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Abstract. We studied numerically the Anderson localization of light in the two-dimensional square photonic lattice, optically induced in both linear and nonlinear dielectric media. We investigated the combined influence of nonlinearity and disorder on Anderson localization in bulk media, as well as when there is interface between the linear and nonlinear medium. In bulk, there exist strongly nonlinear regimes in which localization is less pronounced than in the linear regime. We found that in the defocusing nonlinear regime, the localization is always less pronounced than in the linear regime. We also studied surface-localized modes near the interface between linear and nonlinear dielectric media with an optically induced photonic lattice, and observed the threshold for their existence. We demonstrate suppression of the localization at the interface for lower disorder levels, compared with both pure linear and pure nonlinear media. [DOI: 10.1117/1.OE.51.8.088001]

Subject terms: light localization; disorder; photonic lattice; interface.

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1 Introduction

Quantum effects in the transport properties of electronic systems have become one of the central issues of condensed matter physics. Likewise, the phenomenon of localization of the electronic wave function in random media, the so-called Anderson localization, is also one of the basic concepts in solid-state physics.[1] Originally introduced to explain localization of electronic wave functions as they propagate through disordered crystals,[1] Anderson localization was soon extended to many other fields of physics, such as acoustics,[2] Bose-Einstein condensates,[3] and optics.[4,5] Lately, the transverse localization of light as it propagates in disordered optically induced photonic crystals has become a hot topic of research.[6,7]

Despite its 50-year history,[8,9] Anderson localization still excites much interest in a variety of systems, including light waves. With respect to the localization of light, many issues are still not fully understood, such as the nonlinear Anderson localization. A competition between nonlinearity and disorder was investigated experimentally in fiber arrays[10] and in disordered two-dimensional photonic lattices.[6] Lahini et al.[11] experimentally demonstrated the effect of medium's nonlinearity on the light localization and concluded that the nonlinearity in a disordered medium favors localization to occur within a shorter distance. Recently, it was investigated theoretically and numerically the effect of focusing and defocusing nonlinearities on Anderson localization in highly nonlocal media.[12]

Surface states, a special type of waves localized near an interface separating two different materials, have recently attracted special attention, owing to novel physics and possible interesting applications in all-optical switching and sensing.[13,14] In a number of studies the properties of light propagating along the interface between linear and nonlinear media have been explored.[15–17] There are also recent investigations concerning interface nonlinear effects in random media.[18,19] In our recent work, we investigated different aspects of transverse localization of light in two-dimensional (2-D) photonic lattice: how lattice corner and edge influence Anderson localization of light,[20] the effect of dimensionality crossover on transverse localization of light,[21] and the influence of interface, as well as phase sleep defect on the localization process.[22] But here, we investigate the influence of medium's nonlinearity on Anderson localization of light, as well as different combinations of interface between linear and nonlinear dielectric media with optically induced disordered photonic lattices and compare them with pure linear or nonlinear media (Fig. 1).

First, we analyzed the effect of medium's nonlinearity on the transverse Anderson localization of light in a 2-D square photonic lattice. We studied localization in the nonlinear regime, with both focusing and defocusing nonlinearity, and compared them with the localization in the linear regime. A systematic quantitative study of the dependence on both the strength of disorder and the strength of nonlinearity on the Anderson localization in such a system is presented. Here, we consider the Kerr-type cubic nonlinearity. There exist strongly focusing nonlinear regimes, where the localization is less pronounced than in linear regime. However, in the defocusing regimes localization is always less pronounced than in the linear regime.

Second, we extend these concepts to the light localization at the interface between linear and nonlinear dielectric media with optically induced disordered photonic lattices. We reveal that the localization at the linear-nonlinear interface

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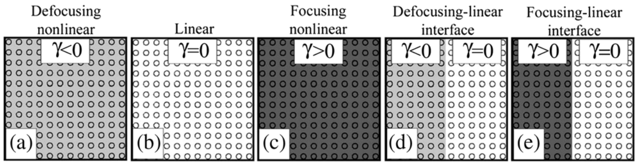

Fig. 1 Sketch of a photonic lattice with different strength of nonlinearity (a)–(c), and an interface between linear and nonlinear dielectric areas (d)–(e).

occurs if the strength of disorder and (or) the strength of the nonlinearity exceed a critical value. We observed threshold curves for the existence of surface-localized modes in the region of these two parameters: disorder level and medium's nonlinearity in the nonlinear part of the medium.

2 Theoretical Model and System Geometry

We study the localization of light in an optically-induced photonic lattice and describe the propagation of a beam along the z-axis using an effective nonlinear Schrödinger equation for the complex electric field amplitude E:

$$i\partial_z E = -\Delta E - \gamma|E|^2 E - VE, \tag{1}$$

where z is the propagation coordinate, Δ is the transverse Laplacian, and γ is the dimensionless nonlinearity coefficient. V is the transverse lattice potential, defined as a sum of Gaussian beams with peak intensity V_0. The propagation equation is solved numerically by employing a numerical approach developed earlier.[23] To study Anderson localization effects, we realize disorder using random lattices. Random lattice intensity V_{0r} takes the values between $V_0(1 - Nr) < V_{or} < (1 + Nr)$, where r is the random number generator from the interval [0,1], and N determines the degree of disorder. We quantify the disorder level by the ratio between the intensity of the random lattice and the intensity of the periodic lattice. Transverse localization was investigated experimentally in a triangular lattice.[6] In our analysis we used a square photonic lattice, and longer propagation distance than in the experiment (20 mm).

To consider propagation of light at the interface between linear and nonlinear dielectric media with an optically induced photonic lattice [Fig. 1(d) and 1(e)], we use a system of equations for the electric field amplitude E in the two regions:

$$i\partial_z E = -\Delta E - \gamma|E|^2 E - VE, \qquad -\infty \le x \le 0 \tag{2}$$

$$i\partial_z E = -\Delta E - VE, \qquad 0 \le x \le \infty. \tag{3}$$

We launched a narrow Gaussian beam positioned at the interface. We use $\gamma > 0$ for the focusing nonlinearity and $\gamma < 0$ for the defocusing nonlinearity. The linear regime means that the nonlinearity is turned off, $\gamma = 0$. The study of Anderson localization requires many realizations of the randomized system. In our system, they are realized by starting each simulation with different seeds for the random number generator.

3 Linear Versus Nonlinear Localization

Our interest is to consider the influence of nonlinearity on the Anderson localization. We investigated localization effects in the linear regime and compared them with the localization in both the focusing and defocusing nonlinear regimes. To observe the effect of Anderson localization, we increased the level of disorder. For quantitative analysis of the phenomenon, we used the standard quantities for the description of Anderson localization: the inverse participation ratio $P = \int I^2(x,y,L)\mathrm{d}x\mathrm{d}y/[\int I(x,y,L)\mathrm{d}x\mathrm{d}y]^2$, and the effective beam width $\omega = P^{-1/2}$.

To compare Anderson localization in different regimes, we measured the effective beam width at the lattice output for different disorder levels. The output beam waist is changed along propagation distance. At weak disorder the beams first expand diffusively, before localization sets in. However, when stronger disorder is introduced, the behavior is somewhat different: the localization is reached after a short propagation distance. Many realizations of disorder are needed to measure such quantities. We took 100 realizations of disorder for each disorder level. The typical results are summarized in Fig. 2(a). As one starts increasing the strength of medium's nonlinearity (from defocusing to focusing), the effective beam width gets smaller, as the level of disorder is increased [Fig. 2(a)]. However, with the further increase

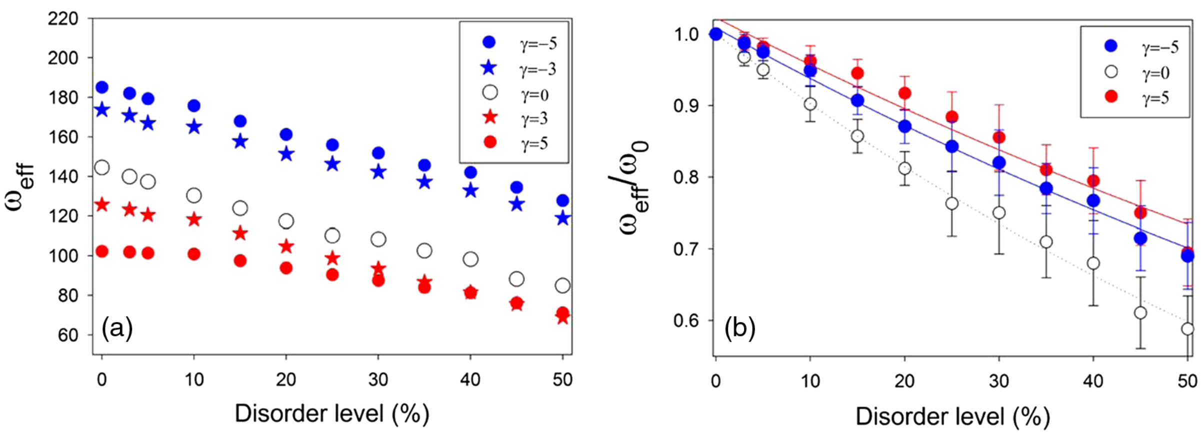

Fig. 2 Comparison between Anderson localization in the linear regime and different nonlinear regimes. (a) Effective beam width at the lattice output versus disorder level, for different strength of nonlinearity. (b) Averaged effective beam widths normalized to their input values. Points are ensemble averages and lines are least square fits through the points. Error bars depict the spread in values coming from statistics. Physical parameters are: crystal length $L = 20$ mm, input lattice intensity $V_0 = 1$, lattice period $d = 15$ μm, input beam intensity $|E_0|^2 = 0.5$, input beam FWHM $= 13$ μm.

in the focusing nonlinearity, the interplay of nonlinearity and disorder leads to different conclusions. At a certain value of focusing nonlinearity, a threshold is reached, after which the disorder produces little influence on the localization. For the parameters we used, this strong nonlinearity threshold, when the disorder ceases to produce significant effect on the localization process, is $\gamma \approx 5$. In such a focusing regime, the localization is less pronounced than in the linear regime.

This is clearly visible when we measure the averaged effective beam width. Averaged effective widths, normalized to the corresponding input values, are presented as functions of the disorder level in Fig. 2(b). Error bars in the figure depict the spread in values coming from different runs. We present only one representative value of the nonlinearity strength, in both the focusing and defocusing regimes ($\gamma = 5$ and $\gamma = -5$). In the case of defocusing nonlinearity, the localization is always less pronounced than in the linear regime. The effective beam width decreases faster in the linear regime, compared with the defocusing nonlinear regime, as the level of disorder is increased. In the focusing case, the situation is more complex, and the strength of localization depends on the strength of nonlinearity. This sharp difference between the linear and nonlinear regimes is a sign of competition between the disorder and the nonlinearity influences on the localization process. Effective beam width as a function of nonlinearity coefficient γ, is presented in Fig. 3(a), for different disorder levels. Examples of output localized modes in different nonlinear regimes are presented for the 20% disorder level Fig. 3(b) and 3(c), and for 50% disorder Fig. 3(d) and 3(e).

4 Localization at the Interface Between Linear and Nonlinear Dielectric Media

Next, we extended the investigation of light localization to the interfaces between linear and nonlinear dielectric media in optically induced square photonic lattice. First, we varied the medium's nonlinearity in the nonlinear part of the medium and increase the level of disorder, keeping all other parameters fixed. By increasing the level of disorder, the output intensity beam profile narrows down, and the exponentially-decaying tails become a direct indication of the localization. The output intensity profiles are averaged over an ensemble of different realizations for each disorder level, and fitted with the appropriate functions. The cross-over to localization is evident by the transition from a broad Gaussian-shaped profile to an exponentially-decaying intensity profile of the form $I \sim \exp(-2|x|/\xi)$. Here, ξ stands for the localization length.

The localization regions for both focusing-linear and defocusing-linear interfaces are presented in Fig. 4. They are based on the results of many numerical simulations in the plane of two parameters: the medium's nonlinearity and the disorder level. We observed the threshold curve for the existence of surface localized modes at the linear-nonlinear interface in this parameter space. For lower values of the nonlinearity strength γ and lower disorder levels, only non-localized modes exist. When either the medium's nonlinearity or the disorder level is increased, one observes a sharp transition to the localized modes.

We found that different strengths of a medium's nonlinearity can lead to different relative positions of the surface modes. It is shown that the localized modes at the linear-focusing interface are mostly positioned in the focusing nonlinear part, whereas in the case of the linear-defocusing interface they are shifted to the linear part of the medium.

Finally, to estimate how the linear-nonlinear interface affects the localization process, we compared the surface modes with the localized beams in pure linear and pure nonlinear medium. A relevant quantity for the comparison of the localization level for the corresponding cases is the averaged effective width normalized to the corresponding input values, for different cases.

We compared the linear-focusing interface with the pure linear and focusing nonlinear media [Fig. 5(a)], and also the linear-defocusing interface with the pure linear and defocusing nonlinear media [Fig. 5(b)]. In completely linear or nonlinear media, the effective beam width became smaller, as the level of disorder increased. But for both interfaces an increase in the level of disorder led to an enhanced expansion of the beam, and then to localization for further increase in the level of disorder. For lower disorder levels, localization was more pronounced in both completely linear and nonlinear cases than at the interfaces. As the strength of disorder was further increased, the localization effects suddenly become less pronounced in both completely linear and nonlinear cases. In these examples, we use the strength of nonlinearity $\gamma = 5$ for the focusing case and $\gamma = -5$ for the defocusing case.

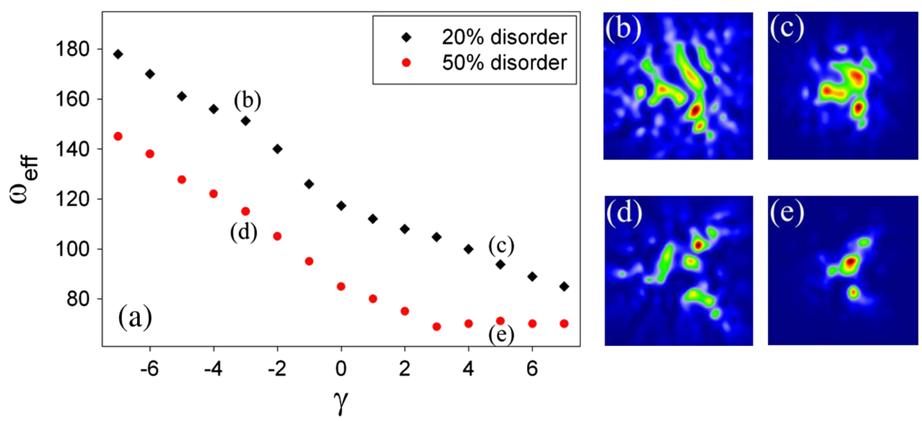

Fig. 3 (a) Effective beam width at the lattice output versus nonlinearity coefficient γ, for different disorder level. Examples of output localized modes in different nonlinear regimes, for 20% disorder (b)–(c), and for 50% disorder (d)–(e). Physical parameters are as in Fig. 2.

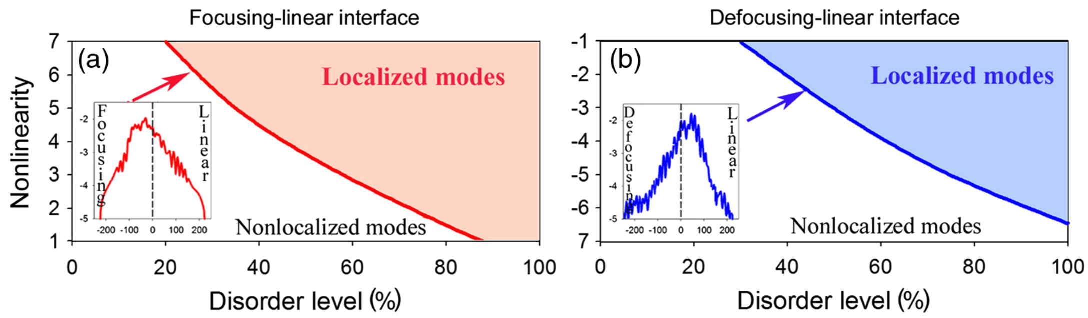

Fig. 4 Threshold curves for the existence of surface localized modes. The strength of nonlinearity in nonlinear part of medium versus disorder level for: (a) focusing and (b) defocusing nonlinearity. Physical parameters are as in Fig. 2.

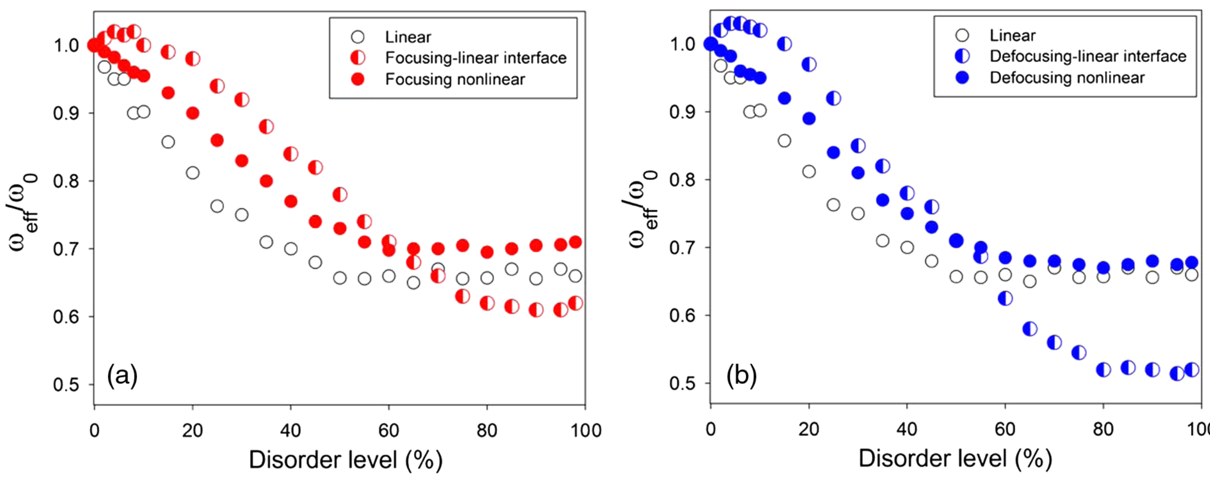

Fig. 5 Comparison between localization in the presence of interface and with no interface. Effective beam width at the lattice output versus disorder level for two different interfaces and for pure linear and nonlinear cases. The widths are normalized to their input values. Parameters are as in Fig. 2.

5 Conclusions

In conclusion, we analyzed numerically the influence of medium's nonlinearity on the Anderson localization of light. We investigated the Anderson localization behavior in both focusing and defocusing nonlinear regimes, and compared them with the localization in the linear regime. In the defocusing nonlinear regime the localization was always less pronounced than in the linear regime. However, in the focusing nonlinear regime, the localization effects depend on the strength of the nonlinearity. We analyzed localized modes near the interface between linear and nonlinear dielectric media with an optically induced photonic lattice. The suppression of localization was demonstrated at the interface for lower disorder levels, compared with both the pure linear and the pure nonlinear medium. We observed the threshold for the existence of surface localized modes at the linear-nonlinear interface within an optically induced photonic lattice.

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