Nondiffracting kagome lattice

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We introduce a generalized approach to generate an elementary nondiffracting beam, whose transverse intensity is distributed corresponding to a two-dimensional kagome structure. Furthermore, we present an effective experimental implementation via a computer controlled phase controlling spatial light modulator in combination with a specific Fourier filter system. Intensity and phase analysis of the kagome lattice beam accounts for an experimental wave field implementation. Altogether, the examined wave field may be a fundament for the fabrication of large two-dimensional photonic crystals or photonic lattices in kagome symmetry using miscellaneous holographic matter structuring techniques.
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The kagome structure is a very famous and prevalent lattice form in nature. Although this kind of structure is well-known in human cultures for centuries, not until the recent years various examples eminently in condensed-matter physics became a subject of numerous theoretical and experimental considerations. In this research area, the kagome lattice occupies a special standing[1,2] as kagome structured atomic and molecular lattices exhibit intriguing behaviors such as magnetic spin frustration[3] or the spin Hall effect.[4] Since a complete band gap[5–7] and flat band structures with applicable propagation characteristics[8] were predicted for a two-dimensional photonic kagome crystal, the nondiffracting kagome lattice beam (KLB) plays an important role in the field of generating photonic crystals as well. In the range of optically induced photonic lattices, localized structures, such as vortices and solitons, were predetermined in discrete and continuous kagome lattices.[9] Having the powerful tool of a KLB in hand, matter structuring to generate a photonic kagome lattice is imaginable in many different ways. Conceivable applications are in the regime of optical induction of photonic lattices in photorefractive crystals,[10–12] optical tweezing in connection with particle assembly on micrometer scale,[13,14] as well as holographic lithography[15] or applying a KLB as an optical atom trap for Bose–Einstein condensates,[2] just to mention a few.

In this letter, we introduce a so far unexploited fundamental nondiffracting beam with a transverse intensity modulation according to the kagome lattice and show simulations of the ideal nondiffracting KLB and results of intensity and phase distributions of the corresponding experimentally implemented wave field. In general, nondiffracting beams, primarily studied by Durnin *et al.*,[16] are characterized by an arbitrarily modulated intensity distribution transverse to the direction of propagation, while the intensity remains constant in the longitudinal direction. As an additional feature, all nondiffracting beams exhibit the intriguing effect of self-healing.[17] Mathematically, the static field distribution of nondiffracting beams is a solution of the time invariant Helmholtz equation, which is separable into a transverse and a longitudinal part.[18] Hence, a set of solutions consists of the product of two independent wave functions: a two-dimensional wave function, accounting for a transverse intensity modulation, and a one-dimensional wave function, depicting the longitudinal field distribution connected to a constant intensity. In literature, four different two-dimensional coordinate systems were termed previously, in which solutions of the transverse Helmholtz equation exist.[17] Thus, fundamental nondiffracting beams are classified by four miscellaneous beam families, namely, *discrete nondiffracting beams* in the Cartesian coordinates, *Bessel beams* in circular cylindrical coordinates, *Mathieu beams* in elliptic cylindrical coordinates, and *Weber beams* in parabolic cylindrical coordinates.[19,20] All these beams share one characteristic: in the case of a fixed wavelength λ, the wave vectors $\mathbf{k}$ of the contributing partial light waves are distributed on the surface of a cone with an opening angle θ. This angle determines the transverse part of each wave vector by $k_t = 2\pi \sin\theta/\lambda$, where $|\mathbf{k}|^2 = k_t^2 + k_l^2$ with k_l denoting the longitudinal component of the wave vector $\mathbf{k}$. In addition, the projection of the wave vectors into the k_x-k_y plane implies the spatial frequencies in the Fourier plane, lying on a ring of radius $k_t = (k_x^2 + k_y^2)^{1/2}$. Figure 1 illustrates schematically a

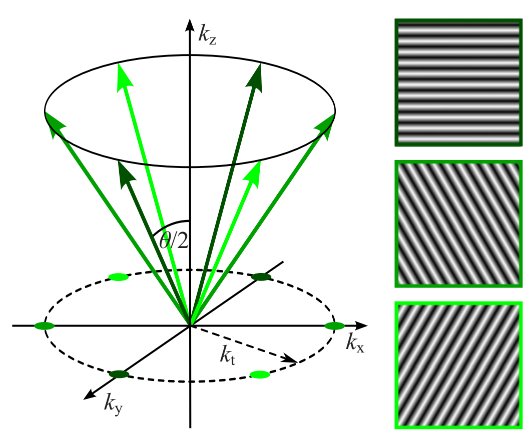

FIG. 1. (Color online) Schematic illustration of the Fourier space spectrum of a sixfold nondiffracting beam. Beam component's direction of propagation lies on the surface of a cone with an opening angle θ. Projection into the k_x-k_y plane illustrates intensity peaks arranged on a ring with radius k_t. Inserted one-dimensional modulated intensities correspond to equally colored arrows [cf. Eq. (1)].

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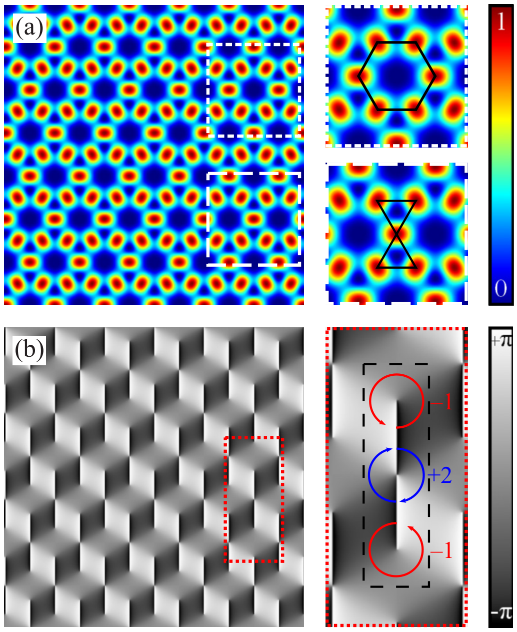

FIG. 2. (Color online) Simulation of intensity and phase distribution of a kagome lattice. (a) Intensity distribution: details show the building blocks of a kagome lattice (upper: hexagon; lower: hour glass). (b) Phase distribution: magnified detail depicts three vortices with total topological charge of zero; dashed rectangle in the right picture implies building block.

cone in Fourier space, whose surface includes the **k**-vectors of six plane waves. In the Fourier plane, one can find six intensity peaks, pictured by the solid circles in the k_x-k_y plane.

In this connection, transverse periodic or quasiperiodic intensity distributions are allocated to the family of discrete nondiffracting beams. The three-dimensional static complex field distribution Ψ of the kagome beam, revealing periodic modulated transverse intensity, takes the form of a summation of three one-dimensionally modulated nondiffracting cosine beams (cf. intensity distributions in Fig. 1, right) in detail

$$\Psi(x,y,z) = \Psi_0 \cdot \{\cos(k_t[x \cos(\pi/6) + y \sin(\pi/6)])e^{2i\pi/3} + \cos(k_t[x \cos(\pi/6) - y \sin(\pi/6)])e^{-2i\pi/3} + \cos(k_t y)\}e^{ik_l z}. \quad (1)$$

In this equation, Ψ_0 establishes the amplitude, which is equal for all plane waves and spatially constant.

Corresponding to Eq. (1), Fig. 2 illustrates the simulated intensity [cf. Fig. 2(a)] and phase distribution [cf. Fig. 2(b)], $I(x,y)=|\Psi(x,y,z)|^2$ and $\phi(x,y,z)=\arg[\Psi(x,y,z)]$. As can be seen in Fig. 2(a), the intensity distribution features the typical kagome structure as a combination of two distinctive shapes building up the kagome lattice: a hexagon consisting of six intensity maxima at the vertices of the shape as well as an hourglass structure composed of five intensity maxima. The appropriate phase distribution of the kagome lattice represents a periodic tessellation of similar building blocks including three vortices of topological charge: −1, +2, and −1. Figure 2(b) depicts the KLB phase distribution and one single building block. Consequently, the whole beam carries no angular momentum as the charge summation of the building block equals zero.

We emphasize that the KLB is a fundamental discrete nondiffracting beam, which, as well as the famous hexagonal and honeycomb nondiffracting lattice beams,[21] has a sixfold symmetry. However, in contrast to the accordant lattices of those beams, the photonic kagome lattice reveals a completely different band structure,[22] intimately connected with differing band gaps, and nonlinear light propagation response. Theoretically, ideal nondiffracting beams are infinitely expanded, carrying infinite energy. Consequently, they are not feasible in experiment. Rather, experimental nondiffracting beams, also termed as pseudonondiffracting beams (PNBs), are restricted to finite apertures of the used optical elements and beam diameters. According to that, there exists a finite volume of interference of all contributory field components forming the PNB, whose volume depends on the structural size and beam apertures. Although the longitudinal expansion of a PNB is finite, it is several times larger than the diffraction length of a wave field with comparable structural size of the modulated amplitude.[17] As one accounts for the finite aperture and volume to simulate nondiffracting beams, the wave fields are handled as Helmholtz–Gauss beams, which are paraxial solutions of the Helmholtz equation.[19] Nevertheless, we solely concentrate on the simulation of ideal nondiffracting beams, receiving the phase information of adequate size for the experimental realization of nondiffracting wave fields with maximum volume, limited only by our optical setup. The procedure is as follows.

An established and highly flexible method to implement complex nondiffracting beams is the utilization of a spatial light modulator (SLM).[23] In order to generate a KLB, we use a setup consisting of a frequency-doubled Nd:YAG (YAG denotes yttrium aluminum garnet) laser at a wavelength of 532 nm and a high resolution phase SLM, which is 15.36 $\times 8.64$ mm^2 in size. The SLM is served by a computer generated phase picture based on Eq. (1). In combination with an appropriate Fourier filter system that solely transmits the first diffraction order of the SLM, this setup can be used to implement any discrete nondiffracting wave field. To investigate the generated field distribution, we employ a camera for transverse intensity examination, on one hand. On the other hand, we analyze the phase distribution of the generated KLB via superimposing the lattice beam with a plane wave reference beam, which is polarized perpendicularly to the lattice beam. In this connection, a set of polarizer and quarter-wave plate allows for the measurement of the Stokes parameters, which, in turn, reveals the beam's phase distribution.[24]

Figures 3(a) and 3(b) depict both the transverse intensity and the phase distribution of an experimentally implemented KLB with $\theta \approx 0.6°$, which reveals a structural size in the regime of tens of micrometers. An excellent agreement of experimental and simulated intensity and phase distributions, respectively, can be observed by comparing Figs. 2 and 3. Thus, all the intensity vertices forming the distinctive kagome lattice can be found clearly in Fig. 3(a). Besides, the fascinating vortex structure is resolved in Fig. 3(b), in which the single and double charged vortices can be identified evidently. As a result, the experimental pictures prove the desired generation of the transverse KLB field distribution.

In the second part of our experimental investigation, we ensure the nondiffracting property of the beam. By mounting the camera onto a translation unit with a total shift distance of 10 cm, we are able to explore the intensity development of the nondiffracting beam in the direction of propagation. Therefore, we reconstruct the intensity distributions of the

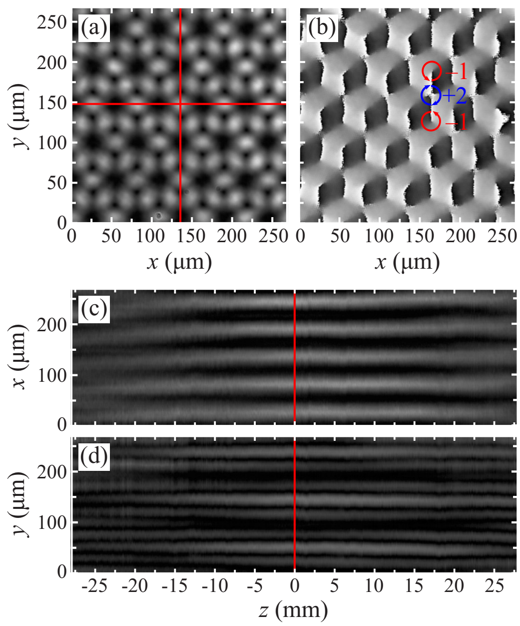

FIG. 3. (Color online) Analysis of the KLB with $\theta \approx 0.6°$. (a) Transverse intensity distribution: lines mark positions of the planes shown in (c) and (d). (b) Phase distribution: vortex positions marked by circle arrows, numbers indicate topological charge. [(c) and (d)] Longitudinal intensity distribution of the *x*-*z* and *y*-*z* intersection plane, respectively. Lines in (c) and (d) indicate the longitudinal beam center [cf. (a)].

generated beam in the *x*-*z* or *y*-*z* plane by stacking appropriate columns or rows of experimental intensity pictures, respectively, belonging to different propagation positions. Figures 3(c) and 3(d) show the resulting intensity distributions of the observed kagome lattice, verifying a nondiffracting beam propagation length of at least 4 cm. To survey our experimental observations, we note that we have the ability to generate a highly extended nondiffracting beam with kagome lattice symmetry. Consequently, this beam can be utilized to structure matter two-dimensionally on a longitudinal scale, which is approximately three orders larger than the transverse modulation.

In conclusion, we have developed a fundamental discrete nondiffracting beam that reveals a kagome lattice structured transverse intensity distribution and features all advantageous properties of a nondiffracting beam such as a translation invariant intensity distribution and the self-healing effect. We have generated and analyzed a KLB experimentally and found that a transverse structure size of some tens of micrometers yields a longitudinal distance of translation invariant intensity of about 4 cm. Due to the very good agreement between computational simulations and experimental observations of the transverse intensity and phase distributions, we are convinced that the demonstrated KLB is highly applicable for photonic applications. All observed properties make this nondiffracting beam a perfect tool for matter structuring in order to create largely expanded photonic kagome lattices.

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