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Liquidity crisis detection: An application of log-periodic power law structures to default prediction

Jan Henrik Wosnitza [a,*], Cornelia Denz [b]

[a] Institute of Business Administration, University of Muenster, Leonardo-Campus 1, 48149 Muenster, Germany
[b] Institute of Applied Physics, Center for Nonlinear Science, University of Muenster, Correnssstraße 2, 48149 Muenster, Germany

HIGHLIGHTS

- This study provides a quantitative description of the mechanism behind bank runs.
- We diagnosed the presence of log-periodic patterns in CDS spreads.
- We investigated univariate classification performances of log-periodic parameters.
- These parameters appear to characterize investor behavior.
- Further, they enable us to draw conclusions of banks' refinancing options.

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ABSTRACT

We employ the log-periodic power law (LPPL) to analyze the late-2000 financial crisis from the perspective of critical phenomena. The main purpose of this study is to examine whether LPPL structures in the development of credit default swap (CDS) spreads can be used for default classification. Based on the different triggers of Bear Stearns' near bankruptcy during the late-2000 financial crisis and Ford's insolvency in 2009, this study provides a quantitative description of the mechanism behind bank runs. We apply the Johansen–Ledoit–Sornette (JLS) positive feedback model to explain the rise of financial institutions' CDS spreads during the global financial crisis 2007–2009. This investigation is based on CDS spreads of 40 major banks over the period from June 2007 to April 2009 which includes a significant CDS spread increase. The qualitative data analysis indicates that the CDS spread variations have followed LPPL patterns during the global financial crisis. Furthermore, the univariate classification performances of seven LPPL parameters as default indicators are measured by Mann–Whitney *U* tests. The present study supports the hypothesis that discrete scale-invariance governs the dynamics of financial markets and suggests the application of new and fast updateable default indicators to capture the buildup of long-range correlations between creditors.

1. Introduction

In economics the opinion prevails that complex financial systems are in general unpredictable [1]. Some econophysicists have nevertheless designed methods in order to predict financial market turmoil. The interactions between microscopic particles dealt with in physics are commonly of short range. But under certain conditions these short ranged interactions develop to long ranged interactions, culminating in cooperative behavior of the physical system. Financial markets also

* Corresponding author. Tel.: +49 251 83 31824.
E-mail addresses: j.w@uni-muenster.de (J.H. Wosnitza), denz@uni-muenster.de (C. Denz).

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exhibit cooperative phenomena, for instance, in terms of crashes when investors decide in bulk to sell their shares. This apparent similarity between physical systems on the one hand and financial market dynamics on the other hand justifies the application of physical methods in order to extract the coarse-grained properties of these large ensembles of market participants [1–3]. In particular, the log-periodic power law (LPPL) has proven very fruitful in forecasting the bursting of speculative bubbles [1].

In a nutshell, the objective of the LPPL ansatz is to observe well defined patterns in price trajectories before critical points in time. In this context, financial markets are regarded as self-organized systems that can drive themselves toward critical points [4]. When financial markets approach their critical points, long-range correlations between traders build up and imitation becomes a leading motive of investment decision [4–6]. This positive feedback mechanism between investors results in a faster-than-exponential price increase decorated by log-periodic oscillations [7]. The faster-than-exponential price increase is defined by a growth rate which increases with time [8]. The log-periodic oscillations manifest themselves as peaks and valleys with progressively smaller amplitudes and greater frequencies that eventually reach the critical point in time t_c at which the average state of the system becomes sensitive to external influences [6]. Any adequate disturbance may trigger a crash at this state. That is why the speculative bubble has the highest probability to burst at t_c. For the sake of illustration, we adopt the example originally given in Ref. [6]: "Consider a ruler put vertically on a table. Being in an unstable position, it will fall in some direction, and the specific air current or slight imperfection in the initial condition are of no real importance. What is important is the intrinsically unstable initial state of the ruler. We argue that a similar situation applies for crashes. They occur because the market has reached a state of global instability. Of course, there will always be specific events which may be identified as triggers of market motions but they will be the indicators rather than the deep sources of the instability" [6].

In order to provide an overview of the literature on log-periodicity, we first have to mention the pioneering works of Feigenbaum and Freund [9] and Sornette et al. [10] who independently of each other discovered ex post LPPL structures in the S&P500 index prior to the crash in October 1987. The hypothesis that the reasons for LPPL structures are rooted deeper in economic mechanisms than only in the dynamics of financial markets has met with skepticism and even triggered a heated discussion between two discoverers [11–14]. The academic community is also divided into two camps on this topic: Although, skeptics mainly acknowledge the presence of LPPL imprints in financial time series, they consider those structures as accidental patterns attributed to the stochastic processes of the financial market [12,15,16]. In contrast, the advocates of the Johansen–Ledoit–Sornette (JLS) model substantiate their point of view of LPPL patterns diagnosing speculative bubbles by offering a wide variety of investigations which can, for example, be partitioned into four groups as follows: First, there is rich literature on the ex post detection of LPPL patterns preceding financial crashes. LPPL structures were, for example, successfully identified in stock market bubbles [6,12,17–22], in real estate price bubbles [8,23], in the 2006–2008 oil bubble [24], and in the US FED Prime Rate [25].[1] Second, a vast body of empirical evidence has accumulated demonstrating that the herding behavior of investors not only results in speculative bubbles with accelerating market overvaluations, but also in anti-bubbles with decelerating market devaluations [27–29]. A third strand of literature went even one step further by claiming that financial crashes can be forecasted by extrapolating the LPPL [1]. The main idea of this research line is to integrate the LPPL structures into a pattern recognition approach in order to predict end times of bubbles and anti-bubbles [1,29–31]. Fourth, researchers have discovered that the presence of LPPL structures is predictive of crashes in real data, but they were unable to establish a link between LPPL patterns and crashes in the synthetic data [9,32,33].

Despite this large number of studies in favor of the LPPL hypothesis, "the statistical significance of these precursors and their predictive power remain controversial in part" [34]. However, an increasing number of case studies in a wide variety of financial time series reinforces one by one the LPPL hypothesis. The detection of LPPL structures in US corporate bond spreads [35], in the credit default swap (CDS) indices [36,37], and in the repurchase agreement market size bubble from 2007 to 2008 [38] can be considered as first applications of the LPPL to credit risk time series. The experiences of the global financial crisis 2007–2009 have shown plainly that high uncertainties about international banks' creditworthiness may induce serious consequences for the real economy [39–42]. Due to significant off-balance-sheet liabilities, it is difficult to estimate the default risk of financial institutions on the basis of conventional methods like balance sheet ratings or within the Black–Scholes–Merton framework [43]. Research on the causes of international banks' deterioration in creditworthiness is therefore of utmost importance for financial market stability. Our main hypothesis is that bankruptcies are not necessarily the consequences of bad business figures, but can also be the result of creditors' positive feedback interaction in analogy to critical phenomena in physics. Thus, it appears to be worthwhile to pursue the idea of LPPL structures as a harbinger of an impending liquidity crisis. We consequently apply the JLS model to explain the CDS spread movements of large banks during the financial crisis 2007–2009 [4,44]. This model is based on two central assumptions: First, CDSs provide an opportunity for creditors to transfer credit risks to counterparties. Hence, the individual creditor in our model has only two possible actions: Either he does hedge or he does not hedge against default by CDSs. Second, no creditor can precisely estimate the probability of default for any debtor. There is always an uncertainty about the obligor's credit standing. That is why each creditor steadily communicates with a limited number of other creditors about the obligor's solvency. The opinions of their networks are crucial for the creditors' future actions.

We perform both, a qualitative and a quantitative investigation on daily data of financial institutions' CDS spreads in order to answer the question whether there is a link between bankruptcies and phase transitions. At first, we calibrate the LPPL

[1] Sornette [26] provides further examples of cases in which LPPL patterns were diagnosed.

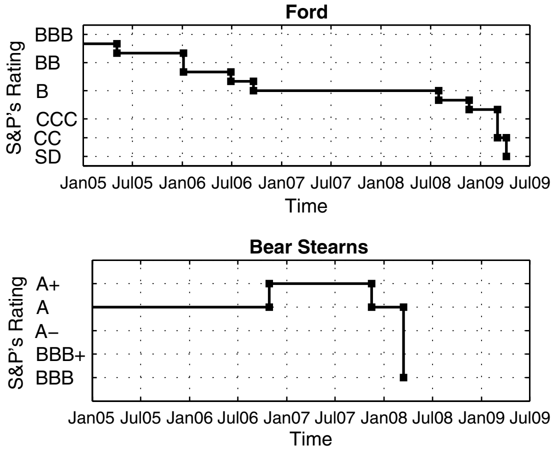

Fig. 1. *Rating history of Ford and Bear Stearns.* For the sake of overview, only some rating classes are displayed.

to CDS spreads of 40 financial institutions. Based on these results, we evaluate the discriminatory power of the parameter values to separate between defaulted and non-defaulted banks using Mann–Whitney U tests. In contrast to the definition of default specified by the Basel Committee on Banking Supervision [45], recipients of federal bailout funds are labeled as defaults whereas financial institutions that did not require public bailouts are labeled as non-defaults in this study.

The present study contributes to the literature twofold: First, this paper provides a quantitative description of the mechanism behind bank runs, and thus ensures better understanding of reasons for insolvencies. Second, the presence of discrete scale-invariance in time series related to credit risk suggests the application of new and fast updateable default indicators to capture the buildup of long-range correlations between creditors.

The remainder of this paper is organized as follows: Section two briefly displays the current knowledge of the company crises theory and shows the incompleteness of this theory. A background section about the LPPL follows in section three. Subsequently, a description of the data set and of the Mann–Whitney U test is presented in section four. The fifth section lists and briefly discusses the empirical results and finally, the sixth section summarizes the most important results, discusses the limitations inherent to the methodology and concludes with implications for theory and practice.

2. Crisis mechanisms for companies on their way to insolvency

The business management theory assigns strategic and operative mistakes as reasons for manufacturers' defaults [46]. The course to insolvency is usually subdivided into four phases which are characterized by the isolated or combined occurrence of an increasing debt-to-equity ratio and liquidity shortages[2]:

1. *Strategic crisis:* The first stage of the crisis mechanism is not as clearly determinable as the stages listed below. In general, this phase is distinguished by a drop of net income, but not necessarily by an increasing debt-to-equity ratio.
2. *Operative crisis:* The second stage is characterized by an increasing debt-to-equity ratio and progressively noticeable losses. As a consequence, liquid assets diminish considerably.
3. *Illiquidity crisis:* The company becomes heavily in debt during this penultimate stage. Since debt rises, the crisis becomes increasingly urgent.
4. *Insolvency:* Finally, the company is unable to meet its financial obligations in part or in full, because of liquidity shortages.

Ford's road to insolvency, for instance, can be differentiated in compliance with the individual phases mentioned above: Standard and Poor's (S&P) justified the first two consecutive downgrades of Ford's creditworthiness illustrated in Fig. 1 on 05/05/2005 and 01/05/2006 by their "skepticism about whether management's strategies will be sufficient to counteract mounting competitive challenges" [48]. Further indication of the strategic crisis is provided by S&P opinion that "the full-year pretax loss of Ford's North American operations could approach $2 billion – before substantial impairment and restructuring charges – although Ford is still expected to be profitable on a net basis in 2005 (before special items) thanks to earnings from its finance unit" [49]. According to S&P, an operative crisis characterized by "mounting cash losses in Ford's North American automotive operations" [50] has unfolded on 07/31/2008. Strictly speaking, S&P decision to lower the rating on Ford Motor Company on 07/31/2008 was driven by "cash outflows (which) will reduce the company's currently adequate liquidity" [50]. Ford's transition to illiquidity crisis was documented by S&P expectation that "Ford's liquidity (will) be significantly reduced the rest of this year and into next year by continued heavy cash use" [51]. Ford's announcement of a debt restructuring on 03/04/2009 induced S&P to demote the company's rating from CCC+ to CC [52]. Given that distressed

2 The theoretical considerations on Section 2 are based on [47].

exchanges belong to S&P default criteria, Ford's rating was lowered to SD (selective default) after completion of the tender offers on 04/06/2009 [53].

Assuming a four stage process in the run-up to insolvency justifies the hope that balance sheet ratios are capable of discriminating between defaulted and non-defaulted companies with a forecast horizon greater than one year. But the sudden decline of creditworthiness of several banks during the financial crisis 2007–2009 arises doubt on the validity of the just presented concept for financial institutions. In contrast to manufacturing firms, financial institutions are subject to bank runs, i.e., to the risk of becoming illiquid due to loss of investors' confidence. Balance sheet ratios usually cannot reflect this risk. Motivated by Bear Stearns' collapse shown in Fig. 1, we posit that illiquidity crisis can also be triggered or at least intensified by the nonlinear interaction of creditors [54]. S&P raised Bear Stearns' credit rating from A to A+ on 10/27/2006 based on the company's "relatively low profit volatility, conservative management, and cost flexibility, as well as long-term improvements in its liquidity and risk management" [55]. This upgrade was compensated by a later rating action on 11/15/2007 reflecting Bear Stearns' first quarterly loss in its history due to a "writedown on its CDO and subprime exposure" [56]. Finally, Bear Stearns spiraled into an existence-threatening crisis on 03/14/2008 since "its liquidity position has substantially deteriorated in the past 48 hours [54]. The company's announcement was followed by S&P downgrade on Bear Stearns' from A to BBB. The minor rating judgment deterioration on Bear Stearns is explained by S&P expectation "that Bear will find an orderly solution to its funding problems" [54]. Over the weekend of 03/15/2008, the US Federal Reserve, the US Department of the Treasury and JPMorgan Chase prepared a rescue takeover to prevent Bear Stearns from going bankrupt [57–59]. JPMorgan completed the acquisition of Bear Stearns on 05/30/2008 [59].

In contrast to Ford Motor Company, whose liquidity crunch was attributed to its North American automotive operations [49–51,60], Bear Stearns' near-bankruptcy in March 2008 was at least partially a different story: This collapse was mainly caused by a run of the counterparties on the investment bank [57,61]. In concrete terms, lenders denied credits and clients withdrew their liquid assets from Bear Stearns due to a lack of trust [61,62].

Skipping or speeding through the strategic or operative crisis, as with Bear Stearns, may shorten the forecasting horizon of default prediction. As a result, balance sheet ratios may be inadequate indicators of defaults triggered by a run of creditors and customers, since they can only be updated annually for the greater part [63]. In the next section we apply the JLS model introduced by Johansen et al. [4] to explain how the nonlinear interaction of creditors may have caused Bear Stearns' failure. Following this model, new default predictors are suggested.

3. Model to explain LPPL patterns in CDS spreads

Johansen et al. [4] proposed a modification of the Ising spin model, in which investors either imitate the opinions of their nearest neighbors or take individual decisions, in order to formalize the information swap between investors which may eventually end up in a cooperative herding behavior of the investors [2,4,12]. According to this model, speculative bubbles emerge from a positive feedback mechanism. As a new area of application, it is demonstrated that this model can be used to explain the CDS spread variations of several banks during the financial crisis 2007–2009.

CDSs enable institutional investors to transfer the credit risk of a reference entity (e.g. bank, corporate or sovereign) from one party to another without exchange of liquidity on transaction date. These financial derivatives serve the purpose of hedging against risks of real business transactions. The design of a standard CDS contract corresponds to that of an insurance: The protection buyer purchases protection against the risk of a credit event by the underlying company from the protection seller. A credit event refers to a legally defined event which usually incorporates bankruptcy, failure-to-pay and sometimes debt restructuring events. To pay for the protection, the buyer of the CDS makes periodic premium payments to the seller either until the specified maturity date of the contract or until a credit event occurs, whichever is earlier. The quoted CDS spread at conclusion of the contract determines the level of the periodic payments. The premium level is paid on the nominal value of the protection. In return, the protection buyer of the CDS gains the right to sell a particular bond issued by the reference entity at par value or to receive the difference between the par value and the market price if a credit event occurs before maturity [64,65].

Roughly summarized, CDS spreads reflect investors' aggregate opinion about the credit quality of the reference entity. Below, $S(t)$ denotes the CDS spread at time t. For the sake of simplicity, we consider CDSs in the following as purely speculative investments. Furthermore we posit that the martingale hypothesis applies for the expectation of the CDS spread $S(t)$—a hypothesis which holds true only under idealized conditions [4]:

$$\mathbb{E}[S(t)] = S(t_0) \quad \forall t > t_0. \tag{1}$$

The point of departure for the model of Johansen et al. [4] is the following Ising spin model [4][3]:

$$s_i = \operatorname{sign}\left(\sum_{j\in N(i)} s_j + \sigma \cdot \varepsilon_i + G\right). \tag{2}$$

A spin-up constituent represents an individual investor i who takes on credit risk ($s_i = +1$) and a spin down constituent represents a trader who has already transferred the default risk to a trading partner via a CDS ($s_i = -1$) [4]. The derivation of

[3] The spins s_i must not be confused with the CDS spread $S(t)$.

the analytical solution (13) necessitates the following two main simplifications: First, each creditor pursues an irreversible investment strategy which means that all investors who have already hedged against default, cannot reassume the credit risk [66]. Second, the model presupposes that the number of insurance providers remains constant during the time when creditors' highly nonlinear behavior evolves. An assumption which obviously stands in opposition to the equilibrium between protection buyers and protection sellers [66]. Eq. (2) says, that the investment decision of a single creditor $i \in 1, \ldots, I$ is dependent on his network's accumulated actions, on an idiosyncratic signal ε_i and on a global influence G. $N(i)$ denotes the number of investors who significantly influence the opinion of creditor i. The scalar multipliers K and σ determine the individual creditors' tendency to imitate and to contradict their nearest neighbors' opinions, respectively [4].

After defining the cumulative distribution function of the time to insolvency $Q(t)$ and the corresponding probability density function $q(t) = \frac{\partial}{\partial t}Q(t)$, we are able to express the hazard rate $h(t)$ [4]:

$$h(t) = \frac{q(t)}{1 - Q(t)}. \tag{3}$$

At the critical point of time $t = t_c$ two scenarios are conceivable with regard to the fate of the debtor: On the one hand the interaction of the creditors could in fact force the debtor into bankruptcy. In consequence, the creditors who hedged against default would receive the agreed payoff and the CDS is taken off the market. On the other hand the debtor could survive the speculative attack [4]. In this case we assume that the CDS spread drops by a fixed percentage κ above the inner credit risk measured by $S_{internal}$. Formally, the spread trend is described by the following differential equation [33]:

$$\mathrm{d}S = \mu(t) \cdot S(t) \cdot \mathrm{d}t - \kappa \cdot [S(t_c) - S_{internal}] \cdot \mathrm{d}j, \tag{4}$$

where $\mu(t)$ denotes the time-dependent drift. Applying the expectation value on both sides of Eq. (4), the martingale hypothesis (1) and the relationship $\mathbb{E}[\mathrm{d}j] = h(t) \cdot \mathrm{d}t$ yields [4]:

$$\mu \cdot S(t) = \kappa \cdot [S(t_c) - S_{internal}] \cdot h(t). \tag{5}$$

Eq. (6) is obtained by inserting (5) in (4):

$$\mathrm{d}S = \kappa \cdot [S(t_c) - S_{internal}] \cdot h(t) \cdot \mathrm{d}t - \kappa \cdot [S(t_c) - S_{internal}] \cdot \mathrm{d}j. \tag{6}$$

As long as no default has occurred, the variable j equals zeros. Thus, Eq. (6) simplifies to:

$$\mathrm{d}S = \kappa \cdot [S(t_c) - S_{internal}] \cdot h(t) \cdot \mathrm{d}t. \tag{7}$$

Under the assumption that $S_{internal}$ depends only weakly on time, the integration of Eq. (7) approximately yields [33]:

$$S(t) \approx S(t_0) + \kappa \cdot [S(t_c) - S_{internal}] \cdot \int_{t_0}^{t} h(t') \cdot \mathrm{d}t'. \tag{8}$$

With Eq. (8) it is easy to see that the time dependence of CDS spreads before t_c is only due to the hazard rate $h(t)$. In our simplified model the susceptibility of the system is defined by the derivation of the expected average state $\mathbb{E}[M] = \frac{1}{I} \cdot \mathbb{E}\left[\sum_{i=1}^{I} s_i\right]$ with respect to the global influence G [4]:

$$\chi = \left.\frac{\partial}{\partial G}\mathbb{E}[M]\right|_{G=0}. \tag{9}$$

The susceptibility provides a measure for the system's response to an infinitesimal global perturbation $\mathrm{d}G$ [4]. In physical systems governed by discrete scale invariance, the susceptibility close to second order phase transitions is approximately a function of the form [4]:

$$\chi(K_c - K) = (K_c - K)^{-\beta} \cdot \left(a + b \cdot \cos\left(\omega \cdot \ln(K_c - K) + \tilde{\phi}\right)\right), \tag{10}$$

where a and b are positive constants, $\beta > 0$ denotes the critical exponent of the susceptibility and K_c stands for the critical value for the coupling strength K: If the coupling strength K is considerably less than K_c, the creditors respond inconsistently to small global perturbations and as a result, there is little effect on the average state of the system. But if the coupling strength K converges toward K_c, the creditors respond identically to small global influences. As a consequence of the investors' homogeneous responsiveness, the system's sensitivity to small global influences becomes extremely high [4]. Since the temporal dependence of the coupling strength K is unknown, we only assume that a first order Taylor expansion around the critical point is possible [4]:

$$K(t) \approx K_c + \tilde{C} \cdot (t_c - t), \tag{11}$$

where $\tilde{C}$ is another constant. The difference $t_c - t$ quantifies the distance between the current state of the system and the critical point. Based on Johansen et al. [4], we assume that the hazard rate follows a similar process as the susceptibility:

$$h(t_c - t) = (t_c - t)^{-\beta} \cdot \left(\tilde{A} + \tilde{B} \cdot \cos\left(\omega \cdot \ln(t_c - t) + \tilde{\phi}\right)\right), \tag{12}$$

with new constants $\tilde{A}$ and $\tilde{B}$. The sensitivity of the financial market to external influences increases while approaching the critical point in time [6]. If β would not lie within the interval (0; 1), the CDS spread would diverge under the condition that no default has occurred yet [4,33]. Substituting Eq. (12) into (8) finally yields the LPPL for the CDS spread development [33]:

$$S(t) = A + B \cdot (t_c - t)^{\alpha} + C \cdot (t_c - t)^{\alpha} \cdot \cos(\omega \cdot \ln(t_c - t) + \phi). \tag{13}$$

The existence of positive feedbacks among creditors leads to a faster-than-exponential CDS spread increase in the form of a finite-time singularity occurring at the critical time t_c. However, this mathematical singularity does not exist in real data, inter alia, because of finite-size effects [67]. Particular importance has been attached to the third summand in the LPPL (13) due to which the faster-than-exponential CDS spread growth is decorated by log-periodic oscillations. As a consequence of these oscillations, LPPL fits can "lock in" on the log-periodic fluctuations and eventually lead to more accurate estimations of the critical times t_c [5]. Within the scope of this article, the faster-than-exponential growth decorated by log-periodic oscillations is referred to as LPPL structures. In addition, it should be noted that the LPPL (13) is only a first order approximation [5]. The log-periodic component in the noisy financial data does of course not correspond to a pure cosine as implied by Eq. (13). Nonetheless, the LPPL (13) has proven successful in capturing the mechanisms of positive feedbacks among investors and thus in modeling the speculative aspects of financial markets [68].

The parameters in Eq. (13) have the following meaning: A represents the CDS spread $S(t)$ in the limit $t \to t_c$ [1]. B governs the polynomial trend over the course of time [18,69] and C quantifies the amplitude of the log-periodic oscillations around this trend [1]. The critical point in time t_c is the most probable moment at which the financial market is transitioning from the speculative bubble into another state, for example, into a crash [4,5,67]. The end of the speculative bubble at t_c does not necessarily imply a crash, but it is the time when a crash is most likely. Apart from the crash scenario at t_c, the bubble may either burst prior to t_c or even land smoothly without any crash [6]. According to prior investigations, bubbles land smoothly in approximately in one third of the cases [30,70]. The critical exponent α quantifies the acceleration of the CDS spreads [8] and thus is promising in capturing the collective organization between investors [71]. α is bounded between 0 and 1 because otherwise the price at t_c would tend toward an infinite value. Substituting $\phi = -\omega \cdot \ln(\tau)$ in the cosinus term in Eq. (13) yields [10]:

$$S(t) = A + B \cdot (t_c - t)^{\alpha} + C \cdot (t_c - t)^{\alpha} \cdot \cos\left(\omega \cdot \ln\left(\frac{t_c - t}{\tau}\right)\right). \tag{14}$$

According to Eq. (14), the parameter ϕ is regarded as time scale [10]. The log-angular frequency ω contains information on how fast the oscillations contract. Sornette et al. [10] suggested that the term $\exp\left(\frac{\pi}{\omega}\right)$ measures the "coupling between traders and the fundamentals of the economy" [10].

The JLS model does not explicitly impose any restrictions on the amplitude of the crash. However, depending on whether the crash amplitude is assumed to be proportional to the spread or to the accretion during the bubble episode, either the spread itself or the logarithm of the spread becomes implicitly the relevant observable [8,19,28]. In the remainder of this paper, we assume that the spread itself is the natural variable.

By way of illustration, we demonstrate the key principles of the model in the following example which is depicted in Fig. 2 [66,72]. At the beginning ($t = 1$) four creditors grant loans to one debtor. Each creditor is assigned a stress value σ_0. This value determines the likelihood that creditor i transfers his default risk to a counterparty in the next instant. Creditor $i = 2$ is the first who hedges against the potential default of the borrower at $t = 2$. At first only creditor $i = 1$ has the privilege to use this information for his own investment decision and since he is aware that his ratings may be, at least in some cases, incorrect this information is an important factor for his own decision. The additional information is reflected by the higher stress value. The creditors $i = 3$ and $i = 4$ do not significantly interact with creditor $i = 1$ and therefore this additional information is withheld from them. Only after creditor $i = 1$ has hedged his exposure, creditors $i = 3$ and $i = 4$ receive the information on the two risk-hedgings which leads to an increase of both stress values at $t = 3$. Next creditor $i = 4$ insures himself against potential losses via a CDS at $t = 4$. Since more and more creditors hedge against a possible insolvency and the number of CDS providers remains constant with respect to time, the CDS spread increases. This leads to higher credit risk in the sense of a self-fulfilling prophecy, because some investors convert CDS spreads into implicit default probabilities to estimate the debtor's creditworthiness [65,73].

4. Used data and applied method

In the present analysis, CDS spreads played a similar role as the concentration of chlorine ions in the examination of the Kobe earthquake [74] or stock market indices in other scientific papers [3,17,75]. More specifically, we analyzed the spread trajectories of senior CDSs with 1-year maturities. Such spreads are daily monitored and available for downloading at Thomson Reuters [76]. The data sample contained daily CDS spreads of 40 international banks from June 2007 until approximately April 2009, which corresponds to about 460 data points per time series. Thereof 20 financial institutions received federal bailouts during the financial crisis 2007–2009. Deviating from the default definitions of the Basel Committee on Banking Supervision [45], banks were labeled as defaults, if they have required public bailouts in the period from 2007 to 2009.

In a first step, the seven parameters in the LPPL (13) were adjusted to each of the 40 CDS spread trajectories by minimizing the squared residuals between the data points and the LPPL function. The first data point of each fitting interval corresponds

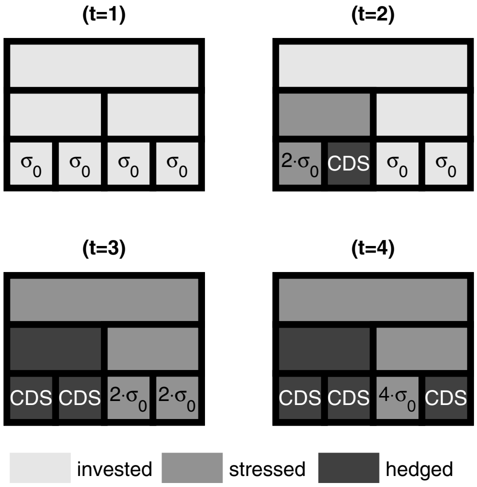

Fig. 2. *Illustrative example of stress transfer.* At first, each creditor is assigned an individual stress value σ_0. A higher stress value usually indicates higher risk aversion which causes an immediate effect on the investors' subsequent action. Creditor $i = 2$ is the first who decides to hedge against a potential default ($t = 2$). His stress is transferred to creditor $i = 1$. Next, Creditor $i = 1$ insures against default transferring stress to traders $i = 3$ and $i = 4$ at $t = 3$. Then, creditor $i = 4$ buys protection at $t = 4$.
Source: Based on [66,72].

to the lowest CDS spread prior to the commencement of the bubble. This procedure discounts the possibility that inceptions of speculative bubbles could be clouded in uncertainty. Instead, we assumed that the interval start points can always be determined objectively. The last data point is equivalent to the highest CDS spread, respectively. Since we use the highest CDS spread as a last data point, our approach runs the risk of biasing the predictive power of the critical point in time t_c. Under the assumption that the critical points in time are close to the last data points and that defaulted banks tend to exhibit the maximum CDS spread earlier than non-defaulted banks, we would artificially introduce discriminative power to t_c by our approach. In order to assess this risk, we conducted a Mann–Whitney U test of the predetermined fitting interval end times. The result indicates that there is no significant difference between the last data points of defaulted and non-defaulted banks at a 5% level. More specifically, the AUROC value of 0.67125 indicates a sub-acceptable discriminative power of the last data points according to Ref. [77]. Thus, we rejected the potential criticism that the discriminative power of the critical time t_c was contaminated by a forward-looking bias.

After specifying the fitting intervals, the LPPL parameters were adjusted to the corresponding CDS spread developments. In order to achieve the best possible fit of Eq. (13) to each trajectory, we applied a multi-start optimization algorithm [78] based on the MATLAB function fmincon.m [79]. This local search algorithm "attempts to find a constrained minimum of a scalar function of several variables starting at an initial estimate" [79]. In order to ensure that the outputted parameter vector corresponds to a global and not only to a local minimum sum of squared residuals, we called fmincon.m 250 times with randomly chosen initial parameter vectors bounded as follows:

- $A > 0$,
- $B < 0$ [1,18],
- $-B \geq \sqrt{1 + \left(\frac{\omega}{\alpha}\right)^2} \cdot |C|$ [1,33],
- $\alpha \in (0; 1)$ [3,4],
- $\phi \in [0; 2\pi]$ and
- $\omega \in [0; 20]$.

We kept the five best outcomes of this search algorithm for each CDS spread trajectory. The constraint $-B \geq \sqrt{1 + \left(\frac{\omega}{\alpha}\right)^2} \cdot |C|$ results from the fact that the hazard rate – as a probability – remains always positive [33]. Furthermore, we only consider parameter sets whose log-frequency ω is smaller than 20, "because a large log-frequency corresponds to many oscillations which most often can be associated with high frequency noise" [6]. These constraints limit the parameter space to be scanned and hence may significantly reduce the computational cost. By slaving the three linear parameters A, B and C to the four nonlinear parameters t_c, α, ϕ, and ω, the LPPL fits become effectively four dimensional [4]. After substituting $f(t) = (t_c - t)^\alpha$ and $g(t) = (t_c - t)^\alpha \cdot \cos(\omega \cdot \ln(t_c - t) + \phi)$ in Eq. (13) we can rewrite this equation as [4]:

$$S(t) = A + B \cdot f(t) + C \cdot g(t) \tag{15}$$

By demanding that the partial derivatives of the cost function with respect to A, B and C are eliminated in a local minimum, we obtain the following linear system of equations [4][4]:

$$\sum_{i=1}^{N}\begin{pmatrix} S(t_i) \\ f(t_i)\cdot S(t_i) \\ g(t_i)\cdot S(t_i)\end{pmatrix} = \sum_{i=1}^{N}\begin{pmatrix} 1 & f(t_i) & g(t_i) \\ f(t_i) & f(t_i)^2 & f(t_i)\cdot g(t_i) \\ g(t_i) & f(t_i)\cdot g(t_i) & g(t_i)^2 \end{pmatrix}\cdot\begin{pmatrix} A \\ B \\ C\end{pmatrix}. \quad (16)$$

For each choice of the nonlinear parameter values t_c, α, ϕ, and ω the equation system (16) has to be solved for A, B and C. The reformulation of the LPPL presented in Ref. [80] opens up the possibility to reduce the dimensionality of the nonlinear optimization problem further from four to three dimensions. For details, the reader is referred to Ref. [80]. Our test design for LPPL structures is of parametric nature, i.e. it is based on a formula which is composed of a power law and of a log-periodic part [6]. This parametric approach allows us to simultaneously extract the log-frequency as well as the power law trend from the financial price trajectory and thus provides a significant advantage over nonparametric approaches [6].

In order to compare the discriminative power of the LPPL parameters, we performed two-sided Mann–Whitney U tests of the null hypothesis H_0 that the parameters of defaulted and non-defaulted banks are independent samples from identical continuous distributions, against the alternative of different medians. Therefore, the banks were ranked according to the seven LPPL parameters, respectively. The hypothesis H_0 of no discriminative power is rejected, if the defaulted banks rank sufficiently higher.[5] This raises the question when the default ranks $(S_1, S_2, \ldots, S_n)$ are sufficiently large. The answer to this question is given by the Mann–Whitney statistic U which is defined in Ref. [81] as[6]

$$U = \frac{1}{N_D\cdot N_{ND}}\cdot\sum_{(D,ND)} u_{D,ND}$$
$$u_{D,ND} = \begin{cases} 1, & \text{if } P_D < P_{ND} \\ \frac{1}{2}, & \text{if } P_D = P_{ND} \\ 1, & \text{if } P_D > P_{ND} \end{cases} \quad (17)$$

where P_D is a LPPL parameter realization of a defaulted bank and P_{ND} is a LPPL parameter realization of a non-defaulted bank. The sum in Eq. (17) is over all pairs of defaulted and non-defaulted banks. It is a well known fact that the test statistic U follows a normal distribution

$$\frac{U-\mathbb{E}[U]}{\sqrt{\mathrm{Var}[U]}} \to \mathcal{N}(0,1). \quad (18)$$

The hypothesis H_0 is rejected and the LPPL parameter judged to be discriminative when U is sufficiently large, this implies $U \geq c$, where c is a constant. It is convention to define the critical value c so that under H_0 the probability of getting a value of U equal or superior to the level of significance α, that is $P_{H_0}(U \geq c) = \alpha$. Here, three different values for α were chosen: $\alpha = 0.1\%$, $\alpha = 1\%$ and $\alpha = 5\%$. The subscript H_0 refers to the fact that the probability is computed under H_0, thus under the assumption that the parameter has no discriminative power. The univariate classification performances of all parameters were evaluated on the total sample [81,83].[7]

[4] The aim is to minimize the cost function $\sum_{i=1}^{N}(S(t_i) - A - B\cdot f(t) - C\cdot g(t))^2$ with respect to A, B and C. The necessary condition for a local minimum states that the first partial derivatives as to A, B and C are zero at the minimum:

$$\sum_{i=1}^{N} S(t_i) - A\cdot N - B\cdot\sum_{i=1}^{N} f(t_i) - C\cdot\sum_{i=1}^{N} g(t_i) = 0$$
$$\sum_{i=1}^{N} S(t_i)\cdot f(t_i) - A\cdot\sum_{i=1}^{N} f(t_i) - B\cdot\sum_{i=1}^{N} f(t_i)^2 - C\cdot\sum_{i=1}^{N} g(t_i)\cdot f(t_i) = 0$$
$$\sum_{i=1}^{N} S(t_i)\cdot g(t_i) - A\cdot\sum_{i=1}^{N} g(t_i) - B\cdot\sum_{i=1}^{N} f(t_i)\cdot g(t_i) - C\cdot\sum_{i=1}^{N} g(t_i)^2 = 0.$$

[5] Here it is assumed that high parameter values are associated with companies of low credit quality. Likewise, if low parameter values characterize poor creditworthiness, H_0 is rejected when the defaults rank sufficiently low.

[6] Eq. (17) provides an interesting interpretation of the Mann–Whitney statistic U: Suppose one defaulter and one non-defaulter are selected from the population of the defaulted and non-defaulted banks, respectively. Suppose further that based on the parameter values of both samples, one has to decide which bank is the defaulter. For different values of the parameters, one would presume that the bank with the lower (or higher) parameter value is the defaulter. In the case of identical parameters, one faces a fifty–fifty chance. The probability of making a correct decision is given by the Mann–Whitney statistic U [81].

[7] All calculations within the scope of this study were carried out on the computers of the Morfeus GRID at the University of Muenster, with the use of condor [82].

Table 1
List of non-bailout banks. LPPL parameters for financial institutions which did not require public bailouts in the period from 2007 to 2009.
Source: Author's calculation.

Names of banks	A	B	C	t_c	α	ϕ	ω
Australia & NZ BKG GP	$2.57 \cdot 10^{+2}$	$-6.35 \cdot 10^{+1}$	$-2.42 \cdot 10^{+0}$	03/10/2009	$2.11 \cdot 10^{-1}$	2.18	5.54
Banco Bilbao Vizcaya ARG	$3.58 \cdot 10^{+5}$	$-3.56 \cdot 10^{+5}$	$-8.83 \cdot 10^{+1}$	08/03/2009	$5.87 \cdot 10^{-4}$	6.25	2.37
Banco STDR CTL HISP SA	$2.34 \cdot 10^{+5}$	$-2.33 \cdot 10^{+5}$	$-1.98 \cdot 10^{+1}$	01/22/2010	$5.64 \cdot 10^{-4}$	3.12	6.63
Barclays bank PLC	$6.32 \cdot 10^{+2}$	$-2.20 \cdot 10^{+2}$	$+9.26 \cdot 10^{+0}$	04/12/2009	$1.65 \cdot 10^{-1}$	0.75	3.93
Commonwealth bank of AUS	$1.41 \cdot 10^{+2}$	$-8.36 \cdot 10^{+0}$	$+4.51 \cdot 10^{-1}$	03/10/2009	$4.33 \cdot 10^{-1}$	2.81	8.02
Credit Suisse Group	$8.42 \cdot 10^{+2}$	$-4.22 \cdot 10^{+2}$	$-1.23 \cdot 10^{+1}$	03/26/2009	$1.09 \cdot 10^{-1}$	5.05	3.74
Daiwa Securities GP INC	$8.82 \cdot 10^{+2}$	$-2.37 \cdot 10^{+2}$	$+1.86 \cdot 10^{+1}$	03/11/2009	$2.15 \cdot 10^{-1}$	0.84	2.73
Deutsche bank AG	$1.84 \cdot 10^{+2}$	$-1.20 \cdot 10^{+1}$	$-1.45 \cdot 10^{+0}$	03/13/2009	$4.30 \cdot 10^{-1}$	0.23	3.52
HSBC bank PLC	$2.20 \cdot 10^{+2}$	$-2.70 \cdot 10^{+1}$	$-2.66 \cdot 10^{+0}$	03/15/2009	$3.34 \cdot 10^{-1}$	0.85	3.36
Intesa Sanpaolo SPA	$1.99 \cdot 10^{+5}$	$-1.98 \cdot 10^{+5}$	$+2.87 \cdot 10^{+1}$	08/24/2009	$5.62 \cdot 10^{-4}$	6.02	3.88
Mediobanca SPA	$6.95 \cdot 10^{+2}$	$-3.66 \cdot 10^{+2}$	$-1.07 \cdot 10^{+1}$	04/11/2009	$1.02 \cdot 10^{-1}$	5.95	3.52
Mitsubishi UFJ FIN GRP INC	$9.46 \cdot 10^{+1}$	$-1.68 \cdot 10^{+1}$	$-1.46 \cdot 10^{+0}$	11/21/2008	$2.63 \cdot 10^{-1}$	0.00	2.99
National Australia bank	$2.57 \cdot 10^{+7}$	$-2.57 \cdot 10^{+7}$	$+6.67 \cdot 10^{+0}$	05/30/2009	$2.38 \cdot 10^{-6}$	4.55	7.49
Nordea bank AB	$1.70 \cdot 10^{+7}$	$-1.70 \cdot 10^{+7}$	$-7.88 \cdot 10^{+0}$	04/06/2009	$3.14 \cdot 10^{-6}$	2.39	4.17
Rabobank	$1.37 \cdot 10^{+5}$	$-1.37 \cdot 10^{+5}$	$-2.01 \cdot 10^{+1}$	04/11/2009	$5.72 \cdot 10^{-4}$	4.01	3.88
Skens Banken AB	$3.53 \cdot 10^{+3}$	$-3.02 \cdot 10^{+3}$	$-1.46 \cdot 10^{+1}$	04/12/2009	$2.44 \cdot 10^{-2}$	0.36	4.56
Standard CHT bank	$3.64 \cdot 10^{+2}$	$-2.21 \cdot 10^{+1}$	$-6.37 \cdot 10^{+0}$	04/01/2009	$4.74 \cdot 10^{-1}$	5.75	1.46
Svenska HANDBKN	$3.84 \cdot 10^{+2}$	$-1.82 \cdot 10^{+2}$	$-7.33 \cdot 10^{+0}$	03/19/2009	$1.14 \cdot 10^{-1}$	4.05	2.51
Unicredito Italiano SPA	$4.40 \cdot 10^{+5}$	$-4.40 \cdot 10^{+5}$	$-1.57 \cdot 10^{+1}$	03/10/2009	$1.21 \cdot 10^{-4}$	1.08	3.40
Westpac banking CORP	$1.03 \cdot 10^{+2}$	$-3.92 \cdot 10^{+0}$	$-2.46 \cdot 10^{-1}$	03/10/2009	$4.96 \cdot 10^{-1}$	6.28	7.91

Table 2
List of bailout banks. LPPL parameters for financial institutions which did require public bailouts in the period from 2007 to 2009.
Source: Author's calculation.

Names of banks	A	B	C	t_c	α	ϕ	ω
Allied Irish Banks	$2.84 \cdot 10^{+7}$	$-2.84 \cdot 10^{+7}$	$-8.51 \cdot 10^{+1}$	03/10/2009	$4.91 \cdot 10^{-6}$	5.65	1.64
Bank of America CORP	$2.71 \cdot 10^{+7}$	$-2.71 \cdot 10^{+7}$	$-6.11 \cdot 10^{+1}$	04/04/2009	$5.09 \cdot 10^{-6}$	2.98	2.03
Bank of Ireland	$1.80 \cdot 10^{+3}$	$-9.61 \cdot 10^{+2}$	$-3.60 \cdot 10^{+1}$	03/08/2009	$1.04 \cdot 10^{-1}$	6.28	1.45
Bear Stearns COS	$1.48 \cdot 10^{+8}$	$-1.48 \cdot 10^{+8}$	$-1.33 \cdot 10^{+2}$	03/14/2008	$1.45 \cdot 10^{-6}$	3.17	1.25
Bradford & Bingley PLC	$2.80 \cdot 10^{+3}$	$-7.39 \cdot 10^{+2}$	$+1.22 \cdot 10^{+2}$	10/08/2008	$2.45 \cdot 10^{-1}$	4.69	1.26
Citigroup INC	$4.73 \cdot 10^{+5}$	$-4.72 \cdot 10^{+5}$	$-3.18 \cdot 10^{+1}$	03/11/2009	$3.30 \cdot 10^{-4}$	4.53	4.89
Erste group bank AG	$4.13 \cdot 10^{+7}$	$-4.13 \cdot 10^{+7}$	$+1.86 \cdot 10^{+1}$	03/10/2009	$2.35 \cdot 10^{-6}$	0.36	4.07
Fortis NL	$8.15 \cdot 10^{+7}$	$-8.15 \cdot 10^{+7}$	$-1.16 \cdot 10^{+2}$	12/07/2008	$2.03 \cdot 10^{-6}$	0.90	1.27
IKB DT INDUSTR bank AG	$3.30 \cdot 10^{+5}$	$-3.05 \cdot 10^{+5}$	$+5.65 \cdot 10^{+3}$	04/20/2008	$1.58 \cdot 10^{-2}$	0.36	0.85
JPMorgan chase & CO	$2.80 \cdot 10^{+2}$	$-7.38 \cdot 10^{+1}$	$+2.85 \cdot 10^{+0}$	03/10/2009	$1.96 \cdot 10^{-1}$	1.53	5.08
KBC group NV	$5.62 \cdot 10^{+2}$	$-1.22 \cdot 10^{+2}$	$-1.12 \cdot 10^{+1}$	03/14/2009	$2.44 \cdot 10^{-1}$	4.84	2.64
Landsbanki ISLE HF	$5.45 \cdot 10^{+3}$	$-3.61 \cdot 10^{+3}$	$+1.25 \cdot 10^{+2}$	10/02/2008	$6.65 \cdot 10^{-2}$	4.57	1.62
Lloyds TSB bank PLC	$1.46 \cdot 10^{+7}$	$-1.46 \cdot 10^{+7}$	$-9.26 \cdot 10^{+0}$	03/14/2009	$3.47 \cdot 10^{-6}$	1.96	4.46
RAIF ZNTRLBK OSTER AG	$6.07 \cdot 10^{+5}$	$-6.06 \cdot 10^{+5}$	$-3.44 \cdot 10^{+1}$	03/07/2009	$1.73 \cdot 10^{-4}$	2.45	3.05
Sberbank	$1.92 \cdot 10^{+4}$	$-1.26 \cdot 10^{+4}$	$-3.21 \cdot 10^{+3}$	10/29/2008	$9.55 \cdot 10^{-2}$	6.28	0.36
Swedbank AB	$1.41 \cdot 10^{+6}$	$-1.41 \cdot 10^{+6}$	$+2.37 \cdot 10^{+1}$	03/19/2009	$5.80 \cdot 10^{-5}$	2.71	3.45
UBS AG	$8.94 \cdot 10^{+5}$	$-8.93 \cdot 10^{+5}$	$+1.52 \cdot 10^{+1}$	03/15/2009	$9.60 \cdot 10^{-5}$	4.30	5.64
VTB Bank	$2.73 \cdot 10^{+3}$	$-6.28 \cdot 10^{+2}$	$-1.86 \cdot 10^{+2}$	10/27/2008	$2.89 \cdot 10^{-1}$	3.33	0.94
Washington MUT INC	$1.58 \cdot 10^{+9}$	$-1.58 \cdot 10^{+9}$	$-7.58 \cdot 10^{+2}$	09/24/2008	$9.21 \cdot 10^{-7}$	5.93	1.60
Wells Fargo & CO	$2.41 \cdot 10^{+7}$	$-2.41 \cdot 10^{+7}$	$-3.49 \cdot 10^{+1}$	04/04/2009	$3.84 \cdot 10^{-6}$	2.51	2.15

5. Application of the model to the financial crisis 2007–2009

LPPL structures are evident for all 40 banks during the time period from June 2007 until around April 2009. The optimal parameter combinations for each of the 40 banks are shown in Table 1 and in Table 2.

Instead of verbalizing these two tables in detail, we focus on summarizing typical parameter ranges for default and non-default banks, respectively. In doing so, we have constructed box-plots of the non-linear parameters t_c, α, ϕ, and ω in Figs. 3–6. These box-plots allow comparisons between the univariate distributions of the default banks and of the non-default banks. On each box, the central (red) mark represents the median and the box's edges correspond to the 25th and 75th percentiles, respectively. Every point outside of the 25th and the 75th percentile is regarded as an outlier. Outliers are plotted individually and are denoted by (red) plus signs (+). Typical parameter ranges for default and non-default banks are, for example, bounded by the 25th and the 75th percentiles.

Next, we investigate whether our ω-values confirm the hypothesis of an universal log-angular frequency close to nine as was postulated, for example, in Refs. [13,84,85]. As can be seen from Fig. 6, our results indicate that the log-angular frequency within the analyzed CDS spread trajectories tend to be smaller than the proposed value of $\omega \approx 9$. The overall median ω amounts to 3.3824. The median ω of the non-default population is equal to 3.8097 and within the default population it is

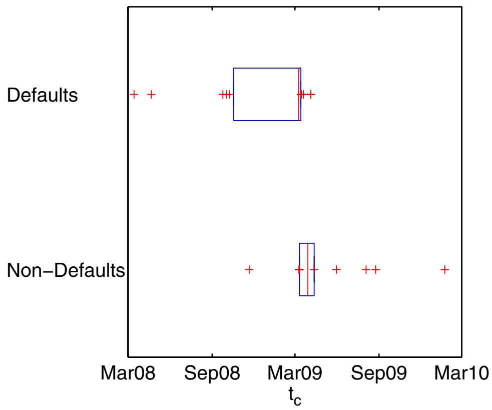

Fig. 3. *Box-plot of the LPPL parameter t_c.* The central (red) marks denote the median t_c's within the populations of the default banks (03/09/2009) and of the non-default banks (03/29/2009), respectively. The box's edges represent the 25th and 75th percentiles, respectively. Every point outside of the 25th and the 75th percentile is regarded as an outlier. Outliers are plotted individually and are denoted by (red) plus signs (+). Typical parameter ranges for default and non-default banks are defined by the 25th and the 75th percentiles, i.e. [10/18/2008; 03/14/2009] for default banks and [03/11/2009; 04/12/2009] for non-default banks. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

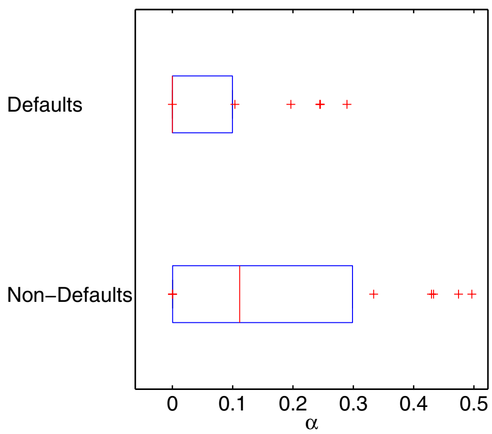

Fig. 4. *Box-plot of the LPPL parameter α.* The central (red) marks denote the median α's within the populations of the default banks ($1.3471 \cdot 10^{-4}$) and of the non-default banks (0.1116), respectively. The box's edges represent the 25th and 75th percentiles, respectively. Every point outside of the 25th and the 75th percentile is regarded as an outlier. Outliers are plotted individually and are denoted by (red) plus signs (+). Typical parameter ranges for default and non-default banks are defined by the 25th and the 75th percentiles, i.e. [$3.6527 \cdot 10^{-6}$; 0.0996] for default banks and [$5.6766 \cdot 10^{-4}$; 0.2986] for non-default banks. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

1.8325. However, we acknowledge that the universal log-angular frequency of $\omega \approx 9$ was substantiated by numerous analyses in a wide variety of financial time series. LPPL structures with a log-angular frequency close to nine have, for example, been identified in stock market trajectories [25,85], in commodity price time series [25,85,86], in foreign exchange rates [25], and in the US Fed Prime Loan Rate [25]. We will discuss possible reasons for the difference in the log-angular frequency between the CDS market and the other mentioned markets in Section 6.

As a typical example, the CDS spread development as well as the corresponding LPPL (13) of the Raiffeisen Zentralbank Österreich AG during the late-2000 financial crisis are shown in Fig. 7. From our point of view, this figure is sufficient to convince the reader of the LPPL behavior. However, the LPPL trend exhibits stochastic fluctuations which can usually be traced back to external events, such as Bear Stearns' near-bankruptcy on 03/14/2008 and the state-financed rescue under cover of JPMorgan on 03/17/2008. The noise peak on 12/08/2008 probably reflects the turmoil over Raiffeisen Zentralbank's preparation for bailout money at the end of November 2008. The LPPL trend in Raiffeisen Zentralbank's CDS spreads was temporarily disrupted by industrial nations' announcements to launch economic stimulation programs amounting to billions of dollars in early 2009. Except for the just mentioned exogenous shocks, Fig. 7 strongly indicates the LPPL patterns in Raiffeisen Zentralbank's CDS spread variations during the global financial crisis 2007–2009.

The main focus of this work is the individual analyses of the LPPL parameters A, B, C, t_c, α, ϕ, and ω with respect to their discriminatory powers to separate default banks from non-default banks. The classification accuracies of the parameter sets resulting in the best fits are detailed in Table 3. According to Hosmer and Lemeshow [77], U-values are categorized as

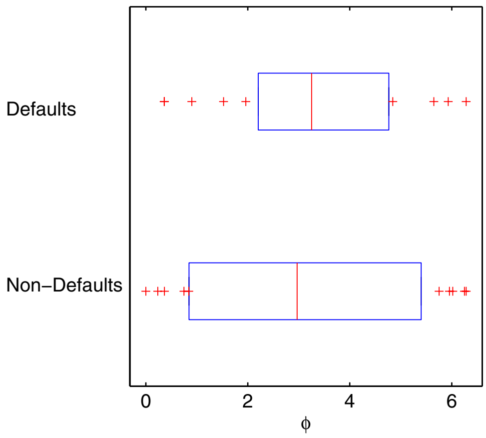

Fig. 5. *Box-plot of the LPPL parameter ϕ*. The central (red) marks denote the median ϕ's within the populations of the default banks (3.2506) and of the non-default banks (2.9666), respectively. The box's edges represent the 25th and 75th percentiles, respectively. Every point outside of the 25th and the 75th percentile is regarded as an outlier. Outliers are plotted individually and are denoted by (red) plus signs (+). Typical parameter ranges for default and non-default banks are defined by the 25th and the 75th percentiles, i.e. [2.2048; 4.7666] for default banks and [0.8470; 5.3995] for non-default banks. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

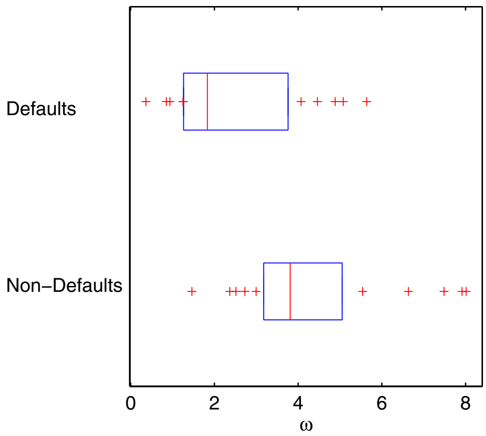

Fig. 6. *Box-plot of the LPPL parameter ω*. The central (red) marks denote the median ω's within the populations of the default banks (1.8325) and of the non-default banks (3.8097), respectively. The box's edges represent the 25th and 75th percentiles, respectively. Every point outside of the 25th and the 75th percentile is regarded as an outlier. Outliers are plotted individually and are denoted by (red) plus signs (+). Typical parameter ranges for default and non-default banks are defined by the 25th and the 75th percentiles, i.e. [1.2630; 3.7604] for default banks and [3.1788; 5.0527] for non-default banks. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

follows [77]:

$$\begin{aligned} U = 0.5 &\Leftrightarrow \text{no discrimination,} \\ 0.7 \le U < 0.8 &\Leftrightarrow \text{acceptable discrimination,} \\ 0.8 \le U < 0.9 &\Leftrightarrow \text{excellent discrimination,} \\ U \ge 0.9 &\Leftrightarrow \text{outstanding discrimination.} \end{aligned}$$

Excellent classification performances are, for example, obtained by the two linear parameters A and B. These results are traced back to the fact that higher CDS spreads are equivalent to higher probabilities of default [65]: A converges for $t \to t_c$ toward the CDS spread $S(t)$. The time derivative of the power law trend $B \cdot \alpha \cdot (t_c - t)^{\alpha-1}$ is controlled by parameter B. Hypothesizing that banks of low credit quality are characterized by higher and faster growing CDS spreads than banks of high credit quality, explains the excellent discriminative powers for A and B. The relatively low discriminatory power of C indicates that there is no fundamental difference between the log-periodic oscillations' amplitudes of low and high credit-standing banks. By considerations of Section 3, the non-existent classification accuracy of the time scale ϕ does not come as a surprise. The excellent discriminatory power of t_c is attributed to the fact that banks of low credit quality have in general shorter time distances to default $t_c - t$ than banks of high credit quality. Furthermore, the acceptable classification accuracies of α and ω are suggestive of LPPL structures' ability to discriminate between banks of low and high credit quality and therefore substantiate the hypothesis of Sornette et al. [10] that these parameters capture the collective organization of investors [10,71].

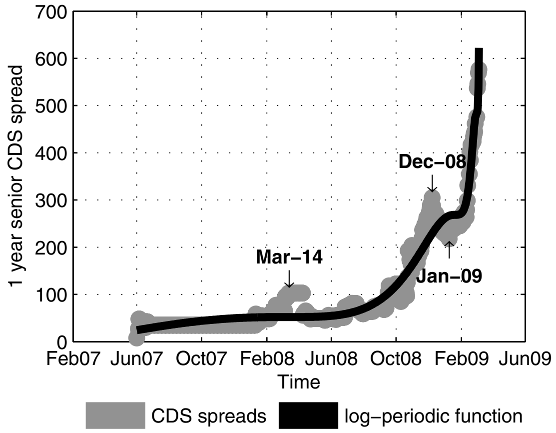

Fig. 7. *LPPL patterns in the CDS spread development of Raiffeisen Zentralbank Österreich AG.* The LPPL "locks in" on the market fluctuations and remains quite well in phase with the CDS spread development over the period from June 2007 until March 2009. The LPPL trend of the CDS spread is, however, superimposed by noise: The first marked noise peak is most likely attributed to Bear Stearns' near-bankruptcy on 03/14/2008. Raiffeisen Zentralbank's preparation in November 2008 for participating in the Austrian bank aid package signaled the start of massive noise superposition culminating on 12/08/2008. Passing economic stimulus programs in early 2009 had a temporary calming effect on the credit market which is not reproduced by the LPPL.

Table 3
Classification performances of LPPL parameters that result in the minimum sums of squared residuals. The Mann–Whitney *U* measures the discriminatory power of the LPPL parameters that result in the minimum sums of squared residuals. Values close to 1.0 correspond to outstanding discrimination whereas values close to 0.5 correspond to no discrimination.

Parameters	Mann–Whitney *U*
A	0.8025**
B	0.8025**
C	0.6225
t_c	0.8225***
α	0.7375*
ϕ	0.5525
ω	0.7675**

*** If the difference of medians is significant at the 0.1% level.
** If the difference of medians is significant at the 1.0% level.
* If the difference of medians is significant at the 5.0% level.

The determination of the four non-linear parameters in Eq. (13) is very sensitive to noise. In addition to the global minimum which is characterized by the parameter set that leads to the smallest sum of squared residuals, our fitting algorithm generates many local minima [32]. Some of these local minima lead to LPPLs which are graphically (almost) indistinguishable from the global minimum LPPL. As a result, widely varying estimates of the four non-linear parameters may circulate in literature for the same data set, all of which may be equally valid [32]. In order to demonstrate the robustness of our results with respect to noise, we saved not only the best, but the five best parameter sets for each CDS spread time series. Subsequently, we selected randomly one of the five parameter sets for each of the 40 banks and conducted a Mann–Whitney *U* test for each LPPL parameter. This procedure was repeated five times and the results are summarized in Table 4. This table clearly shows that our results are remarkably robust with respect to noise.

6. Conclusion

In order to investigate whether LPPL structures contain valuable information on banks' default risk, we proceeded as follows: First, we employed the JLS model to explain how positive feedback mechanisms between creditors can lead to liquidity shortages at banks. Second, we established the existence of LPPL structures guiding the CDS spread development of 40 banks during the late-2000 financial crisis. Our results for the log-angular frequency ω indicate that the CDS market is characterized by smaller values of ω than, for example, stock markets. We suggest to attribute the difference in the log-angular frequencies to the fundamental differences between CDS markets and stock markets: For one thing, the CDS market exhibits a completely different market structure than the equity market, since CDSs are traded over-the-counter. For another thing, the CDS market is dominated by financial institutions. Only 0.7% of all CDSs are held by non-financial customers [87]. In contrast, households' direct ownership of Australian equities amounts to 16% [88]. After calibrating the LPPL (13) to each

Table 4
Classification performances of randomly selected LPPL parameters. The parameter sets resulting in the five best fits to each CDS spread evolution were stored. One of these five parameter sets was randomly selected for each bank. Finally, the discriminatory powers of these LPPL parameters were univariately measured by the Mann–Whitney *U*. This procedure was repeated five times. Each column corresponds to one iteration. Values close to 1.0 correspond to outstanding discrimination whereas values close to 0.5 correspond to no discrimination.

Parameters	Mann–Whitney *U*s for randomly selected parameter sets				
A	0.8225***	0.8075***	0.8050**	0.7575**	0.8175***
B	0.8275***	0.8075***	0.8075***	0.7600**	0.8150***
C	0.5100	0.5225	0.6700	0.5000	0.5625
t_c	0.8100***	0.7875**	0.8050**	0.7700**	0.7475**
α	0.7775**	0.7725**	0.7550**	0.7075*	0.7775**
ϕ	0.6025	0.6300	0.6250	0.5125	0.6250
ω	0.7975**	0.7775**	0.7425**	0.7350*	0.7750**

*** If the difference of medians is significant at the 0.1% level.
** If the difference of medians is significant at the 1.0% level.
* If the difference of medians is significant at the 5.0% level.

of the 40 time series, we evaluated the univariate classification performances of the best-fitting LPPL parameters in a third step. A corresponds to the CDS spread immediately prior to the date of default. Since CDS spreads reflect investors' aggregate opinion about the credit quality of the underlying bank, they represent reliable indicators of whether the reference entity is going to default in the near future. Therefore, the excellent discriminative power of A makes perfect sense. Likewise, B is basically a measure of the slope in the CDS spread. A rapid rise in the CDS spread is surely indicative that the underlying reference is on the brink of ruin. The low discriminatory power of C indicates that there is no fundamental difference between the log-periodic oscillations' amplitudes of low and high credit-standing banks. t_c represents the point in time when a liquidity shortage is most probable. Since default banks tend to have earlier critical points in time than non-default banks, it is highly plausible that t_c has reached the highest univariate discriminative power. Both the critical exponent α and the log-angular frequency ω obtained acceptable discriminative powers. Sornette et al. [10] claimed that α and ω capture the collective organization of investors [10,71]. The non-existent discriminative power of ϕ can be traced back to the fact that ϕ denotes a time scale.

Under the assumption that LPPL structures are generated by the mechanism presented in Section 3, LPPL patterns are the observable signature of alternating positive and negative feedbacks ending up in the final "rupture" [22]. In the scope of this study, systematic differences in LPPL parameters between banks of low and of high credit quality were discovered. These differences appear to characterize investor behavior when banks are at risk of insolvency. This enables us to draw conclusions of banks' refinancing options from the investor behavior quantified by LPPL parameters. Assuming that the level of interpretability of default indicators constitutes a major criterion for selection in practice, not only the critical exponent α and the log-angular frequency ω, but in particular the critical time t_c seem to hold great promise in differentiating defaulted from non-defaulted banks. The application of all LPPL parameters is, however, associated with short-term prediction horizons, because the long-range correlation between creditors may quickly build up. Therefore, these parameters are particularly suited to complement traditional balance sheet analysis. Further investigations demonstrated the robustness of the discriminative powers in differentiating between banks of low and high credit quality with respect to noise.

The strength of our results is on the one hand limited by our default definition and on the other hand by our data set size. The first problem is as follows: Some banks that received bailout money during the financial crisis might not have faced serious financial difficulties. Instead, they might have been urged to take bailout funds in order to disguise which banks have really been in financial straits in avoidance of further bank runs. The second problem relates to the limited number of insolvencies among major financial institutions even during the global financial crisis 2007–2009. Instead of taking large financial institutions' defaults into account, governments all over the world bailed out systematically important institutions [89–91]. To minimize the reduction of our data set, we decided to refrain from incorporating credit event definition underlying CDSs. Instead, we defined bailout banks as defaults and non-bailout banks as non-defaults, respectively, to proceed our investigation.

Although the LPPL (13) quantifies the herding behavior of creditors, this approach cannot capture exogenous influences on the financial system [92]. Of course, real financial markets are exposed to many external factors such as sudden outbreaks of war or unexpected interventions by central banks. Due to our extreme perspective that all CDS spread fluctuation during the late-2000 financial crisis were endogenously triggered by the collective behavior of the investors, there are discrepancies between the real data and the LPPL.

Despite these limitations, our investigation provides further arguments in favor of discrete scale invariance governing the fluctuations of financial markets. Our results support the hypothesis that herding behavior among investors can limit the refinancing options of banks which, in the worst case, can end up in insolvency.

Appendix

For the sake of completeness, we show that the constraint introduced by von Bothmer and Meister [33] may be replaced by the stronger condition presented by Sornette and Zhou [1]. Starting from Eqs. (8) and (13), the following relationship can be derived:

$$h(t) \approx -\alpha \cdot B \cdot (t_c - t)^{\alpha-1} - \alpha \cdot C \cdot (t_c - t)^{\alpha-1} \cdot \cos(\omega \cdot \ln(t_c - t) + \phi) + \omega \cdot C \cdot (t_c - t)^{\alpha-1} \cdot \sin(\omega \cdot \ln(t_c - t) + \phi). \quad (19)$$

In compliance with Ref. [33], we require the following inequality to hold:

$$0 \le -\alpha \cdot B \cdot (t_c - t)^{\alpha-1} - \alpha \cdot C \cdot (t_c - t)^{\alpha-1} \cdot \cos(\omega \cdot \ln(t_c - t) + \phi) + \omega \cdot C \cdot (t_c - t)^{\alpha-1} \cdot \sin(\omega \cdot \ln(t_c - t) + \phi). \quad (20)$$

Since $t_c - t \ge 0\, \forall t \le t_c$, Eq. (20) simplifies to:

$$\begin{aligned} 0 &\le -\alpha \cdot B + C \cdot (\omega \cdot \sin(\omega \cdot \ln(t_c - t) + \phi) - \alpha \cdot \cos(\omega \cdot \ln(t_c - t) + \phi)) \\ &= -\alpha \cdot B \pm |C| \cdot |\omega \cdot \sin(\omega \cdot \ln(t_c - t) + \phi) - \alpha \cdot \cos(\omega \cdot \ln(t_c - t) + \phi)|. \end{aligned} \quad (21)$$

In the next step, we have to prove

$$|\omega \cdot \sin(\omega \cdot \ln(t_c - t) + \phi) - \alpha \cdot \cos(\omega \cdot \ln(t_c - t) + \phi)| \le \sqrt{\alpha^2 + \omega^2}, \quad (22)$$

which is easily done starting from the fact that the negative of a squared real number is trivially smaller than or equal to zero:

$$\begin{aligned} 0 &\ge -(\omega \cdot \cos(\omega \cdot \ln(t_c - t) + \phi) + \alpha \cdot \sin(\omega \cdot \ln(t_c - t) + \phi))^2 \\ 0 &\ge -\omega^2 \cdot \cos^2(\omega \cdot \ln(t_c - t) + \phi) \\ &\quad - 2 \cdot \alpha \cdot \omega \cdot \cos(\omega \cdot \ln(t_c - t) + \phi) \cdot \sin(\omega \cdot \ln(t_c - t) + \phi) \\ &\quad - \alpha^2 \cdot \sin^2(\omega \cdot \ln(t_c - t) + \phi). \end{aligned} \quad (23)$$

Adding $\alpha^2 + \omega^2$ on both sides of Eq. (23) and subsequently using $\sin^2(x) = 1 - \cos^2(x)$, we obtain:

$$\begin{aligned} \alpha^2 + \omega^2 &\ge \big(\omega^2 \cdot \big(1 - \cos^2(\omega \cdot \ln(t_c - t) + \phi)\big) \\ &\quad - 2 \cdot \alpha \cdot \omega \cdot \cos(\omega \cdot \ln(t_c - t) + \phi) \cdot \sin(\omega \cdot \ln(t_c - t) + \phi) \\ &\quad + \alpha^2 \cdot \big(1 - \sin^2(\omega \cdot \ln(t_c - t) + \phi)\big)\big) \\ \alpha^2 + \omega^2 &\ge \omega^2 \cdot \sin^2(\omega \cdot \ln(t_c - t) + \phi) \\ &\quad - 2 \cdot \alpha \cdot \omega \cdot \cos(\omega \cdot \ln(t_c - t) + \phi) \cdot \sin(\omega \cdot \ln(t_c - t) + \phi) \\ &\quad + \alpha^2 \cdot \cos^2(\omega \cdot \ln(t_c - t) + \phi) \\ \alpha^2 + \omega^2 &\ge (\omega \cdot \sin(\omega \cdot \ln(t_c - t) + \phi) - \alpha \cdot \cos(\omega \cdot \ln(t_c - t) + \phi))^2 \\ \sqrt{\alpha^2 + \omega^2} &\ge |\omega \cdot \sin(\omega \cdot \ln(t_c - t) + \phi) - \alpha \cdot \cos(\omega \cdot \ln(t_c - t) + \phi)|. \end{aligned} \quad (24)$$

Substituting inequality (22) in (21) finally yields:

$$-\alpha \cdot B \ge |C| \cdot \sqrt{\alpha^2 + \omega^2}. \quad (25)$$

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