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Disorder-induced localization of light near edges of nonlinear photonic lattices

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ABSTRACT

We investigated the influence of edges and corners on the Anderson localization of light in disordered two-dimensional photonic lattices that are optically induced in nonlinear saturable photorefractive media. A systematic quantitative study of gradual transition from corner to bulk Anderson localization in truncated two-dimensional photonic lattices was carried out. We analyzed numerically the localization at several corners and edges of the square and triangular photonic lattices and compared them with the localization in bulk medium. We found that, for strong disorder, corners and edges effectively suppress Anderson localization, as compared to the bulk, but to a varying degree.

1. Introduction

The study of light propagation in disordered photonic lattices has attracted a renewed interest in Anderson localization [1], due to novel opportunities for observing light localization in random media. Owing to the analogy of photonic lattices to solid state systems and thanks to the fact that longitudinally invariant disorder is more easily realized in lattices [2,3], experimental activities in the study of disorder-induced localization have recently taken a new turn [4–6]. The concept of Anderson localization can be extended to mutually incoherent counterpropagating beams, where the dynamical localization of time-changing beams can take place [7]. Recently, Anderson localization in a system with photonic quasicrystals has excited much interest [8].

Wave localization relies on random defects present in an otherwise periodic structure. The edges joining planar interfaces produce a strong impact on the soliton formation [9]. Since the truncation of the lattice represents a major distortion in the periodicity, one would expect that the presence of boundary would enhance the localization. However, this is not so. Recent experimental study of light localization near the edge of a truncated one-dimensional photonic lattice revealed that higher level of disorder is required near the boundary to obtain similar localization as in the bulk, indicating that surfaces suppress the localization effects [10]. We have confirmed that finding in a truncated nonlinear two-dimensional square lattice, induced in a photorefractive medium [11], and here we further sharpen the claim by considering gradual transition through different possible corners of different possible lattices.

Thus, in this paper we investigate gradual transition in the Anderson localization near the boundaries, from corners to a bulk, by considering localization at several corners of disordered square and triangular photonic lattices. This study is an extension of the broader discussion of effects of surfaces on the transverse Anderson localization of light in two-dimensional optically induced photonic lattices of finite extend [11]. We reveal that Anderson localization in nonlinear truncated lattices is much different from the localization in disordered linear lattices. We demonstrate that the character of localization near the surfaces is in fact nontrivial, and that it depends on the strength of disorder and on the geometry of the surface.

To stress again, one might expect that a surface, representing a major defect in the lattice akin to a domain wall, should advance localization; however, this turns out not to be the case. Corners and edges effectively reduce Anderson localization, so that stronger disorder is needed near the boundary to obtain the same localization as in the bulk. As mentioned, this surprising result is nonetheless consistent with the experimental observations reported earlier for one-dimensional lattices. We further observe that in the square lattice the suppression is more pronounced at the convex 90-degree corner, relative to the interface and the 270-degree concave corner. The same conclusion applies to the edge and various corners of the triangular lattice. However, the localization in the triangular lattice is less pronounced than the localization in the square lattice, for identical other parameters.

2. Modeling disorder

To study the localization of light near the boundary of an optically-induced photonic lattice, we describe the propagation of a beam

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along the z axis by the nonlinear paraxial wave equation for the field amplitude of light F:

$$i\frac{\partial F}{\partial z} = -\Delta F - \Gamma F\frac{|F|^2+V}{1+|F|^2+V} \quad (1)$$

where Δ is the transverse Laplacian, Γ is the dimensionless coupling constant, and V is the transverse lattice potential. The nonlinearity is chosen of the saturable photorefractive type, allowing an easy extrapolation to the Kerr regime. A scaling $x/x_0 \to x$, $y/x_0 \to y$, $z/L_D \to z$ is utilized for the dimensionless equation, where x_0 is the typical FWHM beam waist and L_D is the diffraction length. The propagation equation is solved numerically by employing a numerical approach developed earlier [12,13].

Disorder is introduced into the model by presenting the potential as $V = V_p + rV_d$, where V_p is the perfect periodic potential and V_d is an appropriately chosen random potential. Parameter r, $(0<r<1)$, is a deterministic real parameter that controls the degree of disorder in the lattice. Random potential is generated by multiplying V_p at each (x,y) point by a pseudo-random number of uniform distribution; this ensures that V_d is not varied along the propagation direction. In this manner one is certain of dealing with the transverse Anderson localization, in accordance with the experimental conditions [2,3,5]. The input peak amplitude of the random potential V_{d0} is kept equal to the input peak amplitude of the periodic potential V_{p0}, so that the level of disorder can be controlled by a single parameter r. The degree of disorder is increased by simply increasing the parameter r. Multiplying V_p by a pseudo-random number is one way to introduce disorder; the other is to multiply the distance between the lattice sites by a pseudo-random number, keeping the intensity at each site fixed. The most general disorder is obtained by making both the intensity at lattice sites and the distance between them change irregularly.

In our simulations we use experimental data from Ref. 5: 10 mm long Ce:SBN crystal and the lattice spacing $d = 23\,\mu\text{m}$, but vary the coupling constant Γ and the lattice and beam input intensities. Input probe beam is launched at the respective lattice sites at the surface or in the corner.

3. Localization in truncated photonic lattices

Here, localization effects near the edge and corners of disordered square and triangular photonic lattices are investigated and compared with the localization in the bulk. Anderson localization becomes apparent by simply increasing disorder. Fig. 1 depicts a typical example of localization in the square lattice. The cases when disorder is not present are also shown, for comparison (Fig. 1(a)–(d)). Localized states in Fig. 1(e)–(h) are with 60% of disorder ($r = 0.6$). An example of Anderson localization in the triangular lattice is presented in Fig. 2.

For quantitative analysis we utilize the standard quantities used in the description of Anderson localization: the inverse participation ratio $P = \int I^2(x,y,z)dxdy/[\int I(x,y,z)dxdy]^2$, and the effective beam width $\omega_{eff} = P^{-1/2}$ [2]; here $I = |F|^2$ is the local light intensity of the probe beam. Since Anderson localization is essentially a statistical phenomenon, many realizations of disorder are needed to measure ensemble averages for the quantities of interest. Different disorder realizations are produced by starting each simulation with different random number generators. Even though different realizations lead to different transverse distributions of the probe beams, the measured values of P and ω_{eff} stay close to each other. Fig. 3 presents the averaged effective width at the lattice output as a function of the disorder level, for different locations in square (a) and triangular (b) lattice. The averaged effective widths in Fig. 3 are taken over 50 realizations of disorder for each disorder level. The effective beam width decreases as the level of disorder is increased, displaying similar tendency as in the experiment [2]. It should be stressed that the effective beam width decreases faster in the bulk as compared to the boundary, as the level of disorder is increased. Apparently, the localized modes tend to get squeezed at sharp corners, becoming elongated and yielding larger effective widths. This squeezing effect is more easily discerned in the triangular lattice, with its more varied corners.

For the square photonic lattice, the beam propagation in the corner 90° displays least localization, followed by the beam at the interface, and then in the corner 270°. Similar results are observed in the localization at the corners of the triangular lattice. To compare localization in the square and triangular lattices, we use identical parameters for both lattices. It is seen that the localization in the triangular lattice is less pronounced than the localization in the square lattice. This is the consequence of different lattice site arrangements for different kinds of lattices.

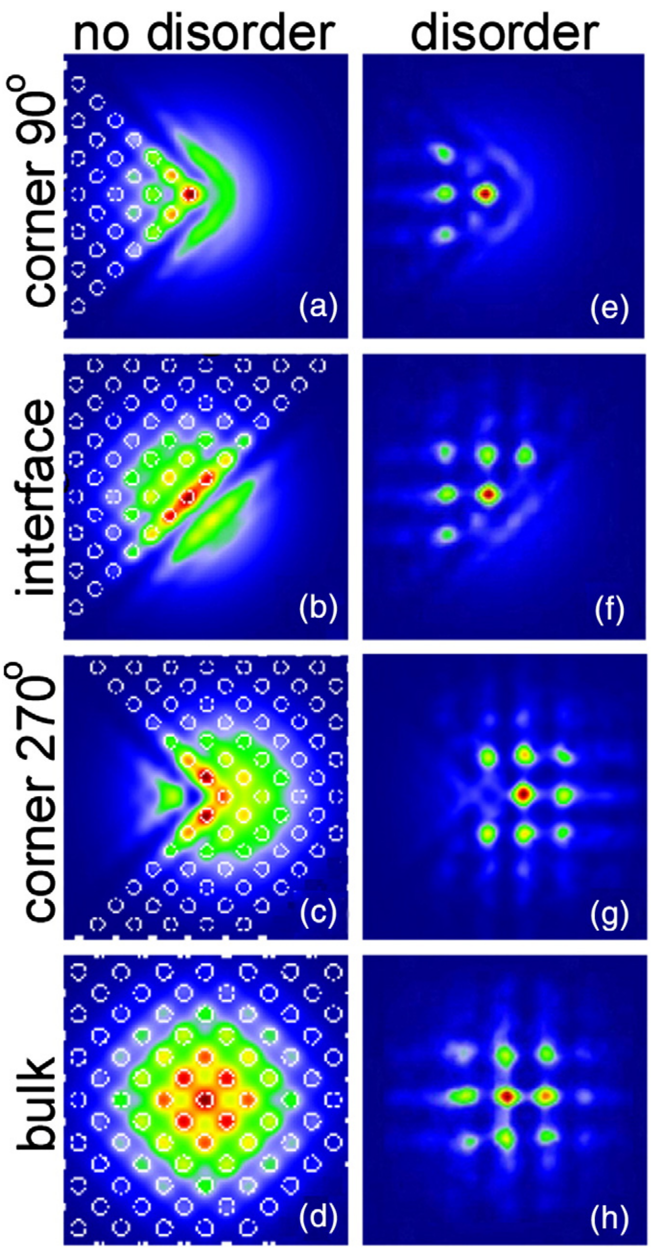

Fig. 1. An example of Anderson localization in a truncated square photonic lattice. Localized modes are shown at 60% disorder: (e) in the corner 90°, (f) near the interface, (g) in the corner 270°, and (h) in the bulk. The left column (a)–(d) presents the corresponding cases without disorder. Input beams are centered on the central lattice site. The layout of lattice beams is shown in the left column, by open circles. Physical parameters are: $\Gamma = 7$, input beam intensity $|F_0|^2 = 0.1$, $V_{p0} = V_{d0} = 1$, input beam $FWHM = 5d$.

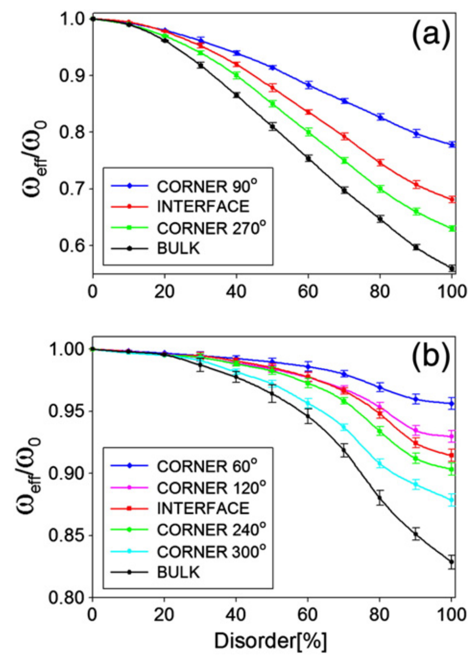

Fig. 3. Effective beam width at the lattice output versus the disorder level for different corners, interface, and bulk modes for (a) square photonic lattice, (b) triangular photonic lattice. The widths are normalized to their input values. Error bars depict the spread in values coming from the statistics. Parameters are as in Fig. 1.

Also, we investigate localization effects in different limits of the model. This is accomplished by varying the input peak amplitude of the lattice potential. As the input peak amplitude is increasing, the dependence of ω_{eff} on the degree of disorder changes. The monotonous decrease in Fig. 3 ceases and a minimum is reached. For different locations in the lattice, the minima are reached at different levels of disorder: first at the convex corners, then at the interface, followed by the concave corners, and finally in the bulk. In principle, Anderson localization in nonlinear media is not only qualitatively (and quantitatively) different from the linear media, but it also depends on the type of the medium nonlinearity.

It is of interest to consider the influence of the crystal length on the beam localization. We investigate the effective beam width as a function of the propagation distance (Fig. 4), for different locations in both lattices. Localization is more pronounced for longer propagation distances *i.e.*, for larger crystal lengths. Also, increasing the strength of nonlinearity makes the localization more pronounced (not shown). These two parameters, the coupling constant and the propagation distance, produce similar effects on the localization. It is seen in Fig. 4 that the beam width displays self-focusing oscillations, which are less pronounced as the level of disorder is increased. No initial diffusive broadening is observed, since we are in the strongly nonlinear regime of the saturable model. Still, higher percentage of disorder is needed to observe similar localization near the edge, as compared to the bulk.

4. Conclusions

We have investigated gradual changes in Anderson localization of light at several types of corners of square and triangular photonic lattices. We have analyzed numerically how the edges and corners of truncated

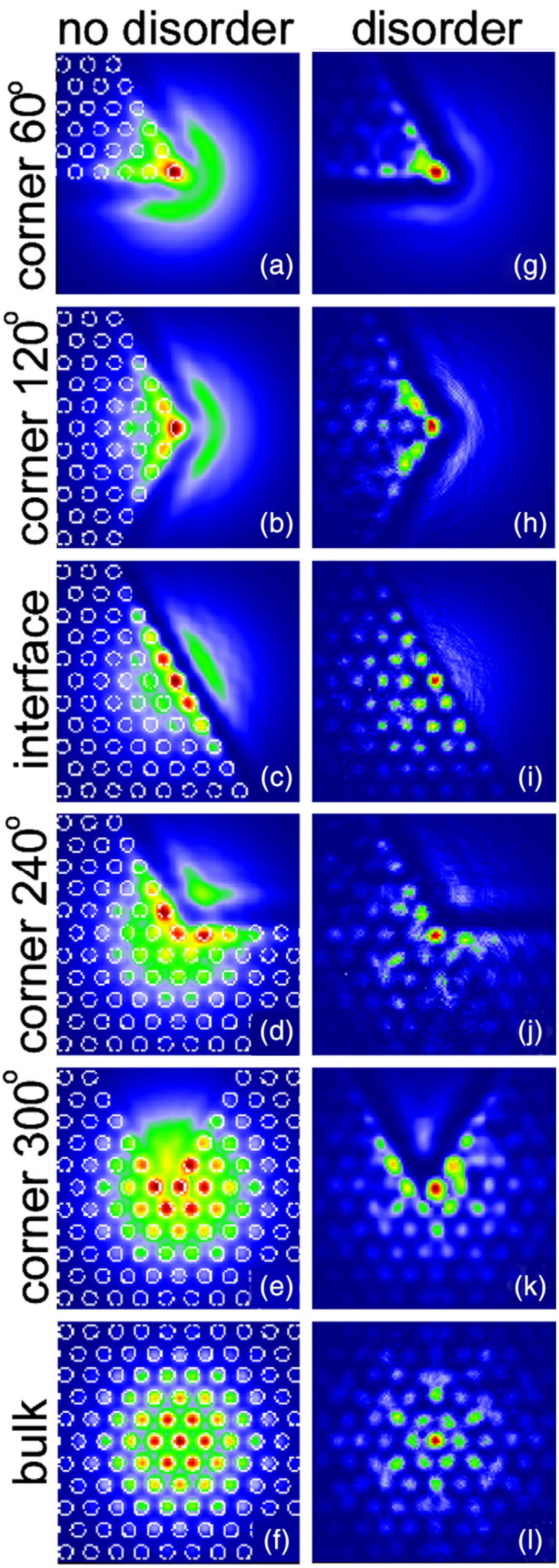

Fig. 2. Anderson localization in a truncated triangular photonic lattice. Localized modes are shown at 60% disorder: (g) in the corner 60°, (h) in the corner 120°, (i) near the interface, (j) in the corner 240°, (k) in the corner 300°, and (l) in the bulk. The left column (a)-(f) presents the cases without disorder. The figure layout and the parameters are the same as in Fig. 1.

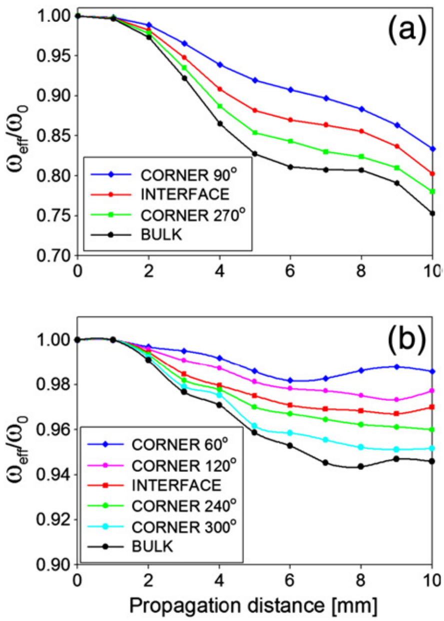

Fig. 4. Effective beam width versus the propagation distance for different localized modes at 60% disorder, for (a) square photonic lattice, (b) triangular photonic lattice. Parameters are as in Fig. 1.

two-dimensional photonic lattices modify disorder-induced localization. We have demonstrated that this effect is nontrivial, and that it depends on the strength of disorder. We have found that the corners and edges effectively suppress Anderson localization, so that a higher level of disorder near the boundaries is required to obtain similar localization as in the bulk. We have revealed that Anderson localization in the convex corners is less effective than the localization at the edge, whereas the localization in the concave corners is more effective.

We have also observed that various aspects of localization depend on the model employed to describe the medium nonlinearity. In principle, Anderson localization in nonlinear media is qualitatively and quantitatively different from the localization in linear media. Typical linear effects, such as the initial diffusive broadening, are absent in the self-focusing nonlinear media. On the other hand, a variety of nonlinear effects, such as beam breathing along the propagation direction, become more noticeable. In the linear regime, a higher level of disorder is necessary to observe the same localization as in the nonlinear regime, for the corresponding geometries.

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