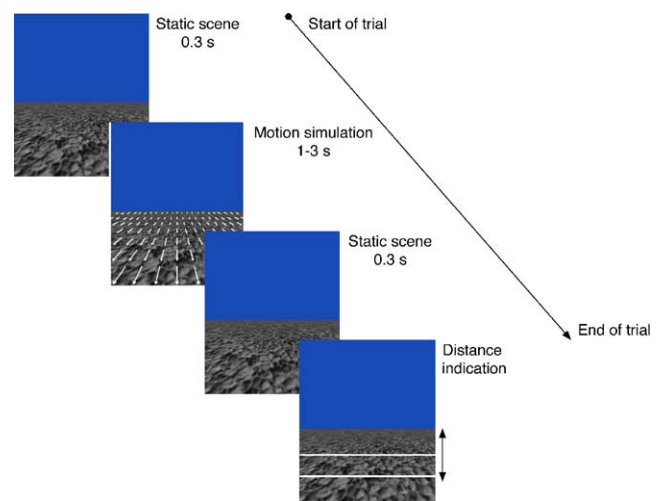


Fig. 1. The temporal sequence of events in each trial. In this example, the motion is simulated on the textured ground plane. The environment is first presented statically for 300 ms. Afterwards ego-motion is simulated for 1–3 s. The white arrows symbolise the flow field experienced by the observer. After the motion stopped, the static scene is again presented for 300 ms. Then the two indicator lines are presented in the static environment and the subject is allowed to adjust the interval between the indicator lines to reflect the distance travelled in the simulation.

we used the means of each subject and each condition. In the result sections we indicate the mean slopes and the range of the confidence interval. The offsets of the regression lines represent a constant error which may result from a range effect. As our main concern was to investigate the relationship between simulated and indicated distances these constant errors were left out from further analysis. As a further indication for an abstract distance gauge we investigated whether subjects perceived identical travel distances as identical when simulated with different combinations of forward velocity and motion duration. As described above, we simulated four travel distances each with two different forward velocities and motion duration. If the subjects used the perceived travel distances rather than a velocity or duration judgement, identical travel distances should be indicated identically.

## 2.4. Environments

### 2.4.1. Textured ground plane

We mapped a 3.08 eye height  $\times$  3.08 eye height texture pattern (Iris Performer type “gravel”) on a virtual ground plane (153.85 eye height  $\times$  153.85 eye height; Fig. 2A). To avoid recognition of identical texture objects between two successive trials we shifted the starting point of the simulation left- or rightward by a random amount. The textured ground plane provides ample static depth cues, contained in gradients of texture density and texture size towards the horizon (Cutting, 1997). It also provides dynamic depth cues in the motion sequence, most notably motion parallax and the change of size of texture elements as they approached the observer. It is also conceivable that the trajectories of the ground plane elements were used as a cue towards depth structure or travel distance. An overview of the included depth cues of this and the following scenes is given in Table 2. The mean luminance of the scene was 3.1 cd/m<sup>2</sup>.

### 2.5. Dot plane 1

Dot plane 1 consisted of 3300 white light points (Fig. 2B). First, these light points were positioned on a grating

Table 2  
Different depth cues contained in the four environments

	Density gradient	Change in size	Motion parallax	Trajectories
Textured ground plane	+	+	+	+
Dot plane 1	+	–	+	–
Dot plane 2	–	–	+	–

A plus marks the presence of a cue, a minus its absence. “Density gradient” refers to the increase of texture density towards the horizon. “Change of size” is the looming of objects as they approach the observer. “Motion parallax” is the scaling of visual velocity of an object with its distance from the observer. “Trajectories” means that objects can be tracked as they cross the screen.

every 0.77 eye heights within 19.23 eye heights in front of the observer and every 2.31 eye height to either side within a distance of 11.54 eye heights. Thereafter, the position of each light point was shifted randomly up to 1.92 eye heights to either side and forward/backwards. To reduce the depth information of the scene we limited the lifetime of the light points. With a probability of 10% each point would vanish and reappear randomly in the scene in each frame. With a frame rate of 72 Hz the mean lifetime of each dot was 139 ms. Therefore, on average 970 light points were visible on the screen. The limitation of the dot’s lifetime ensures that the subjects could not get information about the travel distance from trajectories of the light points. Additionally, the size and luminance of the light points remained constant during movement simulation, eliminating size change as a distance cue. Dynamic depth cues were provided by motion parallax. In the static scene, the gradient of texture density towards the horizon still served as depth cue. Frame rate was 72 Hz. Mean luminance was 2.0 cd/m<sup>2</sup>.

### 2.6. Dot plane 2

Dot plane 2 consisted of 150 white light points on a black background (Fig. 2C). The points were evenly distributed on the lower part on the screen. During movement simulation the dots moved as if they lay on a ground plane, i.e. they obeyed the pattern of motion

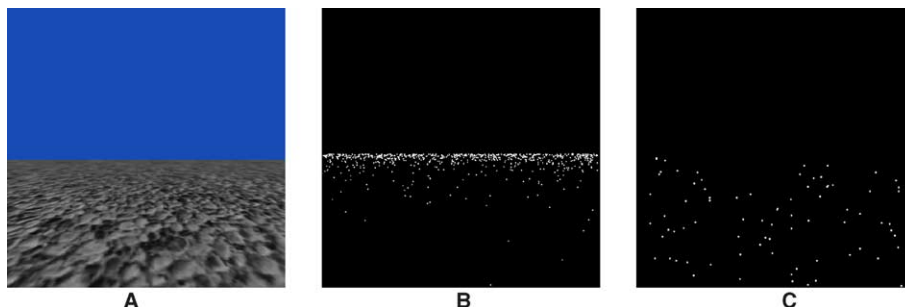


Fig. 2. Screenshots from the three environments: (A) textured ground plane; (B) dot plane 1; (C) dot plane 2. For a detailed description see text.

parallax. Without motion simulation the dot pattern provided no information about the distance between observer and light points or about the structure of the environment. Furthermore the lifetime of the light points was limited in the same way as described for dot plane 1. The mean luminance was  $0.6 \text{ cd/m}^2$ .

### 2.6.1. The virtual indicator lines

The indicator lines spread over 15.38 eye heights to both sides of the observer's virtual position and had a thickness of 2 pixels. The luminance and size of the light points remained constant regardless of the distance to the observer's virtual position.

The subjects controlled the movement of the adjustable line by moving a computer mouse. We used the vertical co-ordinates of the invisible mouse pointer position on the screen (ranging from 0 to 1024 pixel) and calculated the corresponding virtual position on the simulated environment (ranging from 0 to 11.54 eye heights). Therefore, changing the position of the mouse pointer by one pixel altered the position of the line by 0.01 eye heights on the virtual ground plane (11.54 eye heights/1024 pixel). The physical distance between the two lines in the virtual scene was calculated after the subject indicated the decision with a button press.

## 3. Experiment 1: Distance indication in a static scene

In the first experiment we wanted to investigate whether human subjects possess an abstract distance gauge derived from optic flow fields and whether they are able to indicate this estimate in a static scene.

### 3.1. Methods

We simulated the reference motion with four different velocities and four different simulation durations (see Table 1) resulting in 16 movement conditions. The subjects had to indicate the perceived travel distances in terms of a virtual interval on the ground. Indication was in the same virtual scene as the movement simulation. The textured ground plane and dot plane 1 were presented statically during the interval adjustment phase. In the case of dot plane 2, static presentation of the light points did not convey any depth information. Because of this lack of depth information in a static scene, distance estimation in terms of an interval was impossible. Therefore, dot plane 2 continuously simulated forward motion during adjustment of the interval. This forward motion simulation provided dynamic depth information in terms of motion parallax. The simulated velocity of this motion was the same as in the reference motion. To investigate the influence of this motion simulation on distance estimation, we performed control experiment 1. In this experiment the textured

ground plane served as the virtual environment but was presented moving also during the indication of the travel distance.

Each virtual environment was separately tested in a block of 160 trials. One block lasted approximately 20 min. We first run the experiment on the textured ground plane, then on dot plane 1, followed by the experiment on dot plane 2. To investigate whether or not the subjects showed a practice effect we repeated the experiment on the textured ground plane afterwards. In this control experiment 2, each condition was tested only five times to reduce the duration of the experiment.

Five subjects (24–30 years of age, three males and two females) participated in the experiments, including one author. Two subjects (*ps* and *jl*) had never before participated in psychophysical experiments.

### 3.2. Results

Fig. 3 shows the mean results over all subjects obtained in the experiment with motion simulation on the textured ground plane (upper left panel), dot plane 1 (upper right), and dot plane 2 (middle row). The results of both control experiments are illustrated in the bottom row of Fig. 3. The correlation coefficients between the indicated and simulated distances vary between 0.61 and 0.78 in the different experiments and therefore indicate that the subjects possess an abstract distance gauge derived from optic flow fields. The fitted linear regressions were an accurate description of the data points (all  $r^2$  above 0.92). The slopes were 0.51 ( $\pm 0.06$ ), 0.67 ( $\pm 0.14$ ), 0.76 ( $\pm 0.16$ ), 0.72 ( $\pm 0.1$ ), and 0.79 ( $\pm 0.08$ ) for the textured ground plane, dot plane 1, dot plane 2, control experiment 1, and control experiment 2, respectively. The values in brackets show the width of the 95% confidence interval of the calculated slopes (see Section 2.3). Subjects always undershot the simulated travel distances but the undershoot depended on the virtual environment. A two-way-ANOVA showed that the slopes of the fitted regressions obtained with the experiment on the textured ground plane, dot plane 1, and dot plane 2 were significant different ( $p < 0.05$ ). The results obtained with control experiment 2 (repetition of the textured ground plane) showed a significant steeper slope of the fitted linear regression compared to the first experiments on the textured ground plane (two-way-ANOVA,  $p < 0.05$ ). Therefore, the reproduction of the first experiment reduced the error in distance estimation and the subjects showed a practise effect. The slope of the fitted linear regression to the data obtained with control experiment 1 (motion simulation during distance indication on the textured ground plane) was not significantly different from that obtained with control experiment 2 (two-way-ANOVA,  $p > 0.05$ ). We used the results of control experiment 2 as reference data for the textured ground plane without motion

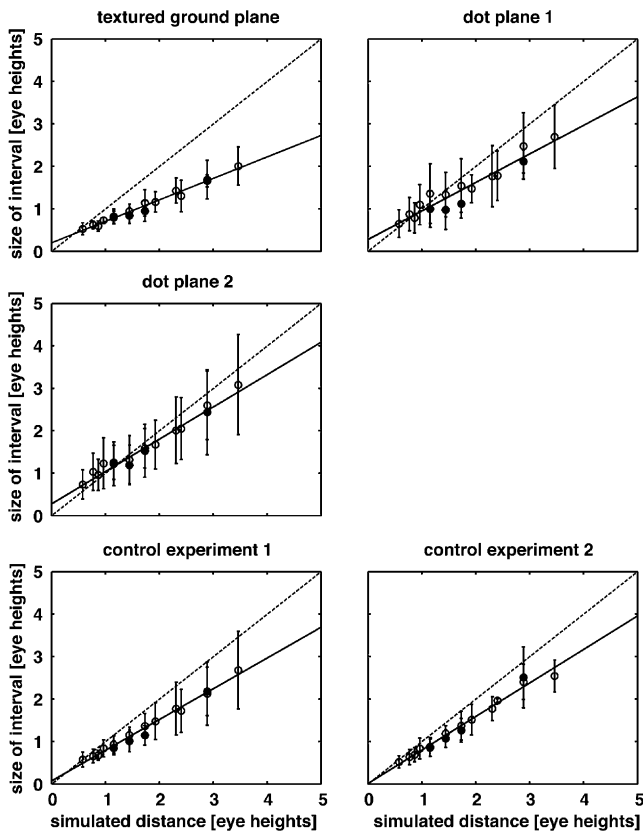


Fig. 3. Comparison of the pooled results of all subjects between the virtual environments. The mean size of the adjusted interval is plotted as a function of the simulated motion distance. The upper left panel shows the results of the experiment on the textured ground plane. The upper right panel and the middle row show the results on dot plane 1 and dot plane 2, respectively. In the bottom row the results of the control experiments are shown. Each circle corresponds to the mean perceived distance across all subjects. Error bars indicate the standard deviation between subjects. Filled circles indicate results of self-motions simulated with high translation velocities and short simulation durations. Open circles correspond to the same simulated travel distances obtained with lower translation velocities and longer simulation durations. The solid line is the fitted regression. The dashed line indicates perfect responses.

simulation during distance indication because of the observed practice effect. Thus the motion simulation during distance indication did not influence the subject's distance judgement.

#### 4. Experiment 2: Under- vs. overshoot

The results of experiment 1 clearly showed that human subjects *undershot* the simulated travel distances. A different study investigating human subject's ability to estimate travel distances on the basis of optic flow fields was performed by Redlick et al. (2001). They simulated a virtual corridor in the dimensions of a real corridor familiar to the subjects (2 m width, 2.5 m height and 50 m length). The walls of the virtual corridor were

painted with vertical stripes that changed their colour every two seconds to prevent subjects from tracking particular bars. The virtual scene was presented in a head mounted display ( $84^\circ \times 65^\circ$  viewing angle) and was calibrated to the real world corridor. The experimental procedure started with the presentation of a virtual movement target (width = 2 m, height = 2.5 m) in a distance either 4, 8, 16 or 32 m in front of the observer's position. The movement target then disappeared and motion in the forward direction was simulated with constant velocities of 0.4, 0.8, 1.6, 3.2 or 6.4 m/s. The subject's task was to report in terms of a button press when they thought they had reached the position of the movement target. In contrast to our findings, Redlick et al. (2001) found an *overshoot* of the travel distances. With the following experiments we wanted to investigate the reason for this discrepancy.

#### 4.1. Methods

There were three main differences between the experiments performed by Redlick et al. (2001) and ours: First, Redlick et al. used a head mounted display to present the virtual scene. The orientation of the display was tracked and the scene rendered accordingly. We used a projection screen with constant gaze simulation. Second, in our experiment 1 the reference distances were first presented in terms of a simulated self-motion and afterwards indicated in a static scene. In the experiments of Redlick et al. the reference distances were first presented in a static scene in terms of a motion goal and afterwards indicated in terms of a self-motion simulation. Third, Redlick et al. used large distances and long durations for the motion simulation. The distances between the movement targets and the observer ranged from 4 to 32 m (2.5–20 eye heights with an assumed eye height of 1.6 m) and were approached with different self-motion velocities (ranging from 0.25 to 4 eye heights/s). In our study the travel distances ranged from 0.58 to 3.46 eye heights (see Table 1).

To test for the influence of the differences in the type of projection (head mounted display vs. screen projection) and the different simulation of the reference distance (static movement target vs. optic flow) we reproduced the experiments of Redlick et al. with our experimental set-up in experiment 2A. We therefore first simulated the movement target in terms of an indicator line. Then the line vanished and we simulated motion in the forward direction. Speed, duration, and target distances were taken from Redlick et al. (2001). The subject's task was to press a mouse button when they thought they reached the position of the movement target. To investigate whether the size of simulated travel distance and the translation velocities were the reason for the difference in the observed error in distance estimation (over- vs. undershoot), we performed experiment

2B. In this experiment we used the parameters of experiment 1 of the present work for motion simulation. Thus, experiment 2B (first static distance, than motion simulation) differed from experiment 1 (first motion simulation, than static distance) only in the temporal sequence of the procedure. Four subjects participated in this experiments. All subjects previously participated in experiment 1.

4.2. Results of experiment 2A

Fig. 4 shows the results of experiment 2A, the reproduction of the experiment of Redlick et al. (2001). We fitted linear regressions to the data points for each subject and translation velocity. If subjects correctly perceived and indicated the travel distances the linear regression should have a slope of 1 (black dashed line in Fig. 4). Slopes smaller than 1 indicate an under-

slopes larger 1 an overshoot of the simulated travel distance.

The correlation coefficients were above 0.93 for three of four subjects. These subjects were able to transfer the distance perceived in a static scene to an estimate of travel distance based on virtual self-motion information. For subject *ps*, the linear regressions were not an accurate description of the data ( $p > 0.1$  for all velocities), even though the slopes followed the same trend as those of the other subjects. Therefore, we omitted the results of the subject *ps* from further analysis.

The linear regressions fitted to the data of the three remaining subjects showed p-values below 0.05 and formed therefore an adequate description of the data. All slopes were greater than 1, indicating an overshoot of the travel distances. Fig. 4 shows that with increasing translation velocity the slopes of the fitted linear regressions decreased. A two-way-ANOVA showed that this decrease of the slopes is significant for subjects *jl* and *kg* ( $p < 0.05$ ) but not for subject *hf* ( $p = 0.5$ ).

The pooled data of the three subjects *hf*, *jl* and *ks* are shown in Fig. 4 “Pooled data”. Depending on the translation velocity the slopes of the regressions varied between 2.08 and 1.41. The differences between the slopes of the regression lines were significant (two-way-ANOVA,  $p < 0.05$ ). Thus, increasing translation velocity of the simulated self-motion reduced the distance overshoot up to a level of 40%. The distance overshoot corresponded to the results of Redlick et al. (2001).

In this experiment, variations of translation velocity are coupled with variations of the duration of the motion simulation. Subjects might have misjudged the simulation duration or the translation velocity. We plotted the mean ratios of the indicated and simulated travel distances as a function of the duration required for a correct distance indication in Fig. 5. The duration required for a correct distance indication is the time the subjects had to wait before pressing the button during the self-motion simulation to indicate the simulated distance without errors and undershoot. Note that an overshoot of the distance travelled during the motion simulation (this is the indicated distance!) means that the subjects associate a short distance of the motion simulation with a larger distance in a static scene. The different translation velocities in Fig. 5 are line encoded corresponding to Fig. 4. If the subjects’ distance estimate is independent of the simulation duration, the indices should remain constant. Fig. 5 shows that the indices decreased with increasing duration for each self-motion velocity. Thus, subjects increasingly overshoot their travel distances with increasing duration of the self-motion. Differences in distance indication depending on the used translation velocity were significant (one-way-ANOVA,  $p < 0.05$ ) for the pooled data of the subjects *hf*, *jl*, and *kg*.

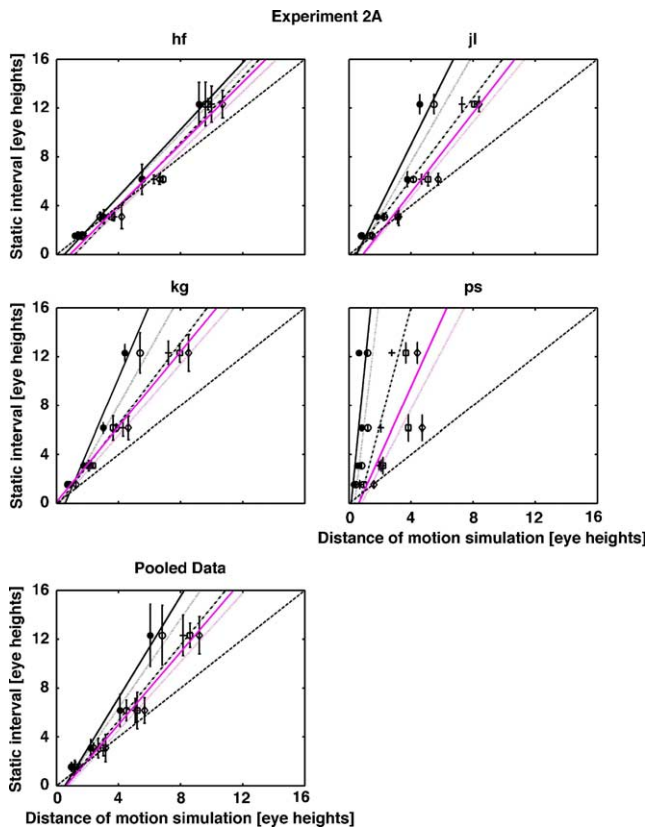


Fig. 4. Single subject’s results and pooled data obtained with experiment 2A. The travel distances are plotted as a function of the presented distances between the movement target and the virtual position of the observer. Each symbol shows the mean of five trials for the single subject results. In the pooled data, each symbol represents the mean of 15 trials. Error bars show the standard deviation. The translation velocities are: filled circles with solid black line: 0.15 eye heights/s, open circles with dotted line: 0.31 eye heights/s, cross with slash-dot line: 0.62 eye heights/s, squares with solid grey line: 1.23 eye heights/s and diamonds with dotted grey line: 2.46 eye heights/s. The dashed black line visualises accurate distance estimation, the other lines are the fitted linear regressions to the data.

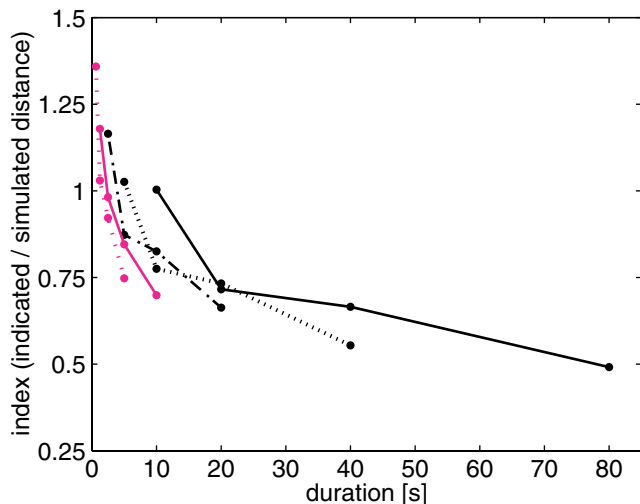


Fig. 5. Influence of the simulation duration on distance estimation. The mean ratios of the indicated and simulated distances are plotted as a function of the necessary true duration of the motion simulation for the correct distance. Translation velocities are: solid black line: 0.15 eye heights/s, dotted black line: 0.31 eye heights/s, slash-dotted line: 0.62 eye heights/s, solid grey line: 1.23 eye heights/s and dotted grey line: 2.46 eye heights/s.

#### 4.3. Results of experiment 2B

The aim of experiment 2B was to investigate whether the translation velocity and the range over which the movement targets were presented had an effect on distance estimation. Secondly, we wanted to compare the presentation of a reference distances before movement simulation with interval adjustment after movement simulation in experiment 1.

The slopes of the fitted linear regressions (Fig. 6) varied among the four subjects: for subjects *hf* and *jl* the slopes were below the correct value of 1 (0.73 ( $\pm 0.07$ ) and 0.6 ( $\pm 0.04$ )). This indicated an undershoot of the traversed distance of the self-motion simulation. For subject *kg* the slope was 0.98 ( $\pm 0.07$ ) indicating correct performance, and for subject *ps* it was 1.11 ( $\pm 0.25$ ), indicating an overshoot of the covered distance of the self-motion simulation. The slope of the pooled data of all subjects was 0.64 ( $\pm 0.1$ ), indicating an average undershoot of the simulated self-motion by about 36%. Despite the individual differences in slope and offset the main effect that slopes are steeper in experiment 2A than in experiment 2B is consistent for each subject.

Subjects indicated identical travel distances identically independent of the translation velocity of the self-motion simulation. This was also the case for the pooled data.

#### 4.4. Discussion

In experiment 2A, which used the parameters of Redlick et al., an overshoot by about 40% occurred. This re-

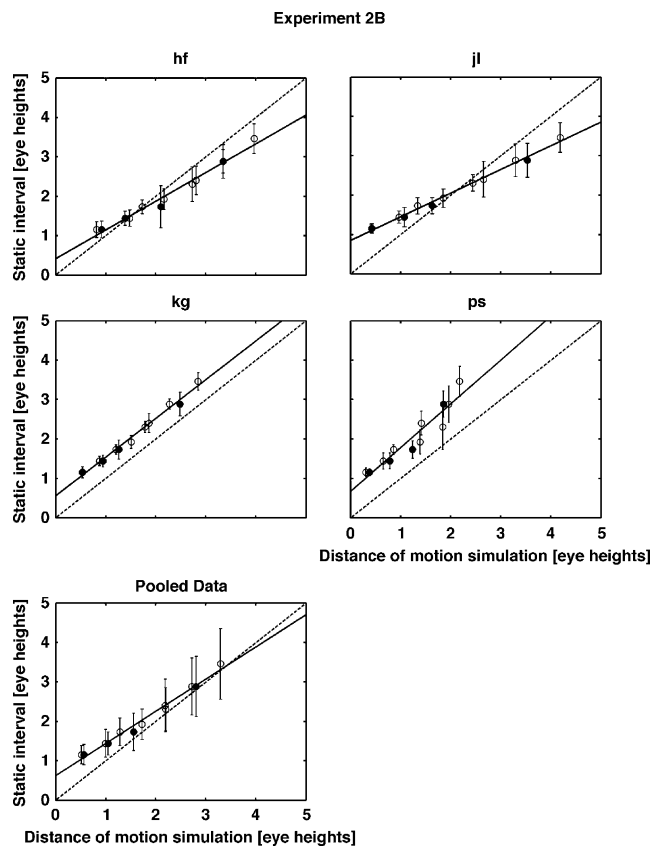


Fig. 6. Results of experiment 2B. The travel distances are plotted as a function of the presented distances between the movement target and the virtual position of the observer. Each circle in the single subject plot shows the mean over ten trials. Error bars indicate the standard deviation. In the pooled data, each circle shows the mean over 40 trials. Filled circles indicate data obtained with high translation velocities and short simulation durations. Open circles correspond to the same simulated travel distances obtained with lower translation velocities and longer simulation durations. The solid line corresponds to the fitted linear regression. The dashed line indicates perfect performance.

sult corresponds to the distance estimation error Redlick et al. (2001) described. Therefore, the differences in presentation mode between our study and that of Redlick et al. were not critical for the distance estimation task.

Experiment 2B used the same experimental paradigm (first movement target in a static scene, afterwards distance indication in terms of motion simulation) as experiment 2A but with shorter virtual distances of the motion target to the observer's virtual position and shorter simulation durations. Distances were now undershot by about 36%. This amount of undershoot corresponds to the results obtained in experiment 1 (about 27%). Thus both experimental paradigms gave the same the error of distance estimation.

The differences between experiments 2A and 2B concerned only the simulated travel distances and self-motion velocities. Depending on the duration for a correct distance indication, either an overshoot (long duration) or an undershoot (short duration) of the travel



distance was observed. Redlick and co-workers also described this relationship between duration of the self-motion simulation and error in distance estimation: with increasing translation velocity the error in distance estimation decreased, although this decrease of the error was not significant. A constant acceleration of the simulated self-motion in the Redlick study led to systematic changes of the indicated distance: lower accelerations led to higher overshoot of the travel distance than higher acceleration. Or, in other words, longer durations of the self-motion simulation for a correct distance indication led to stronger overshoot of the travel distance than shorter durations of the self-motion simulation.

The experiment of Redlick et al. (2001) and our experiments 2A/B clearly show that the duration of the motion simulation has a strong influence of the observed error in distance estimation derived from optic flow. Possible explanations for the influence of the motion duration will be discussed in the general discussion.

### 5. Experiment 3: Different modes of distance indication

The previously described experiments could not explain why the subjects undershot the simulated travel distances. A possible explanation for the occurrence of the undershoot is the way in which subjects indicated the perceived traversed distances. In experiment 1 the perceived distances of self-motion simulations were indicated in terms of a virtual interval on the ground. In experiments 2A/B the subjects indicated the perceived distance to a movement goal in terms of a self-motion. Both types of distance indication led to an undershoot of the travel distance for short durations.

In the following experiments we test different types of distance indication: active controlling of the motion simulation, indicating the travel distance in multiple eye heights, and active walking of the perceived distance without visual information.

#### 5.1. Experiment 3A: Active reproduction of the motion simulation

Bremmer and Lappe (1999) investigated distance reproduction with active control of a visually simulated self-motion. They instructed human subjects to indicate the perceived distance of a visually simulated self-motion in terms of a second self-motion simulation. The subjects actively controlled the translation velocity of the second self-motion with a force transducer joystick. Bremmer and Lappe found accurate reproduction of the reference distance. In experiment 3A we now ask whether or not such an active reproduction of the travel distance can improve the subject's ability to indicate the perceived travel distance in a static scene.

#### 5.1.1. Methods

We used the experimental set-up and parameters for the motion simulation as described in Section 2. After the reference motion simulation ended, the subject's task was to virtually travel the same distance covered by the reference motion again in the virtual environment. To this end subjects could control their translation velocity in the forward direction by varying the pressure on a force detector (SpaceBall 3003, Spacotec, IMC). When they felt that they had travelled the same distance subjects indicated the end of the motion simulation with a button press. After the active reproduction of the perceived distance the subjects had to indicate the travel distance again, this time in terms of a virtual interval on the ground as in experiment 1. The procedure of the adjustment of the interval was the same as described in Section 2.2. The temporal sequence of the stimuli is illustrated in Fig. 7. To ensure that subjects were familiar with the use of the force detector they were allowed to steer through the virtual environment without any instructions prior to the data collection. The relationship between the force on the SpaceBall and the corresponding translation velocity was adjusted until subjects felt comfortable in the control of the motion simulation. The textured ground plane served as the virtual environment. Four subjects participated in this experiment. The experiment was performed in two blocks. In each block each condition was tested five times.

#### 5.1.2. Results

In Fig. 8 the results for the single subjects and for the pooled data of all subjects obtained with experiment 3A

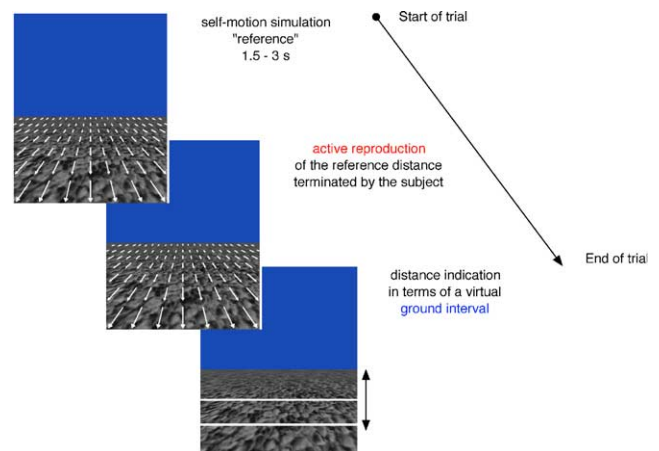


Fig. 7. The temporal sequence of events in experiment 3A. All self-motions are simulated on the textured ground plane. First, the reference self-motion is simulated for 1.5–3 s. The white arrows symbolise the flow field experienced by the observer. Afterwards the subjects actively reproduce the distance of the reference motion with the help of a control device in a second self-motion simulation. At the end of each trial the participants indicate the traversed distance of the reference motion in terms of a virtual interval on the ground as in experiment 1.

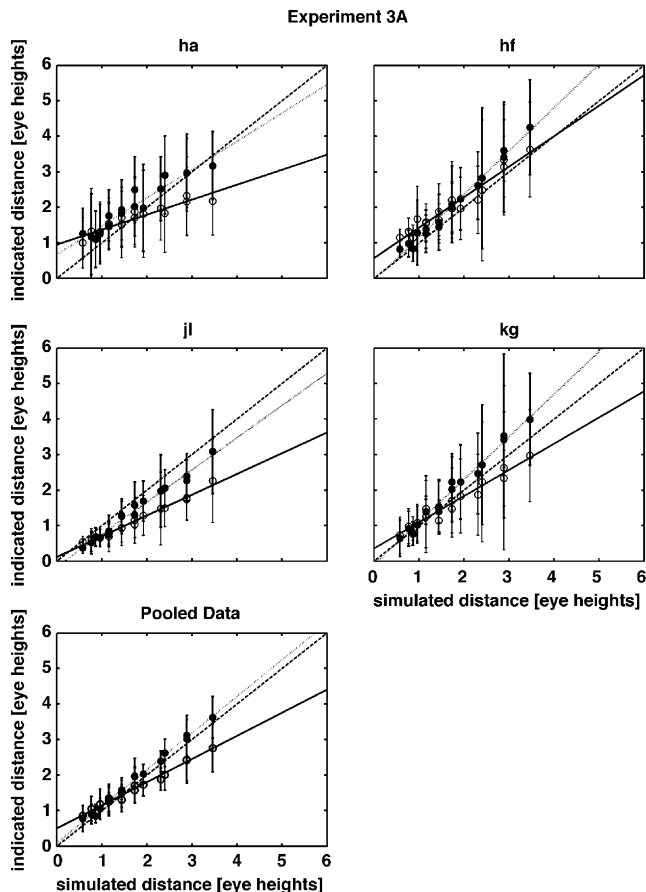


Fig. 8. Results of experiment 3A. The indicated distances are plotted as a function of the simulated travel distances. In the single subject plots, each circle marks the mean of 10 trials. In the plot of the pooled data, each circle marks the mean over 40 trials. The error bars are standard deviations. The solid lines correspond to the fitted linear regressions to the data points. The dashed lines indicate perfect distance estimation. The results obtained with different ways of indicating the traversed distance were symbol encoded: filled circles with dotted line: active reproduction of the traversed distance in terms of a second self-motion simulation; open circles with solid line: distance indication in terms of a virtual interval on the ground.

are illustrated. Filled data points illustrate the results obtained with active reproduction of the motion simulation, open data points correspond to results obtained with interval adjustment. All fitted linear regressions were adequate descriptions of the data.

**5.1.2.1. Results: Active reproduction.** The correlation coefficients between the actively reproduced distances and the simulated distances of the self-motion varied between 0.72 and 0.95 for the single subject's results. This means that all participants were able to indicate the perceived distance of the self-motion simulation with an actively controlled second motion simulation. The slopes of the linear regression fitted to the data obtained with the active reproduction of the reference distance varied between 0.8 and 1.2. The slope of the pooled data was 1.03 ( $\pm 0.05$  width of the 95% confidence interval) and

therefore showed accurate distance estimation. The indicated and simulated distances of the reference motions were highly correlated ( $\rho = 0.85$ ). Identical travel distances (1.15, 1.44, 1.73, and 2.88 eye heights) were indicated by the subjects with same interval sizes (see Fig. 8, filled circles of the four distances mentioned above). Consistent with Bremmer and Lappe (1999) our subjects were thus able to indicate the travel distances in terms of a second self-motion rather accurately.

**5.1.2.2. Results: Interval adjustment.** Also in the interval task, correlation coefficients between the perceived distances indicated in terms of the virtual interval on the ground and the simulated travel distance were high among subjects (0.69–0.91). For the pooled data the correlation coefficient was 0.73. Also the single subject data as well as the pooled data showed no difference in distance indication of identical travel distances simulated with different translation velocities and motion duration. The slopes of the regression, however, were much lower than for the reproduction task. Among the single subject results slopes varied between 0.42 and 0.86. Thus the traversed distances of the reference motion simulation were undershot by 14–58%. The slope of the linear regression fitted to the data of all subjects was 0.65 ( $\pm 0.05$  width of the 95% confidence interval). On average, subjects showed therefore an undershoot of the travel distance of 35%. The slope was not significant different from the slope of the regression, obtained on the textured ground plane in experiment 1, which did not involve active reproduction of the travel distance (two-way-ANOVA,  $p > 0.05$ ).

## 5.2. Experiment 3B: Distance indication in terms of eye heights

### 5.2.1. Methods

In this experiment we asked four subjects to provide metric judgements about the travel distances in terms of eye heights. We created and presented the virtual environment (textured ground plane) as described in the general methods (see Section 2). We simulated distances of 1, 2, 3, and 4 eye heights. Subjects had no information about the maximum travel distance. They were instructed to use the range from 1 to 9 on a computer keyboard for their distance estimation. The translation velocity was pseudorandomly chosen between 0.38 and 1.92 eye heights/s in each trial. The simulation duration was given by the ratio of the travel distance and the translation velocity. We presented each of the four travel distances 10 times.

### 5.2.2. Results

In Fig. 9 the indicated distances are plotted as a function of the simulated travel distances. All regressions were an adequate description of the data ( $p < 0.05$ ).

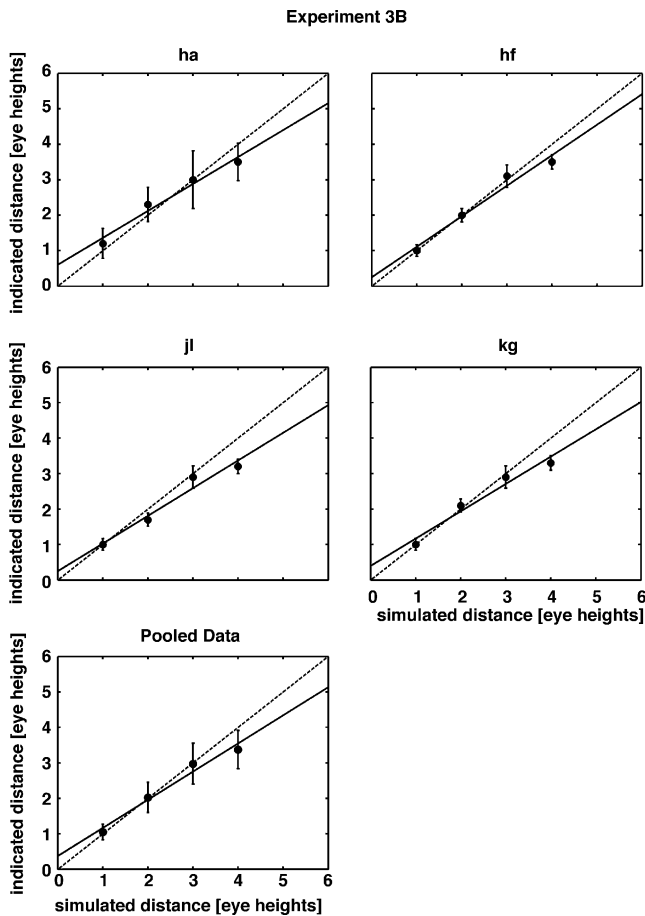


Fig. 9. Results of experiment 3B. Indicated travel distances in terms of eye heights are plotted as a function of the simulated travel distance. Each circle marks the mean over 10 trials for the single subject plots and the mean over 40 trials in the plot of the pooled data. The error bars indicate standard deviation. The solid lines are the fitted linear regressions to the data. The dashed lines show hypothetical distance estimation data without errors.

For the single subjects correlation coefficients varied between 0.83 and 0.94. The pooled data of all subjects showed a correlation coefficient of 0.88. This means that the subjects were able to indicate their distance judgement in terms of eye heights.

The slopes of the regressions varied between 0.76 and 0.86 for individual subjects. The fitted regression to the pooled data had a slope of 0.79 ( $\pm 0.07$  width of the 95% confidence interval; note, however, that three of the four data points fall on a slope of one). Thus, subjects on average undershot the traversed distances of the self-motions by 21%. This result corresponded to the results obtained in experiment 1, 2B, 3A and 3B of this work.

### 5.3. Experiment 3C: Indication of travel distances by active walking

#### 5.3.1. Methods

In this experiment the subjects viewed the virtual environment (textured ground plane) on a head

mounted display (Sony Glastron PLM-S700E; 30° horizontal field of view; vertical refresh rate 60 Hz) with a spatial resolution of 800 × 600 pixel. The participants stood at one end of a 7 m long and 1.5 m wide catwalk in a totally dark room. At the beginning of each trial we visually simulated a self-motion. At the end of the self-motion simulation, the subjects had to walk the same distance as the motion simulation on the catwalk. During the walking the head mounted display turned black. Thus, the subjects had no visual feedback about the self-motion. To avoid injuries (the catwalk was 0.6 m above the ground) we spanned a rope over the catwalk serving as a hand guide for the subjects. Half a meter before the end of the catwalk a knot (which the subjects could feel in their hands) was fixed to the rope indicating the end of the catwalk. No subject walked this far. When the subjects thought they walked the same distance as traversed during the self-motion simulation they gave a short verbal indication. We marked the position of the heel of the nearest foot to the starting point. The walking distance was measured at the end of the experiment with a yardstick. We measured the eye height of each subject before the experiment and adjusted the virtual horizon corresponding to this eye height. Nine distances (between 0.58 and 2.40 eye heights) were tested. One of them (1.44 eye heights) was simulated with two different combinations of translation velocities and simulation duration. We pseudorandomly presented each distance twice in a block of trials. Each of the three subjects participated in three blocks.

#### 5.3.2. Results

The walked distances are plotted as a function of the simulated travel distances in Fig. 10. To test whether the subjects were able to indicate the perceived distances of the self-motion simulations in terms of active walking we first calculated the correlation coefficients between the indicated and simulated distances. For the single subject's results the correlation coefficients varied between 0.83 and 0.86. For the pooled data of all subjects the correlation coefficient was 0.84. Thus subjects were able to indicate the perceived travel distances of a visually simulated self-motion in terms of actively walking the same distance. But again, the subjects undershot the traversed distances of the self-motion simulations: the slopes of the fitted linear regressions to the data of the single subjects varied between 0.57 and 0.84. The slope of the regression fitted to the pooled data of all subjects was 0.71 ( $\pm 0.07$  width of the 95% confidence interval). Thus, subjects undershot the traversed distances of the self-motion simulation by 29%. Two subjects indicated the same simulated travel distance of 1.44 eye heights with different combinations of translation velocities and simulation duration significantly different (Wilcoxon-signed-rank test,  $p < 0.05$ ). Also for the pooled data difference between the indicated

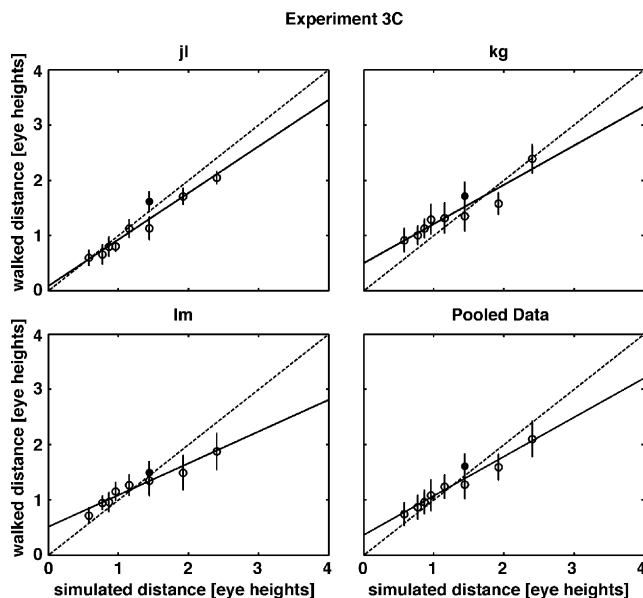


Fig. 10. Results of experiment 3C. The walked distances are plotted as a function of the simulated distances. Each circle shows the means over 6 trials in the single subject plots and the mean over 18 trials in the “pooled data” plot. The error bars show standard deviation. Filled circles indicate data obtained with higher translation velocity (0.96 eye heights/s) and shorter simulation duration (1.5 s) than the corresponding open circles (0.58 eye heights/s and 2.5 s). In both cases travel distance was 1.44 eye heights. The solid lines correspond to the fitted linear regressions to the data. The dashed lines indicate hypothetical data of exact distance estimation without errors.

distances of the same simulated travel distance was significant (Wilcoxon-signed-rank test,  $p < 0.05$ ). In all cases the faster translation velocities of the same travel distance resulted in indication of larger distances.

#### 5.4. Discussion

Even after subjects accurately reproduced the distance of the simulated self-motion with a second motion in experiment 3A the error in interval adjustment on the ground remained. Bremmer and Lappe (1999) reported that the subjects reproduced the velocity profile of the simulated self-motion to reproduce the travel distance. The same strategy was used when subjects perceived and reproduced the travel distances of real self-motions with only proprioceptive and vestibular information about the movement (Berthoz, Israel, Georges-Francois, Grasso, & Tsuzuku, 1995). This strategy cannot be used for the interval adjustment task in the second part of experiment 3A. Here, the subjects undershoot the travel distances. We therefore conclude that the subjects either did not build up an internal representation of the travel distance when they indicated it in terms of an actively controlled self-motion or they misperceived the presented optic flow field. A misperception of the flow field would occur in both motion simulation and therefore lead to the same travel distances in the motion condi-

tions, whereas the indication of the travel distances in a static scene would lead to errors in distance indication.

In experiment 3B we instructed the subjects to indicate the travel distance of the self-motion simulation in terms of eye heights. As described in the introduction, our simulation are not ambiguous with respect to the travel distances of a self-motion in terms of eye heights. Thus, if the error in distance estimation was based on a misperception of the environmental structure, distance indication in terms of eye height should be unaffected by this misperception. But also this way of indicating the traversed distance led to the same undershoot of the travel distance. The error in distance undershoot corresponded to that obtained in the first experiment.

In experiment 3C, subjects actively walked the perceived distances of the motion simulation after they saw the simulation. Again, the subjects undershot the distances of the self-motion simulations. The error was similar to that obtained with indication of the travel distance in terms of a virtual interval (experiment 1). Loomis, Da Silva, Fujita, and Fukusima (1992) also asked subjects to blindfoldedly walk a perceived distance. Similar to Redlick et al. (2001), they first presented the goal distance by a static target in the scene rather than a movement simulation. Loomis et al. found only small errors in the accuracy of the reproduced distances. This indicates that blindfold walking can be an accurate method to indicate perceived distance. The most important difference between the two mentioned studies is the presentation type of the reference distance: the presentation in a static scene in the Loomis et al. study vs. optic flow simulation in a dynamic scene in the experiments of this work. Thus, the undershoot observed in experiment 3C must result from the distance estimate from the motion estimation. In conclusion, the results of experiments 3A-C suggest that a mispercept of the visually simulated self-motion, not the task used for distance indication, is the source of the observed error in distance estimation.

## 6. General discussion

In previous work we asked subjects to judge distances derived from optic flow fields by discriminating travel distances of two visually simulated self-motions (Bremmer & Lappe, 1999; Frenz et al., 2003). This method was chosen, because optic flow fields are ambiguous with respect to travel distances if no scales are displayed in the scene. For a ground plane environment the optic flow becomes non-ambiguous in terms of eye height, even if a direct translation into meters is still subject to a scale factor. This allows to ask whether optic flow derived distance measures can be related to intervals on the ground plane. Our results show that subjects can use an ego-motion perception derived from optic flow over a ground plane to indicate the travel distances

also in a static interval (experiments 1, 3A, 3B) or with actively walking without visual information (experiment 3C). The indicated distances were linearly correlated with the simulated travel distances. This linear correlation is consistent with many studies that investigated motion based distance estimation in other modalities, such as walking the distance that a reference self-motion covered (Kearns, Warren, Duchon, & Tarr, 2002; Loomis et al., 1993; Witmer & Kline, 1998), steer on a mobile robot using vestibular information (Berthoz et al., 1995), or to navigate in virtual environments (Bremmer & Lappe, 1999; Kearns et al., 2002; Peruch, May, & Wartenberg, 1997; Redlick et al., 2001; Sun et al., 2004; Witmer & Kline, 1998). In two of these studies (Berthoz et al., 1995; Bremmer & Lappe, 1999) the authors investigated the subject's strategy for correct distance indication. They showed that subjects reproduced the velocity profile of the reference motion. This strategy does not necessarily involve an internal representation of the travel distance. In our present study, an internal distance gauge is essential as optic flow was only available during the presentation of the reference distance but absent during distance indication. From the present data we conclude that human subjects do not only have a representation of the velocity profile of previous self-motions but possess an abstract distance measure.

Although statically indicated distances were linearly correlated with travel distances the indicated distances did not match the true travel distances. In most of our experiments, perceived travel distance was undershot by 20–36%. As experiments 2A and 2B showed, the duration of the motion simulation has a strong influence on the error in distance perception. Short durations up to 3 s as used in experiment 1, 2B and 3 result in an *undershoot* of the perceived travel distance. Longer durations up to 80 s as used by Redlick et al. (2001) in experiment 2A result in an *overshoot* of the perceived travel distance. The influence of travel duration on distance perception was also studied in the experiments of Witmer and Kline (1998). They observed that distances between ca. 1.9 and 53 eye heights in virtual environments were judged with less error when traversed with lower translation velocities (0.54 eye heights/s) compared to higher velocities (1.07 eye heights/s). In both cases the distances were undershot, however. The authors suggested that subjects associate long travel durations obtained with slow velocities with larger distances. Another possibility is that the duration is misperceived such that short duration are perceived longer and long durations shorter.

The question why travel distances were misperceived in virtual environments cannot be fully answered from the present experiments. However, we can exclude a number of possibilities. Distance undershoot is not dependent on the way subjects indicate the perceived

distances (experiment 3), the projection arrangement or the depth cues in the virtual scene (experiment 1) or the presentation sequence of the stimuli (experiment 2). Could the distance undershoot be related to a misperception of visual space in general? Perceived visual space in static scenes, pictures, or virtual environments is increasingly compressed with increasing distance (for example: Beusmans, 1998; Cuijpers, Kappers, & Koenderink, 2000, 2002; Foley, 1980; Foley, Ribeiro-Filho, & Da Silva, 2004; Frenz & Lappe, submitted for publication; Indow, 1991; Wagner, 1985). The correlation between physical and perceived distance in static scenes is non-linear. In a separate study (Frenz and Lappe, submitted for publication) we investigated perceived distance in the static scenes used. We found that compressed space perception cannot be the reason for the undershoot observed in the present study because perceived distances in the range used here were essentially uncompressed. Therefore, the misperception of the travel distances cannot be fully explained by a misperception of visual space. However, as the distances used by Redlick et al. (2001) were much larger than the distances we used, and consequently related to stronger compression of the perceived visual space, the overshoot of perceived travel distance in that case may be influenced by an undershoot of the perceived distance to the movement target in the static scene. On the other hand, the decrease of the error in distance judgement with decreasing motion duration shows that time effects are also important.

Distance misjudgement may also result from erroneous perception of simulated translation velocity. Driving simulator studies have shown that both speed underestimation (e.g. Törnros (1998)) and speed overestimation (e.g. Godley, Triggs, & Fildes, 2002) in virtual scenes can occur when driving behaviour in real cars and driving simulators is compared. But a misperception of the translation speed is not in line with our finding that same travel distances are indicated identically, independent of the translation velocity and motion duration. For long trial durations, however, motion adaptation might lead to a decrease of perceived speed which may contribute to the observed overshoot of the distance in that case.

Under natural conditions humans can access more information than visual to control self-motion. In particular, proprioceptive and vestibular information are important. The combinations of these signals might lead to a more veridical estimate of travel distance. This was indeed found by Harris, Jenkin, and Zikovitz (2000) and Kearns et al. (2002) who concluded that vestibular/proprioceptive information can dominate over optic flow. Sun et al. (2004), on the other hand, suggested that visual information plays a dominant role in distance estimation when coupled with proprioceptive (but not vestibular) information. In their experiments subjects

had to ride a stationary bicycle to receive proprioceptive information while their self-motion was simulated visually. Relation between the two signals was varied. Sun et al. found that although the proprioceptive information was not used for distance estimation directly it facilitated the perception of the motion. Presumably, the integration of the different sources of information depends on the reliability of the individual signals in the respective context.

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