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Nonlinear optical beams carrying phase dislocations

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Abstract
We describe different types of self-trapped optical beam carrying phase dislocations, including *vortex solitons* and ring-like *soliton clusters*. We demonstrate numerically how to create such nonlinear singular beams from the interaction of several fundamental optical solitons. Mutual trapping of several solitons can be regarded as a synthesis of '*soliton molecules*', and it corresponds to a transfer of an initial orbital angular momentum of a system of solitons to a spin momentum of an optical vortex.

Keywords: phase dislocation, singular optics, optical soliton

1. Introduction

Phase dislocations carried by the wavefront of a light beam are associated with the zero-intensity points where the light intensity vanishes. The phase of the wave is twisted around such points creating a structure associated with *an optical vortex*. Optical beams with phase dislocations play an important role in linear singular optics [1].

In self-focusing nonlinear media, an intense finite-extent laser beam becomes localized due to the self-trapping mechanism which can compensate for the beam diffraction. The nonlinear self-action of light may result in the formation of stationary structures with both intensity and phase remaining unchanged along the propagation direction. Such self-trapped stationary structures of light beams are termed *spatial optical solitons* [2]. When such solitons have phase singularities, they determine the internal structure of the beam; they can be stabilized by the light self-action generating nonlinear self-trapped optical beams carrying phase dislocations. Examples of such beams include *vortex solitons* [3–5] with point screw dislocations, *multipole vector solitons* [6] with π-edge dislocations, and more complicated '*necklace*'-type beams [7–9] and *soliton clusters* [10] with a combination of a screw dislocation at the beam centre and, generally, ϑ-edge dislocations, where ϑ is the phase jump between neighbouring peaks in the intensity distribution [10].

The fundamental optical solitons show a fascinating combination of the properties of classical wavepackets together with a number of particle-like properties demonstrated in their elastic and inelastic interactions and mutual scattering, when each of the solitons preserves its identity. Moreover, the coherent interaction between the solitons depends strongly on a relative phase which provides an additional degree of freedom to control the interaction. We may draw an analogy between the spatial soliton and the 'atom of light', and then the soliton collision and interaction can be treated in terms of the effective forces acting between these effective 'atoms'. Following this concept, the higher-order multi-hump optical beams can be regarded as bound states of 'atoms' trapped by a common potential induced in a nonlinear medium. A balance of the interaction forces acting between the solitons is the necessary condition for the formation of the soliton 'clusters' or 'molecules of light'.

In this paper we investigate the excitation of higher-order beams, including optical vortices and soliton clusters, through the inelastic soliton scattering and mutual trapping of initially well separated fundamental solitons, the effect resembling a synthesis of 'soliton molecules'. In addition, we propose the application of this effect in the context of 'soliton algebra' [11] regarding the fundamental spatial solitons as the information carriers, and the transformation of an optical pattern induced by the soliton interaction as all-optical soliton switching.

2. Optical vortex solitons and soliton clusters

Optical vortices were introduced as the first example of a stationary light beam with the phase twisted around its core;

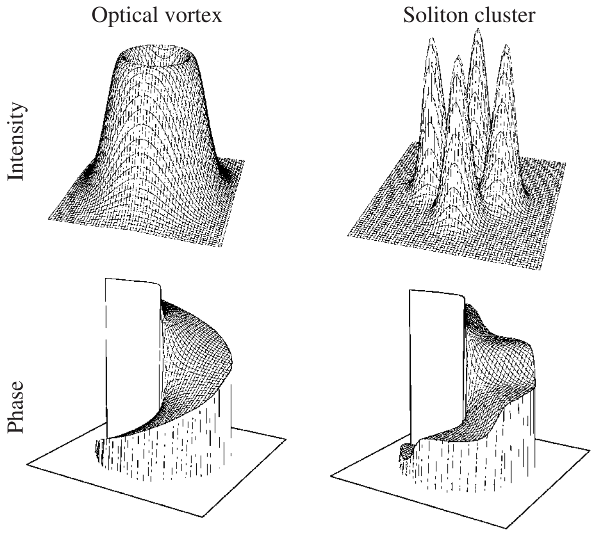

Figure 1. Intensity and phase distributions for the optical vortex soliton and soliton cluster composed of four fundamental beams. Note that in terms of the azimuthal coordinate $\theta = \tan^{-1}(y/x)$, the vortex phase is given as a *linear function* $m\theta$ with integer m, while the staircase-like phase of the cluster is a *nonlinear* phase dislocation.

the twist is proportional to 2π with integer m, the so-called topological charge of the phase dislocation [3]. The physical model analysing the evolution of the slowly varying field envelope E is described by the nonlinear Schrödinger equation,

$$\mathrm{i}\frac{\partial E}{\partial z} + \Delta_\perp E + f(I)E = 0, \qquad (1)$$

where $\Delta_\perp$ is the transverse Laplacian, and z is the propagation distance measured in the units of the diffraction length. Function $f(I)$ describes the nonlinear properties of an optical medium, and it is assumed to depend on the total beam intensity, $I = |E|^2$. The simplest spatially localized solution of equation (1) carrying a phase dislocation, i.e. the vortex soliton, can be written in the form: $E(r, \theta, z) = A(r)\mathrm{e}^{\mathrm{i}m\theta + \mathrm{i}\beta z}$, where $A(r)$ and β are the beam amplitude and propagation constant, respectively, while r and θ are the polar coordinates in the transverse plane.

In self-focusing nonlinear media, such vortex beams are subject to the azimuthal modulational instability, which results in splitting of the doughnut ring-like structure into a certain number of the fundamental solitons. The number of splitters and their dynamics are determined by the topological charge of the phase dislocation corresponding to the beam angular momentum (see, e.g., [4] and references therein).

The simplest higher-order scalar stationary solutions found for the model (1) are a family of the radially symmetric solitons which includes radial modes with nodes in the form of concentric rings. The important characteristic of these states is the *soliton spin* determined as a ratio of two conserved quantities, the beam angular momentum and the beam power. For the vortex soliton, the spin coincides with the topological charge m of the phase dislocation carried by the vortex. Because of the condition of the field periodicity, the topological charge m is quantized and has integer value. The fundamental spatial solitons and their higher-order radial states have zero spin.

Novel types of higher-order self-trapped optical beam can be introduced as *azimuthally modulated* self-trapped structures in the form of the so-called 'necklace' beams [7, 8]. However, it was found that a combination of the edge-phase dislocation with π-out-of-phase neighbour peaks cannot produce a stationary state, and the structure becomes slowly expanding [7]. Such a stabilization is indeed possible for a more complicated system including the attraction between several incoherent beams [6, 9]. Another approach to this problem is to combine the screw dislocation in the origin of a ring-shaped beam with the edge dislocation within the necklace [8]. The screw dislocation introduces a centrifugal force to the ring, being also responsible for spiralling and mutual repulsion of the solitons in the case of vortex break-up [4], and the edge dislocations prevents noise-induced instability break-up of the ring. Because of a nonzero angular momentum, the whole structure rotates with its propagation. As a result, the stabilization of the ring-shaped multi-hump beams requires a complex phase distribution characterized by a *fractional* value of the soliton spin [8, 9].

A phase distribution required for the formation of quasi-stationary higher-order self-trapped optical beams was found in references [10] where the concept of *soliton cluster* was introduced. In this approach, the azimuthally modulated beam is regarded as a bound state of the interacting fundamental solitons. Because of the phase-sensitive interaction, the requirement of the balance of the interaction forces between the solitons determines the beam phase in the form of a *staircase-like* screw dislocation. Figure 1 compares the vortex phase dislocation (left column) with the phase of a four-soliton cluster, having well defined $\pi/2$ steps between the soliton positions (right column). It was found [10] that a *radially stable* dynamical bound state is formed if these phase jumps satisfy the condition $\vartheta = 2\pi m/N$, with $N \geqslant 4m$ being the number of solitons in the ring.

Stability of the soliton clusters has been tested numerically for different nonlinear media, including cubic saturating, competing cubic self-focusing and quintic self-defocusing, and competing quadratic and cubic self-defocusing nonlinearities [10]. The idea has also been extended to higher dimensions, covering the case of the spatio-temporal vortex solitons and light bullets. The common outcome of these studies is the confirmed robustness of the soliton clusters to random noise and strong radial perturbations. In the latter case, the pulsating states viewed as the radial excitations of a 'soliton molecule' have been observed. Nevertheless, soliton clusters are subject to modulational instability, and they are unstable with respect to azimuthal perturbations. The remarkable feature of this instability is that the number of fundamental solitons flying off the main ring after the splitting is determined mainly by the topological charge m instead of the initial number N of solitons, similar to the case for the vortex solitons. For what follows, we stress the fact that the conservation of the angular momentum of an optical beam in an isotropic medium determines the dynamics of splitters after break-up, so the initial 'spin' angular momentum of the vortex or cluster can be viewed as being *transformed* into the orbital momentum of the spiralling splitters [4].

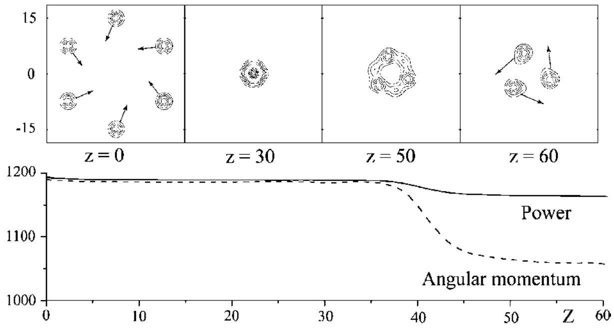

Figure 2. Switching from $N = 6$ to 3 solitons. After reaching the minimum radius at $z = 30$, the *dynamically unstable* three-lobe cluster is formed with its subsequent break-up into three fundamental solitons flying away. Note that symmetry-breaking instability is accompanied by power and angular momentum losses due to nonsolitonic radiation emission.

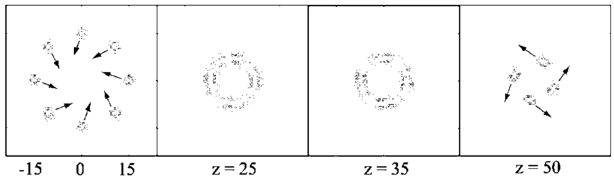

Figure 3. Switching from $N = 8$ to 4 solitons. After reaching the minimum radius at $z = 18$, the ring-like structure creates a single-charged metastable vortex which breaks up subsequently into four fundamental solitons.

3. Soliton molecules

Recent progress in both theoretical and experimental studies of the higher-order optical spatial solitons brought the soliton community to the gates of the direct search for all-optical soliton-based switching schemes, when the initial data carried by the light distribution on the front face of the nonlinear medium can be processed, in a predictable and controllable way, by employing the light self-action effects. One of the examples of such an approach is the recently proposed concept of the 'soliton algebra' [11], based on the instability-induced break-up of optical vortices to a controllable number of the fundamental solitons. This idea also represents an example of a nontrivial approach to the soliton instability, when the symmetry-breaking instability, usually regarded as a serious disadvantage in using spatial solitons, is employed as *a key physical mechanism* for all-optical soliton switching from a given initial state (optical vortex) to the known final state defined by a certain number of fundamental solitons. This approach can be generalized to a broad variety of scalar and vector higher-order *metastable* solitons.

The symmetry-breaking soliton instability may serve as a physical mechanism for all-optical switching with only one disadvantage—it is a one-way process describing the transition from an initial higher-order state (a soliton molecule or cluster) to a number of simple stable states (dipole mode and fundamental solitons). Below, we propose the opposite process, viewed as the excitation or 'synthesis' of higher-order states from a predefined number of initially separated solitons, or 'atoms of light', in a nonlinear bulk medium. Indeed, introducing molecules of light would not be self-consistent without the possibility of mutual trapping of the free atoms or molecule synthesis.

To demonstrate this phenomenon, we numerically propagate the ring-shaped arrays of initially well separated coherently interacting fundamental solitons in a saturable nonlinear medium. Figure 2 shows a characteristic example of a set of six solitons which have their relative phases growing in steps of $\vartheta = \pi/3$ along the ring, being initially directed to collide with each other. This initial condition corresponds to the inversion of the instability-induced ring break-up, so the solitons move towards the ring instead of flying away. We observe a highly inelastic collision of the solitons when they strongly interact, overlapping and losing their identity. Nevertheless, the initial phases of the solitons are tilted in such a way that the total phase of the beams forms a screw dislocation in the ring origin which prevents a simple fusion of all solitons. Instead, the ring-shaped structure is formed. A similar situation is observed in figure 3 with an array of eight solitons and the formation of a metastable vortex ring. Due to large amplitude modulations, these intermediate (or metastable) structures never form ideal stationary states, and they quickly split off to a set of new isolated solitons, with the total number of splitters predefined by the initial conditions. In this way, we were able to produce, as a final state, the patterns with different numbers of solitons by changing initial parameters, including the number and phase tilt of solitons. Generally, the final number of solitons is determined by the ring instability mode with the largest growth rate, and in a saturable medium it is usually twice the topological charge [4]. At the same time, we were able to force the single-charged 'meta-vortex' in figures 2 and 3 to split to three and four fundamental solitons, respectively. The intermediate meta-state shows complex dynamics of the instability development which may continue for several tenths of the diffraction length. Thus, the whole picture of the soliton collision, ring formation, and the ring splitting is somewhat similar to the 'delayed-action interaction' recently reported for interacting composite solitons carrying nonzero angular momentum [12]. We note also that, in addition to the known transformation of 'spin to orbital' angular momentum [4], in figures 2 and 3 we observe a kind of 'orbital–spin–orbital' transformation. The change of the field momentum shown in figure 2 is about 10% of the initial value, and it occurs only at the break-up stage.

Highly unstable ring-shaped beams are formed as a result of mutual trapping and inelastic interaction of coherent solitons, as shown in figures 2 and 3. They represent the intermediate steps in the process of nonlinear switching between the states with different numbers of solitons. In the context of the soliton algebra and all-optical switching, it might be of interest to study in more detail and to determine the quantitative parameters necessary to obtain the final state with a given number of solitons. At the same time, the mutual trapping of coherent solitons is found to be too sensitive to the initial perturbations to produce metastable clusters.

Nevertheless, it is indeed possible to generate higher-order optical beams, i.e. to synthesize the soliton molecules, from interacting fundamental *coherent and incoherent* solitons. In figure 4 we show the excitation of a long-lived four-lobe vector cluster. We note that the four solitons in figure 4(b) at $z = 0$ are directed exactly to the centre so there is no azimuthal tilt

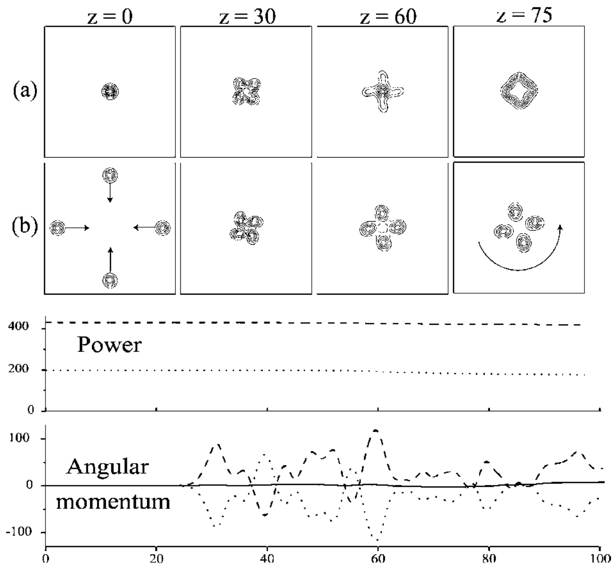

Figure 4. Excitation of a four-lobe rotating vector cluster, consisting of fundamental (row (a) and dotted lines) and modulated (row (b) and dashed lines) mutually incoherent components. Solid line: the total angular momentum. The arrows in row (b) show the initial transverse velocities of interacting 'atoms' and rotation of the final 'molecule'.

of their phase. Because each soliton has a phase growing by a jump $\vartheta = \pi/2$ along the ring, similar to the cluster in figure 1, this initial phase distribution guarantees the appearance of the screw phase dislocation, even the corresponding value of the total angular momentum is very small (a solid line in figure 4). Because only the total angular momentum is conserved, not the partial momenta of the components, there is a freedom for components to symmetrically exchange angular momentum during the beam propagation. After mutual trapping of all solitons at $z \approx 25$ the new vector cluster experiences strong radial oscillations between the states with maximal (e.g. at $z = 30$ and 60) and minimal partial angular momenta in the components; see the diagram in figure 4. At the same time, the $\pi/2$ phase jumps introduced initially between the solitons (edge dislocations) survive these strong oscillations, and the whole cluster preserves its structure for a distance exceeding 100 diffraction lengths. Therefore, we observe the formation of a composite state, the soliton molecule, by colliding simple solitons with a nontrivial phase pattern.

4. Conclusions

We have studied the scattering and mutual trapping of several fundamental solitons and the generation of soliton clusters and vortex solitons—the complex self-trapped states of light carrying phase dislocations in the wavefront. Inelastic collision of solitons has been shown to result in the formation of ring-shaped beams or metastable vortices which subsequently break up creating different numbers of fundamental solitons dictated by the initial conditions. This kind of soliton delayed-action interaction and the nonlinear transformation of the number of fundamental solitons have been analysed in the context of the soliton algebra and all-optical soliton switching. We have also shown that the vectorial interaction between the field components provides an additional mechanism of soliton cluster stabilization. In particular, we have demonstrated the formation of vector soliton clusters from colliding solitons, a process which can be regarded as a synthesis of 'soliton molecules'.

Acknowledgments

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