Vortex solitons at the boundaries of photonic lattices

Dragana Jović,[1,2,*] **Cornelia Denz,**[1] **and Milivoj Belić**[3]

[1] *Institut für Angewandte Physik and Center for Nonlinear Science (CeNoS), Westfälische Wilhelms-Universitt Münster, 48149 Münster, Germany*
[2] *Institute of Physics, P.O. Box 68, University of Belgrade, Serbia*
[3] *Texas A & M University at Qatar, P.O. Box 23874 Doha, Qatar*

jovic@ipb.ac.rs

Abstract: Using numerical analysis we demonstrate the existence of vortex solitons at the edge and in the corners of two-dimensional triangular photonic lattice. We develop a concise picture of their behavior in both single-propagating and counterpropagating beam geometries. In the single-beam geometry, we observe stable surface vortex solitons for long propagation distances only in the form of discrete six-lobe solutions at the edge of the photonic lattice. Other observed solutions, in the form of ring vortex and discrete solitons with two or three lobes, oscillate during propagation in a way indicating the exchange of power between neighboring lobes. For higher beam powers we observe dynamical instabilities of surface vortex solitons and study orbital angular momentum transfer of such vortex states. In the two-beam counterpropagating geometry, all kinds of vortex solutions are stable for propagation distances of the order of typical experimental crystal lengths.

© 2011 Optical Society of America

OCIS codes: (190.5330) Photorefractive nonlinear optics; (190.4350) Nonlinear optics at surfaces.

References and links

1. Yu. S. Kivshar, "Nonlinear Tamm states and surface effects in periodic photonic structures," Laser Phys. Lett. **5**(10), 703–713 (2008).
2. S. Suntsov, K. G. Makris, G. A. Siviloglou, R. Iwanow, R. Schiek, D. N. Christodoulides, G. I. Stegeman, R. Morandotti, H. Yang, G. Salamo, M. Volatier, V. Aimez, R. Ars, M. Sorel, Y. Min, W. Sohler, X. Wang, A. Bezryadina, and Z. Chen, "Observation of one- and two-dimensional discrete surface spatial solitons," J. Nonlinear Opt. Phys. Mater. **16**(4), 401–426 (2007).
3. K. G. Makris, S. Suntsov, D. N. Christodoulides, G. I. Stegeman, and A. Hache, "Discrete surface solitons," Opt. Lett. **30**, 2466–2468 (2005).
4. K. Motzek, A. A. Sukhorukov, and Yu. S. Kivshar, "Polychromatic interface solitons in nonlinear photonic lattices," Opt. Lett. **31**, 3125–3127 (2006).
5. Y. V. Kartashov, V. A. Vysloukh, and L. Torner, "Soliton emission in amplifying lattice surfaces," Opt. Lett. **32**, 2061–2063 (2007).
6. S. Suntsov, K. G. Makris, D. N. Christodoulides, G. I. Stegeman A. Haché, R. Morandotti, H. Yang, G. Salamo, and M. Sorel, "Observation of discrete surface solitons," Phys. Rev. Lett. **96**, 063901 (2006).
7. C. R. Rosberg, D. N. Neshev, W. Krolikowski, A. Mitchell, R. A. Vicencio, M. I. Molina, and Yu. S. Kivshar, "Observation of surface gap solitons in semi-infinite waveguide arrays," Phys. Rev. Lett. **97**, 083901 (2006).
8. D. Mihalache, D. Mazilu, F. Lederer, and Yu. S. Kivshar, "Spatiotemporal surface solitons in two-dimensional photonic lattices," Opt. Lett. **32**, 3173–3175 (2007).
9. X. Wang, A. Bezryadina, Z. Chen, K. G. Makris, D. N. Christodoulides, and G. I. Stegeman, "Observation of two-dimensional surface solitons," Phys. Rev. Lett. **98**, 123903 (2007).

#153920 - $15.00 USD Received 2 Sep 2011; revised 25 Oct 2011; accepted 20 Nov 2011; published 8 Dec 2011

10. A. Szameit, Y. V. Kartashov, F. Dreisow, T. Pertsch, S. Nolte, A. Tünnermann,and L. Torner, "Observation of two-dimensional surface solitons in asymmetric waveguide arrays," Phys. Rev. Lett. **98**, 173903 (2007).
11. A. Szameit, Y. V. Kartashov, F. Dreisow, M. Heinrich, V. A. Vysloukh, T. Pertsch, S. Nolte, A. Tünnermann, F. Lederer, and L. Torner, "Observation of two-dimensional lattice interface solitons," Opt. Lett. **33**, 663–665 (2008).
12. M. Heinrich, Y. V. Kartashov, L. P. R. Ramirez, A. Szameit, F. Dreisow, R. Keil, S. Nolte, A. Tünnermann, V. A. Vysloukh, and L. Torner, "Two-dimensional solitons at interfaces between binary superlattices and homogeneous lattices," Phys. Rev. A **80**, 063832 (2009).
13. Y. V. Kartashov, A. A. Egorov, V. A. Vysloukh, and L. Torner, "Surface vortex solitons," Opt. Express **14**, 4049–4057 (2006).
14. D. Song, C. Lou, K. J. H. Law, L. Tang, Z. Ye, P. G. Kevrekidis, J. Xu, and Z. Chen, "Self-trapping of optical vortices at the surface of an induced semi-infinite photonic lattice," Opt. Express **18**, 5873–5878 (2010).
15. M. Belić, M. Petrović, D. Jović, A. Strinić, D. Arsenović, K. Motzek, F. Kaiser, Ph. Jander, C. Denz, M. Tlidi, and P. Mandel, "Transverse modulational instabilities of counterpropagating solitons in photorefractive crystals," Opt. Express **12**, 708–716 (2004).
16. M. S. Petrović, M. R. Belić, C. Denz, and Y. S. Kivshar, "Counterpropagating optical beams and solitons," Laser. Photon. Rev. **5**, 214–233 (2011).
17. Z. Chen, H. Martin, A. Bezryadina, D. Neshev, Yu. S. Kivshar, and D. N. Christodoulides, "Experiments on Gaussian beams and vortices in optically induced photonic lattices," J. Opt. Soc. Am. B **22**, 1395–1405 (2005).
18. M. S. Petrović, D. M. Jović, Milivoj R. Belić, and S. Prvanović, "Angular momentum transfer in optically induced photonic lattices," Phys. Rev. A **76**, 023820 (2007).
19. A. S. Desyatnikov, Yu. S. Kivshar, and L. Torner, "Optical vortices and vortex solitons," Prog. Opt. **47**, 291–391 (2005).
20. M. S. Petrović, "Counterpropagating mutually incoherent vortex-induced rotating structures in optical photonic lattices," Opt. Express **14**, 9415–9420 (2006).
21. D. M. Jović, S. Prvanović, R. D. Jovanović, and M. S. Petrović, "Gaussian-induced rotation in periodic photonic lattices," Opt. Lett. **32**, 1857–1859 (2007).
22. V. I. Petviashvili, "Equation of an extraordinary solution," Plasma Phys. **2**, 469 (1976).
23. J. Yang, I. Makasyuk, A. Bezryadina, and Z. Chen, "Dipole and quadrupole solitons in optically induced two-dimensional photonic lattices: theory an experiment," Studies Appl. Math. **113**, 389–412 (2004).

1. Introduction

Self-trapped nonlinear surface states (surface solitons) propagating along the interface of two different media have attracted recently a great deal of interest in different optical systems [1, 2]. Nonlinear discrete surface waves in one-dimensional (1D) waveguide lattices have been predicted to exist theoretically [3–5] and were demonstrated experimentally [6, 7]. Two-dimensional (2D) lattice interfaces also support surface solitons [8]. Experimental observation of 2D solitons was reported at the boundaries of a finite optically induced photonic lattice [9, 10], at the interface between the square and hexagonal waveguide arrays [11], and also at the interface between a homogeneous lattice and a superlattice [12].

Special attention has also been devoted to the nonlinear surface vortex solitons. Such solitons are supported at the interface of two different optical lattices imprinted in Kerr-type focusing nonlinear media [13] and are demonstrated experimentally at the surface of an optically induced 2D photonic lattice [14]. In principle, surface solitons should be distinguished from the other surface waves propagating at the interface of different media, such as the surface plasmon polaritons. These surface waves are *linear* in their nature and require widely different media at the interface, e.g. a metal and a dielectric.

In this paper we report on the existence and properties of vortex solitons at the edge and in the corners of two-dimensional photonic lattices. We consider the very general case of the counterpropagating (CP) beams [15, 16], and compare them with the single-propagating beam geometry. However, we present only the single-propagating beam results, as they provide for more varied dynamical behavior. Recent experimental results [14] predict the existence of stable vortex solitons at the edge of 2D square photonic lattice in the form of four-site vortex solitons, in the single-propagating beam geometry. We extend the analysis to vortex beams in the triangular lattice, including edge and corner geometries, and focus more attention to the

#153920 - $15.00 USD Received 2 Sep 2011; revised 25 Oct 2011; accepted 20 Nov 2011; published 8 Dec 2011
(C) 2011 OSA

study of extensively oscillating surface vortex solitons, as well as on the dynamical instabilities of such solitons.

We describe novel types of discrete vortex solitons and the ring surface vortex solitons, localized in the lattice corners or at its edges. Discrete vortex surface solitons are observed in the form of two, three, or six lobe solutions. We demonstrate that in the single-beam geometry, the lattice surface produces a strong stabilizing effect only on the discrete vortex solitons in the form of the six-lobe solutions, enabling them to stably propagate for long distances. Other kinds of discrete solitons are unstable during propagation; oscillations and irregular dynamics are observed while increasing the beam power or the propagation constant. But in the CP geometry, it is possible to find stable CP vector solutions of different kinds for the propagation distances of the order of typical experimental crystal lengths. We also study the orbital angular momentum transfer of the vortex surface states [17, 18].

The paper is structured as follows. Section 2 introduces the model and the basic equations. It also presents the methods of eigenstate determination and numerical integration. Section 3 discusses different vortex surface states. Instabilities of such states are investigated in Sec. 4 and Sec. 5 offers conclusions.

2. Model and basic equations

The behavior of CP vortex beams propagating in photonic lattices is described by the wave equation in the paraxial approximation for the beam propagation in a photorefractive crystal. The model equations in the computational domain are given by [15, 16]:

$$i\partial_z F + \Delta F + \Gamma \frac{I + I_g}{1 + I + I_g} F = 0, \tag{1}$$

$$-i\partial_z B + \Delta B + \Gamma \frac{I + I_g}{1 + I + I_g} B = 0, \tag{2}$$

where z is the propagation distance, F and B are the envelopes of the counterpropagating beams, Δ is the transverse Laplacian, Γ is the dimensionless beam coupling constant, and $I = |F|^2 + |B|^2$ is the total laser light intensity measured in units of the background intensity I_d. Typically, in photorefractive media laser beams interact incoherently, through the change in the index of refraction, caused by the light intensity. Here, I_g is the intensity distribution of the optically induced truncated photonic lattice. In this paper, we concentrate on the triangular lattice.

It is well known that in the homogeneous bulk photorefractive media the vortices are unstable; they tend to break up into filaments that rotate and fly away from each other [16]. The presence of a lattice changes this behavior somewhat [19], in that there appears the tendency of localization of the beam filaments at the lattice cites; however, the instability remains [20, 21]. It is our aim to investigate what happens to the vortices launched at the edges and corners of a lattice.

First, we want to establish the existence of CP vortex solitonic solutions. Owing to their symmetry, the above equations suggest the existence of a fundamental 2D vector soliton solution, in the form $F = u(x,y)cos\theta e^{i\mu z}$, $B = u(x,y)sin\theta e^{-i\mu z}$, where μ is the propagation constant, and θ is an arbitrary projection angle, which controls the relative size of beam components. For the simple CP geometry, we take $\theta = \pi/4$; for the single-beam geometry, we take $\theta = 0$. After the substitution of the presumed solitonic solution in Eqs. (1) and (2), they transform into one, degenerate equation:

$$-\mu u + \Delta u + \Gamma u \frac{|u|^2 + I_g}{1 + |u|^2 + I_g} = 0. \tag{3}$$

Equation (3) presents an eigenvalue problem in the transverse (x, y) plane. In solving this problem, i.e., in finding the solitonic solutions with the corresponding propagation constants, we utilize the modified Petviashvili's iteration method [22, 23]. We determine different classes of vortex surface solitons, by launching vortex beams whose rings are covering one or more lattice sites near the boundaries of the photonic lattice. The topological charge of the input vortex beam is chosen to be 1. In this paper we analyze four different classes of surface vortex solitons: the ring vortex, and discrete solitons consisting of six, three, and two lobes.

Next, to investigate the stability of such solutions in the single beam geometry, we use the vortex surface solitons as input beams in Eq. (1) and solve the equation numerically. For investigating the stability of such solutions in the CP geometry, we use the CP model given by Eqs. (1) and (2). The numerical procedure is based on the fast-Fourier-transform split-step numerical algorithm [15].

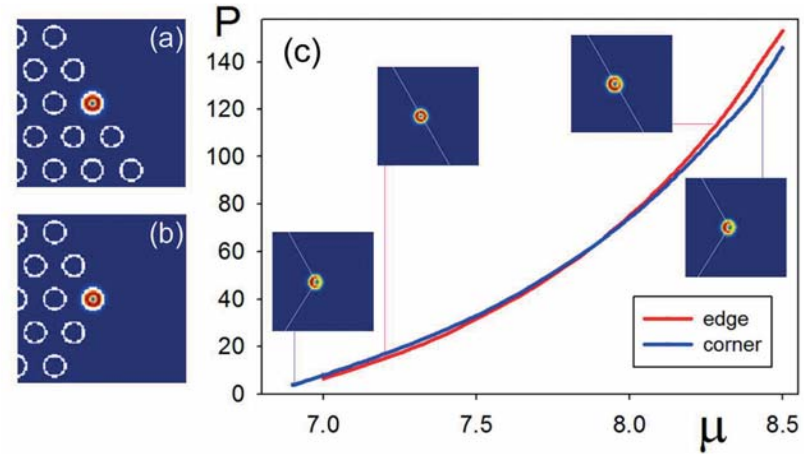

Fig. 1. Narrow surface ring vortex solitons. Input vortex beams are shown at the edge (a) and in the corner (b) of the lattice, with the layout of the lattice beams indicated by open circles. (c) Power diagram for the existence of narrow surface vortex solitons. Insets depict the corresponding intensity distributions for the corner and the edge vortex solitons. The lines in the insets depict the lattice outlines. Parameters: $\Gamma = 11$, the maximum lattice intensity $I_0 = 1$.

3. Vortex surface states

The lattice induces confinement of the filaments approximately at the location of the incident vortex ring and the surrounding lattice sites; so initially we choose the input ring vortex beam to cover the corresponding single lattice site only. For investigating such narrow ring vortex surface states, the incident vortex beams covering only one lattice site are used at the edge (Fig. 1(a)) and in the corner (Fig. 1(b)) of the triangular photonic lattice. Both corner and edge vortex solitons are found in almost the same range of the propagation constant μ, but the existence domain of corner solitons is a bit broader than that of the edge solitons. The corresponding power diagrams are presented in Fig. 1(c), with the characteristic outcomes shown as insets. The beam power for vortex solitons is given by the formula $P = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |u|^2 dxdy$. An essentially new finding is that the surface vortex solitons *can* exist for narrow input beam vortices. However, such surface states with symmetric uniform ring profiles exist only at the edge of the lattice and for lower values of the propagation constant. Asymmetric nonuniform surface vortex states are commonly observed in both corner and edge geometries. Both kinds

#153920 - $15.00 USD Received 2 Sep 2011; revised 25 Oct 2011; accepted 20 Nov 2011; published 8 Dec 2011
(C) 2011 OSA

of narrow ring vortex surface solitons are unstable during propagation in the single-beam geometry, similar to the experimental results [14]. In the CP geometry, the narrow ring solutions may propagate stably, but only for short propagation distances.

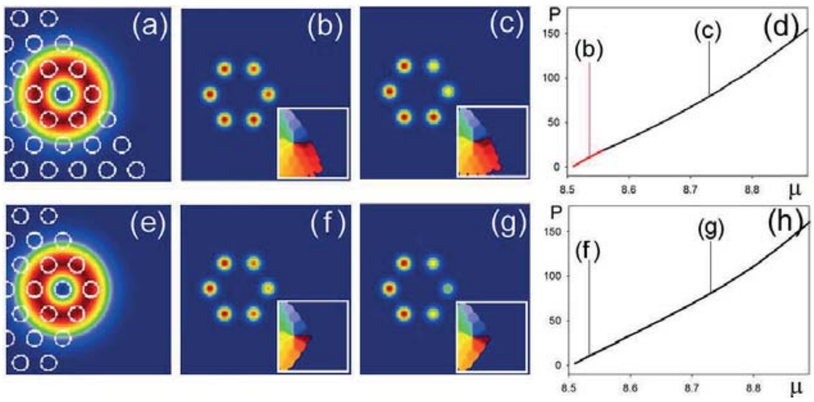

Fig. 2. Six-lobe discrete surface vortex solitons. Top row: The edge surface modes. Bottom row: The corner surface modes. The layout of the lattice beams is only shown in the first column. (a), (e) The input vortex beams. Different kinds of edge (b), (c), and corner vortex solitons (f), (g), are shown with the corresponding power diagrams (d), (h). The phase distributions are presented as insets. The parameters are as in Fig. 1.

Next, we investigate the discrete surface vortex solitons, in the form of multi-lobe solutions. Figure 2 presents the six-lobe discrete vortex solutions at the edge and in the corner of the optically induced triangular lattice. The input vortex beams are presented in the first column at the edge (a) and in the corner (e) of the lattice. For easier comparison, the layout of the lattice beams is again depicted by open circles. For both corner and edge states, the asymmetry of the vortex soliton with higher power is more pronounced than that of the vortex with lower power. The corresponding power diagrams are presented for both the corner and the edge states; they exist in the same range of the propagation constant.

Investigating the stability of such vortex solitons, for the single-beam geometry, we find that only the six-lobe edge solutions with small asymmetry are stable during propagation, and can exist for long propagation distances. Such solutions are observed for lower propagation constants, as well as for lower beam powers (the region marked red on the power line in Fig. 2(d)). In the CP geometry, the stability of all six-lobe solutions is observed for short propagation distances (~ 20 mm).

Finally, we investigate the discrete surface vortex solitons when the input vortex beam is launched in-between the three neighboring lattice sites (Fig. 3(a), 3(e)). Commonly, then the three-lobe soliton solutions are observed. Figure 3 summarizes the results for the three-lobe surface vortex solitons. Again, the asymmetry of the vortex soliton with higher power is stronger than that of the vortex with lower power. Both the corner and the edge solutions are not stable during propagation in the single-beam geometry. The low-power states exhibit regular oscillations, but solutions with the higher power display irregular oscillations. For high beam powers, even chaotic instabilities are observed. In the CP geometry, the stability of all three-lobe solutions is observed only for short propagation distances, such as the typical experimental crystal lengths. Similar results in both the single and the CP beam geometry hold for the surface modes in the form of two-lobe vortex solitons (not shown).

#153920 - $15.00 USD Received 2 Sep 2011; revised 25 Oct 2011; accepted 20 Nov 2011; published 8 Dec 2011
(C) 2011 OSA

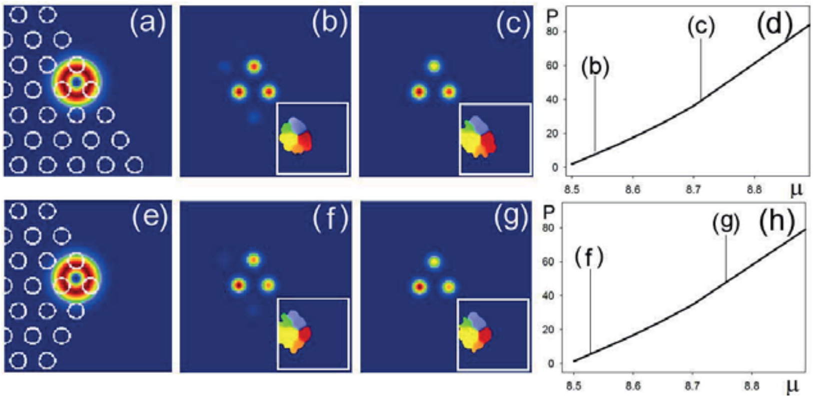

Fig. 3. Three-lobe discrete surface vortex solitons. The figure layout is as in Fig. 2. The parameters are as in Fig. 1.

4. Soliton instabilities

In the end, we display the instabilities of vortex surface solitons, and investigate their orbital angular momentum [17, 18]. The standard definition for the (normalized) z component of the orbital AM is adopted, $L_z(F) = -\frac{i}{2}\int\int dxdyF^*(x,y)(x\partial_y - y\partial_x)F(x,y) + cc$. In Fig. 4(a), a typical example of the ring surface vortex solution in the single-beam geometry is presented as a movie of the intensity distribution along the propagation distance. The ring shape of the vortex beam is broken very fast, and irregular dynamics take place along propagation. This is also confirmed by monitoring the angular momentum of such a vortex beam (Fig. 4(b)).

The most illustrative cases of discrete surface vortex solitons in the single-beam geometry are shown in Fig. 5 as 3D movies along the propagation distance. Figure 5(a) shows typical behavior of an asymmetric six-lobe surface state during propagation. At the beginning each lobe oscillates, slightly changing its peak intensity with the periodic oscillations of the angular momentum. During propagation, the neighboring lobes exchange more power, irregular oscillations take place, and the transfer of angular momentum among the lobes of the vortex, as well as from the vortex to the photonic lattice, is more pronounced (Fig. 5(d)).

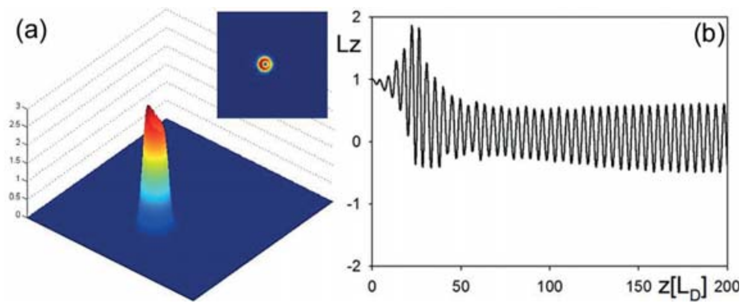

Fig. 4. Dynamical behavior of surface ring vortex solitons. Movies of the intensity distribution are shown along the propagation direction: (a) (Media 1). The inset depicts the corresponding solitonic solutions. (b) The normalized z component of the angular momentum.

In the case of three-lobe discrete vortex solitons, we present the behavior of symmetric edge mode solutions during propagation (Fig. 5(b)). This solution shows regular oscillations, but the oscillations of each lobe around its initial position are more pronounced than the exchange of power between the neighboring lobes. This is also visible when monitoring the angular momentum transfer (Fig. 5(e)). The simplest state is the discrete solution with only two lobes (Fig. 5(c)). During propagation the lobes exchange power, but with a substantial transfer of the angular momentum as well (Fig. 5(f)).

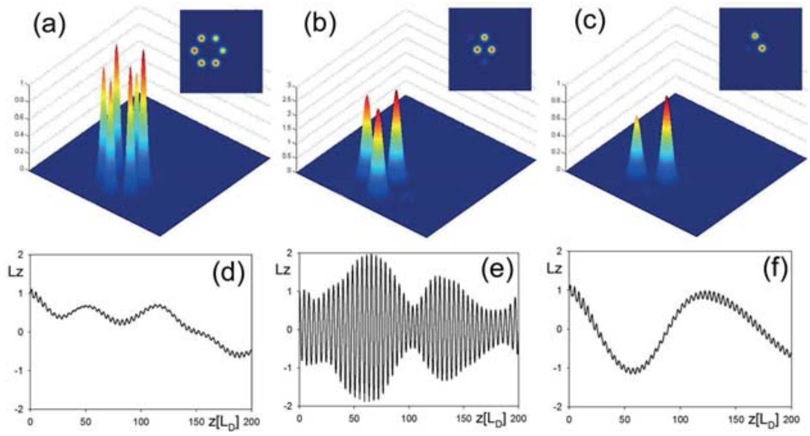

Fig. 5. Oscillating behavior of the edge surface vortex solitons. Top row: The edge vortex solitons. Movies of the intensity distributions are shown along the propagation direction: (a) (Media 2), (b) (Media 3), (c) (Media 4). Insets depict the corresponding solitonic solutions. Bottom row: The normalized z component of the angular momentum for the corresponding vortex solitons.

By further investigating the propagation of vortex solitons in the CP beam geometry, we observe stable solutions only for short propagation distances, corresponding to the typical experimental crystal lengths. Such a stable propagation is found for the solutions in the form of ring vortices, as well as discrete vortices with two, three, and six lobes. But, when the propagation of CP vortex solutions is followed for longer propagation distances, we observe irregular behavior for all kinds of the vortex solutions in the CP beam geometry (not shown).

5. Conclusion

We have studied surface vortex solitons in truncated 2D photorefractive photonic lattices and revealed the existence of novel types of discrete vortex surface solitons, localized in the lattice corners or at the edges. We have developed a concise picture of different scenarios of the vortex solutions behavior, and investigated their stability in the single-beam propagating and the CP beam geometry. In the single-beam geometry, beside the stable six-lobe discrete surface modes propagating for long distances, we have observed various oscillatory vortex surface solitons, as well as dynamical instabilities of different kinds of solutions. Dynamical instabilities occur for higher values of the propagation constant, or at higher beam powers. We also have investigated orbital angular momentum transfer of such solutions during propagation. In the CP geometry it is possible to find stable CP vector solutions for propagation distances corresponding to the typical experimental crystal lengths.

Acknowledgments

This work was supported by the Ministry of Science and Technological Development of the Republic of Serbia (project OI 171036), and the Qatar National Research Foundation (NPRP project 25-6-7-2). Dragana Jović expresses gratitude to the Alexander von Humboldt foundation for the Fellowship for Postdoctoral Researchers.