Anderson localization of light in $\mathcal{PT}$-symmetric optical lattices

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Anderson localization (AL) of light is investigated numerically in a disordered parity-time ($\mathcal{PT}$)-symmetric potential, in the form of an optical lattice. The lattice is recorded in a nonlinear medium with Kerr nonlinearity. We demonstrate enhancement of light localization in a $\mathcal{PT}$-symmetric lattice, as compared to the localization in the corresponding real lattice. The effect of strength of the gain-loss component in the $\mathcal{PT}$ lattice on various regimes of AL is also discussed. It is found that the localization exists and is further enhanced above the threshold strength of the imaginary part of the potential. The influence of nonlinearity and disorder level on the transverse localization of light in such a complex-valued potential is addressed.

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The study of physical systems that do not obey parity ($\mathcal{P}$) and time-reversal ($\mathcal{T}$) symmetries separately but exhibit a *combined* parity-time ($\mathcal{PT}$) symmetry has attracted a great deal of attention during the past few years. There has been an increased interest in such systems in various fields of physics, from quantum field theory and mathematical physics [1–3], to solid-state and atomic physics [4,5] and optics [6–10]. These studies follow the original work by Bender and co-workers [1,2], which demonstrated that non-Hermitian Hamiltonians can have an entirely real eigenvalue spectrum, provided they respect the $\mathcal{PT}$ symmetry. A Hamiltonian is $\mathcal{PT}$-symmetric when its potential $V(\mathbf{r})$—now a complex function—satisfies the condition $V(\mathbf{r}) = V^*(-\mathbf{r})$; i.e., the real part of the complex-valued $\mathcal{PT}$ potential is a symmetric function of position, whereas the imaginary component is antisymmetric.

Among the most interesting features of this class of Hamiltonians is the existence of *phase transition*, arising from a bifurcation in the eigenspectrum of the $\mathcal{PT}$-symmetric potential, as the strength of the imaginary part of the potential is increased; above a threshold, the spectrum becomes complex, leading to gain and loss in eigenfunctions. Convenient complex-valued potentials can be realized in optics, using the refractive index modulation that combines gain and loss regions [6,11]. This is the consequence of the equation describing the propagation of an optical beam in a medium being of the Schrödinger-equation type, with the potential proportional to the distribution of the index of refraction.

Anderson localization (AL) in disordered media, originally predicted 50 years ago [12], has drawn a great deal of attention in the past few years. The phenomenon has been observed in a variety of classical and quantum systems [13–15], including *light waves* [16–18]. AL of light has come to the focus of investigations, especially in nonlinear optics and photonics, due to the emergence of new optical technologies and media, such as disordered photonic crystals and optical lattices, in which the appearance of AL considerably changes the propagation of light [18–21].

In this Letter, we present a numerical study of transverse localization of light in a uniformly disordered two-dimensional complex lattice (Fig. 1). We aim at elucidating the effect of disorder on $\mathcal{PT}$-symmetric potentials, or conversely, the effect of $\mathcal{PT}$-symmetry on AL. We demonstrate *enhancement* of AL in $\mathcal{PT}$-symmetric potential, as compared to the real-valued potential. We find that the existence of phase transition in the perfect lattice does not abruptly or qualitatively change AL. This finding stems from the fact that disorder effectively *destroys* $\mathcal{PT}$-symmetry of the lattice, rendering the existence of the threshold strength of the imaginary component of the potential in the disorder-free lattice irrelevant. As soon as uniform disorder is introduced, $\mathcal{PT}$-symmetry is lost; the eigenvalues are no longer real and there immediately appear spatially distributed gain and loss regions. While disordered $\mathcal{PT}$-symmetric potentials are viable and interesting in nonlinear optics, their significance in addressing fundamental questions of quantum mechanics is rather limited. We also show that there is no localization when only gain or only loss is present in the lattice, under conditions similar to the $\mathcal{PT}$ lattice.

We consider the propagation of a light beam in such a lattice along the z axis, perpendicular to the lattice, using the scaled nonlinear Schrödinger equation for the paraxial optical field amplitude E:

$$i\frac{\partial E}{\partial z} = -\Delta E - \gamma|E|^2 E - VE, \tag{1}$$

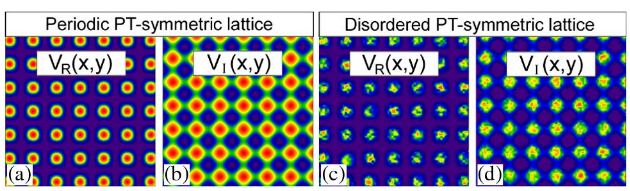

Fig. 1. (Color online) Amplitude distributions of (a) real and (b) imaginary component of the periodic $\mathcal{PT}$-symmetric potential $V(x, y)$. (c)–(d) The same potential with disorder.

where Δ is the transverse Laplacian, and γ is the dimensionless strength of the nonlinearity. Thus, we consider a nonlinear Kerr medium, in which a complex lattice is introduced. The question is, What is the effect of a $\mathcal{PT}$-symmetric potential on AL?

The potential function V is chosen in the form $V(x,y) = V_0\{\cos^2(x) + \cos^2(y) + iW_0[\sin(2x) + \sin(2y)]\}$, where V_0 is the depth of the real component of the potential and W_0 is the relative magnitude of the imaginary component. For such a $\mathcal{PT}$-symmetric lattice, the phase transition point is known to be at $W_0 = 0.5$ [7,8]. We solve the propagation equation by employing a beam propagation method developed earlier [22].

To study localization effects, we realize disorder using random potential depth V_{0r} instead of V_0, which takes values in the interval $V_0(1 - Nr) < V_{0r} < V_0(1 + Nr)$. Here r is the random number generator taken from a uniform distribution on the interval [0,1] and N determines the degree of disorder. We quantify the disorder level by the ratio between the potential depth of the random and the periodic $\mathcal{PT}$-lattice. Many realizations of a randomized system, necessary for studying AL, are realized by starting each simulation with different seeds of the random number generator. All quantities of interest characterizing AL [19] are measured as ensemble averages over 100 disorder realizations. Once randomization is included, the eigenvalues of the propagating modes are no more real-valued, so the slightest disorder causes the loss of $\mathcal{PT}$ symmetry.

First, to estimate how $\mathcal{PT}$-symmetric lattice affects the localization process, we compare localization effects in the $\mathcal{PT}$ lattice ($W_0 = 0.45$) with the corresponding real lattice ($W_0 = 0$). Localization is observed by increasing the level of disorder; for quantitative analysis, we use the effective beam width, defined as $\omega_{\text{eff}} = P^{-1/2}$, where $P = \int |E|^4(x,y,L)\mathrm{d}x\mathrm{d}y / \{\int |E|^2(x,y,L)\mathrm{d}x\mathrm{d}y\}^2$ is the inverse participation ratio [18]. We analyze localization effects in both lattices in the linear regime, and measure the averaged effective width as a function of the propagation distance [Fig. 2(a)]. To discern the influence of crystal length on the localization process more clearly, we consider a longer crystal, $L = 50$ mm. For a weak disorder, the beam first expands diffusively before localization is reached; a broadening of the beam is more pronounced in the real than the $\mathcal{PT}$-symmetric lattice. When stronger disorder is introduced, the localization regime is reached after a short propagation distance, but again with broader beam widths in the real than in the $\mathcal{PT}$-symmetric lattice, especially at longer distances. Also, one can see an enhanced expansion of the beam at the disorder level 20% as compared to disorder level 5% in the case of the $\mathcal{PT}$-symmetric lattice. This effect is a sign of competition between disorder and $\mathcal{PT}$ symmetry in the localization process. As a general conclusion, the localization is more enhanced in $\mathcal{PT}$-symmetric lattices, as compared to the corresponding real lattices.

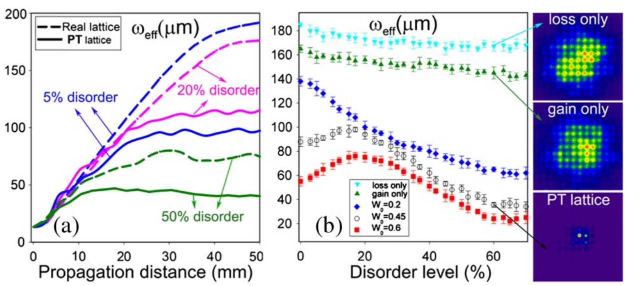

Fig. 2. (Color online) (a) Comparison between localization in real and $\mathcal{PT}$ lattices in the linear regime: effective beam widths versus the propagation distance, for different disorder levels (5%, 20%, 50%). (b) Influence of gain and loss on localization: effective beam width versus the disorder level, in the linear regime, for different gain-loss strengths. Panels to the right depict actual intensity distributions. Physical parameters are the crystal length $L = 20$ mm, $V_0 = 1$, $W_0 = 0.45$, input beam intensity $|E_0|^2 = 0.5$, input beam FWHM = 15 μm, and lattice constant 15 μm.

Next, we study the effect of the strength of the gain-loss component [the imaginary part of $V(x,y)$] in the $\mathcal{PT}$ lattice on the localization process, as well as localization (or even better, the *absence of*) when only gain or only loss is present in the lattice. The effective beam width at the lattice output versus the disorder level is presented in Fig. 2(b), for various cases. For lower values of the gain-loss strength, the reduction in the effective width is similar to the case of the real lattice: the beam width decreases as the level of disorder is increased; in other words, disorder always suppresses transport. Increasing the strength of the gain-loss component leads to an enhanced initial expansion of the beam for lower disorder levels, and more pronounced localization at higher gain-loss strength. Furthermore, AL is even more pronounced above the threshold value $W_0 = 0.5$. In fact, the influence of threshold is diminished. This is not difficult to understand: the appearance of complex eigenvalues at any W_0 induces some eigenfunctions to grow—which increases the *longitudinal* transmission but does not influence the *transverse* localization—and it induces other eigenfunctions to attenuate, which decreases the transmission but enhances the localization. However, when only gain or only loss is present in the lattice, practically no localization is observed (see panels in Fig. 2).

We also study the influence of nonlinearity on AL in complex-valued potentials, by considering three different regimes: linear ($\gamma = 0$), nonlinear focusing ($\gamma > 0$), and nonlinear defocusing ($\gamma < 0$). Measuring the effective beam width at the lattice output enables us to compare localization effects in $\mathcal{PT}$-symmetric lattices in linear and nonlinear media [Fig. 3(a)]. In the linear medium, increasing the level of disorder leads to an enhanced expansion of the beam at lower disorder and then to localization for further increase in disorder. In the presence of focusing nonlinearity, the reduction in the effective width is steeper than in the defocusing and the linear case. Considering the effect of defocusing nonlinearity, one can see that the width of the average output beam is slightly increased, as compared to the linear regime. The corresponding intensity distributions of localized modes in the periodic as well as disordered $\mathcal{PT}$-symmetric lattice are shown in Fig. 3(b). For the same disorder level, the localization effects are most pronounced for the focusing nonlinearity, followed by the linear medium, and then the defocusing nonlinearity.

To demonstrate AL in linear and nonlinear media with a $\mathcal{PT}$-symmetric lattice, we compare averaged intensity profiles on the logarithmic scale, obtained for different

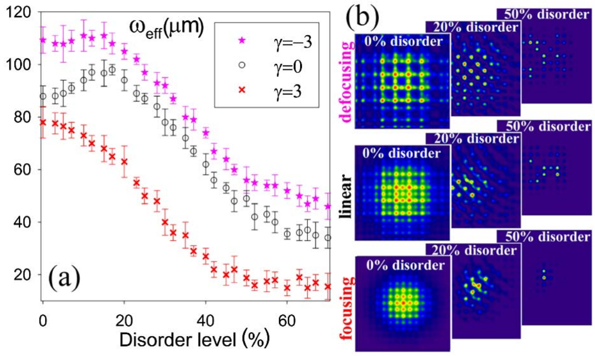

Fig. 3. (Color online) Transverse localization of light in $\mathcal{PT}$-symmetric lattice. (a) Effective beam widths at the lattice output versus the disorder level for the defocusing, linear, and focusing media. The points are ensemble averages and error bars depict the spread coming from statistics. (b) Examples of localized modes: output intensity distributions of the probe beam with no disorder, and for disorder levels 20% and 50%. Parameters are as in Fig. 2.

disorder levels. Increasing the level of disorder causes the output beam profiles to narrow down. The exponentially decaying tails are a direct indication of the level of localization. The output beam profile is the broadest for the defocusing, followed by the linear, and then the focusing medium. An exponentially decaying intensity profile of the form $I \sim \exp(-2|x|/\xi)$ is observed in the focusing medium for 30% disorder level. We measure the localization length ξ by fitting the averaged profile to the said form and determine its value to be $\xi \approx 43$ μm. The same value of ξ is measured along the y axis as well. A higher disorder is necessary to observe the same localization in the linear and the nonlinear defocusing medium, as compared to the focusing medium. In the linear medium, this value of localization length is observed at ~57% disorder level and in the defocusing medium at ~73%.

In summary, we have studied AL of light in $\mathcal{PT}$-symmetric lattices. We have analyzed numerically how the gain-loss component of the $\mathcal{PT}$-symmetric lattice, disorder, and Kerr nonlinearity modify transverse localization of light. Uniform disorder effectively removes the $\mathcal{PT}$ symmetry. We have demonstrated that the presence of $\mathcal{PT}$-symmetric potentials enhances light localization, as compared to the localization in the corresponding real lattice. Above the threshold strength the localization is even further enhanced. However, in the presence of gain only or of loss only, little localization is evident.

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