15 August 2002

Optics Communications 209 (2002) 501–506

OPTICS COMMUNICATIONS

www.elsevier.com/locate/optcom

Multi-component vector solitons in photorefractive crystals

Kristian Motzek [a], Friedemann Kaiser [a], Carsten Weilnau [b], Cornelia Denz [b], Glen McCarthy [c], Wieslaw Krolikowski [c,*], Anton Desyatnikov [b,d], Yuri S. Kivshar [d]

[a] Institute of Applied Physics, Darmstadt University of Technology, D-64285 Darmstadt, Germany
[b] Institute of Applied Physics, Westfälische Wilhelms-Universität Münster, D-48149 Münster, Germany
[c] Laser Physics Centre, Research School of Physical Sciences and Engineering, Australian National University, Canberra ACT 0200, Australia
[d] Nonlinear Physics Group, Research School of Physical Sciences and Engineering, Australian National University, Canberra ACT 0200, Australia

Received 8 May 2002; received in revised form 24 June 2002; accepted 26 June 2002

Abstract

We study the existence and stability of multi-component spatial optical solitons in anisotropic nonlocal photorefractive media. For the case of three components, we find numerically the whole family of composite solitons with two perpendicular dipole components, and show their stability for a wide range of parameters. We confirm our theoretical results by an experimental observation of these novel types of composite optical solitons in a photorefractive strontium barium niobate crystal. © 2002 Elsevier Science B.V. All rights reserved.

PACS: 42.65.Tg; 05.45.Yv; 42.65.Hw

Recently, there has been growing interest in the study of composite (or vector) optical solitons [1,2]. Such solitons are optical beams that consist of several components that self-trap together when propagating in a nonlinear medium. Essential for the formation of this kind of vector soliton is the absence of coherent wave-mixing effects, i.e., the interference of beams must be suppressed during their nonlinearity-mediated interaction. This has first been suggested [3,4] and realized [5] with the beams carrying orthogonal states of polarization. Later, this concept has been generalized to include all kinds of optical self-trapped beams that are mutually incoherent with each constituent beam being a corresponding component of a generalized multi-dimensional "vector beam" [6].

As is well accepted, the fundamental optical soliton induces an optical waveguide in a nonlinear medium and propagates self-consistently as its fundamental mode [7]. In the same way, components of the composite (vector) soliton represent various (higher) modes of the self-induced wave-

* Corresponding author. Tel.: +61-261-253-752; fax: +61-261-250-029.
E-mail address: wzk111@rsphysse.anu.edu.au (W. Krolikowski).

PII: S0030-4018(02)01728-5

guide. Because of their mutual temporal incoherence, mode beating is absent here and the total intensity distribution as well as the induced waveguide structure remain stationary along the propagation direction.

Composite (1+1)-dimensional optical solitons have been first experimentally demonstrated by Mitchell et al. [8]. They typically consist of multi-hump beams propagating in a stationary manner in a self-focusing (or defocusing) medium. In two spatial transverse dimensions, composite solitons were shown to exist in two fundamental forms. The first one displays a cylindrical symmetry and consists of an optical vortex of Laguerre–Gaussian-type co-propagating with a fundamental Gaussian beam [9]. Subsequently, it has been shown that such an object experiences symmetry-breaking instability which transforms it into a stable entity, the so-called *dipole-mode vector soliton* [10]. The latter consists of a dipole, i.e., a beam in the form of two out-of-phase lobes, co-propagating with an elliptically shaped nodeless beam [10]. The natural repulsion of the out-of-phase lobes in the dipole component is counteracted by an attractive force provided by the fundamental component leading to the formation of a stable composite object, slightly elongated along the dipole's axis. These dipole-mode solitons have recently been observed in photorefractive nonlinear crystals [11,12].

As a matter of fact, the dipole-mode vector soliton is only a particular (one of the simplest) example of a large family of *multipole vector solitons* made by an incoherent superposition of a fundamental and various multipole beams [13–15]. However, among various different types it is particularly the dipole-mode vector soliton that displays a surprising robustness with respect to perturbations.

So far only two-component vector solitons have been investigated experimentally in both, one [8] and two spatial dimensions [11–15]. Recently, Desyatnikov and Kivshar [16] considered a new type of composite soliton in an isotropic medium formed by the mutual trapping of two perpendicularly oriented dipole components [16]. They predicted a decay of this structure into three fundamental solitons flying away along the normal directions. Additionally, numerical and experimental studies by Ahles et al. [17] for photorefractive nonlinearity revealed that although such a double dipole vector soliton can propagate in a stable manner over a distance of several diffraction lengths, it is unstable and eventually breaks up. Desyatnikov et al. [18] have shown that the two-dipole soliton can be stabilized by introducing a third, mutually incoherent nodeless component. Similar to its role in the dipole-mode vector soliton, the fundamental component provides an attractive force consolidating the whole structure.

The theoretical analysis of the three-component solitons presented in [18] applies only to an isotropic nonlinearity. On the other hand, photorefractive crystals, which are frequently used to demonstrate the formation of various types of spatial solitons, may exhibit a strong anisotropy and nonlocality in their nonlinear response. Both these effects have been shown to affect significantly the properties of the spatial solitons and their interaction [14,19,20]. Here, we study the general properties of the three-component composite solitons in a photorefractive nonlinear medium. Firstly, we demonstrate the soliton generation and explore numerically the parameter space for which they are stable. Secondly, we show that these solutions may become unstable and display a symmetry-breaking instability. Both the stable soliton formation as well as the breakup of the composite beam have been observed experimentally.

As a fundamental example of multi-component beams of a nontrivial geometry (see discussions in [18]), we consider the propagation of three mutually temporally incoherent optical beams with the slowly varying amplitudes $E(x,y,z)_j$ ($j = 1,2,3$) in a biased photorefractive crystal. In this notion, $j = 1$ represents the fundamental Gaussian beam, $j = 2$ is the dipole beam oriented along the vertical y-axis and $j = 3$ denotes the dipole beam whose axis points along the horizontal x-axis. Assuming that the biasing field is applied along the horizontal axis and the beams propagate along the other axis, the beam evolution is described by the system of equations

$$\mathrm{i}\frac{\partial E_j}{\partial z} + \frac{1}{2}\nabla_\perp^2 E_j = \gamma \frac{\partial \Phi}{\partial x} E_j, \tag{1}$$

where γ stands for the photorefractive coupling coefficient and Φ denotes the electric potential induced in the crystal by the propagating beams. This potential satisfies the following equation (see [21])

$$\nabla^2 \Phi + \nabla\Phi\nabla \ln(1+I) = E_{\text{ext}} \frac{\partial}{\partial x} \ln(1+I), \qquad (2)$$

where the total light intensity $I = \Sigma_j |E_j|^2$ is measured in units of the background (or dark) illumination I_0, and E_{ext} represents the externally applied electric field.

We look for the stationary soliton solutions to the system of equations (1) and (2) in the form $E_j(x,y,z) = a_j(x,y)\exp(\mathrm{i}\lambda_j z)$ with λ_j being the propagation constant of the jth beam. Inserting this ansatz into Eqs. (1) and (2) yields the following set of equations:

$$-\lambda_j a_j + \frac{1}{2}\nabla_{\perp}^2 a_j = \frac{\partial \Phi}{\partial x} a_j. \qquad (3)$$

This can be solved by using an iterative numerical procedure suggested by Petviashvili [22], which is generalized here to describe the case of several beams [23]. As a starting point of our investigations, we calculate a solitary solution with equal power in all its three components and with a maximum intensity of the order of unity. From there, we explore the whole family of the solutions by varying the propagation constants of both dipole components (λ_2, λ_3). The graph in Fig. 1 shows the whole family of solutions by illustrating the power of the single components versus λ_2. It can be seen that the solitary solutions range from structures dominated by the fundamental mode (E_1) and the dipole parallel to the external electric field (E_3), for small values of $\lambda_2 = 0.11$ (see Fig. 1(a)), to structures with power concentrated predominantly in the dipole perpendicular to the external field (E_2), for larger values of $\lambda_2 = 0.17$, as shown in Fig. 1(b). The frames in Figs. 1(a) and (b) depict the transverse intensity profile of the single components of the composite soliton solutions, with dark regions corresponding to high intensity.

Fig. 2 shows the family of soliton solutions for the case when the propagation constant of the dipole parallel to the biasing field (λ_3) is varied. Our simulations show that soliton solutions with very small power in the dipole component parallel

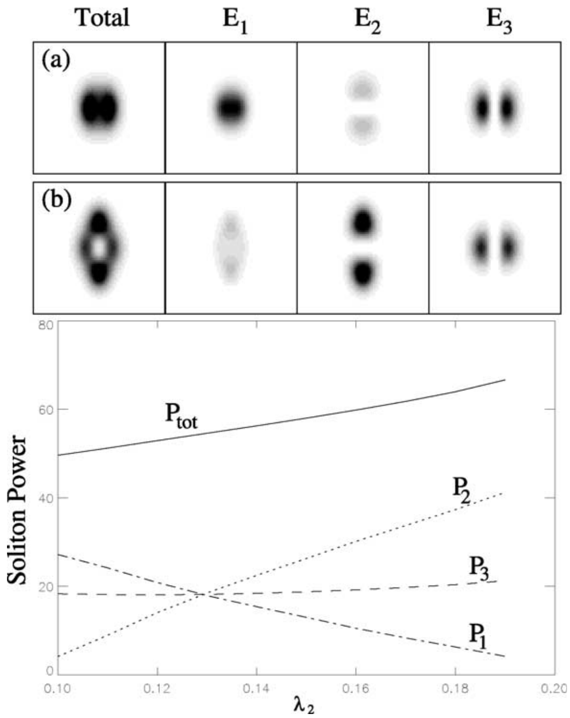

Fig. 1. Partial, P_j, and total, P_{tot}, powers of a three-component composite soliton as a function of λ_2 for $\lambda_1 = 0.2$ and $\lambda_3 = 0.056$. Top: two examples of the soliton structure for (a) $\lambda_2 = 0.11$ and (b) $\lambda_2 = 0.17$.

to the external field do not exist, even as $\lambda_3 \to 0$. This is a consequence of the anisotropic structure of the waveguide induced in the nonlinear crystal by the fundamental beam and the dipole perpendicular to the external field. It appears that such a waveguide does not support the TE_{10} mode, i.e., the mode whose structure corresponds to a dipole parallel to the applied field. In order to support such a mode the corresponding dipole component has to carry significant energy to appreciably modify the waveguide. An example of such a solitary solution is shown in Fig. 2(a). As Fig. 2(b) shows, for large values of λ_3 the powers of the two dipoles, P_2 and P_3 are comparable whereas that of the ground mode, P_1 becomes negligibly small. As a result, the soliton solutions become basically identical to those obtained by considering just two orthogonal dipoles as discussed in [17].

To check the stability of the steady-state soliton solutions described above we propagated them

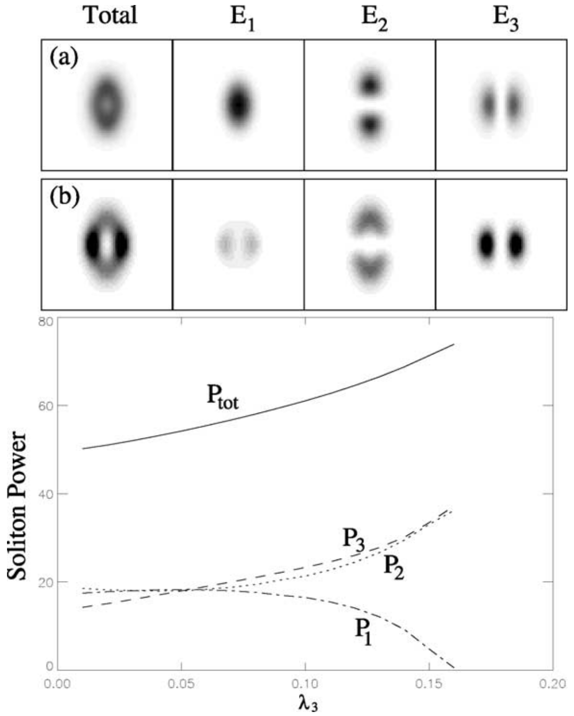

Fig. 2. Partial, P_j, and total, P_{tot}, powers of a three-component soliton as a function of λ_3 for $\lambda_1 = 0.2$ and $\lambda_2 = 0.129$. Top: two examples of the soliton structure for (a) $\lambda_3 = 0.03$ and (b) $\lambda_3 = 0.14$.

numerically using the original system of equations (1) and (2). We found that the solutions shown in Figs. 1 and 2 are stable for λ_2 (λ_3) smaller than 0.15 (0.06). In this region they are rather robust and withstand even strong initial perturbation. When perturbed, the soliton propagates as a single entity, only exhibiting strong internal oscillations of its constituent components. The solutions depicted in Figs. 1 and 2 become unstable for λ_2 (λ_3) exceeding 0.15 (0.06). In this case the solutions are characterized by a relatively weak fundamental Gaussian component (P_1) that is not capable of stabilizing the two orthogonal dipole beams.

We studied the generation of the three-component dipole solitons experimentally using a SBN photorefractive crystal and the experimental setup similar to the one described in [11]. Two mutually incoherent light beams (wavelength of 532 nm) derived from Nd:YAG lasers were transmitted through microscope glass slides to imprint the desired π-phase jumps across the beams. In this way two perpendicularly oriented dipole components were created. They were superimposed and combined with an additional, mutually incoherent Gaussian beam and subsequently focussed onto the input facet of a 10-mm long SBN crystal biased with a DC field. To control the degree of saturation the crystal was illuminated with a wide beam derived from the white light source. The initial (i.e. at the input face of the crystal) degree of saturation was estimated to be of the order of unity in all our experiments. The outgoing light intensity distribution was monitored finally by a CCD camera. In Fig. 3 we show an example of the three-component soliton. The initial power in the fundamental and the dipole beams were 2, 2.2 and 1.8 μW, respectively.

The top row of this figure shows the initial intensity distribution of the constituent components while the bottom row depicts the intensities of each component after 10 mm of propagation through the biased photorefractive crystal. It can be clearly seen that the light self-traps and forms a stable vector soliton. The parameters of this experimentally observed soliton are very close to the numerical example shown in Fig. 2(a) with $\lambda_3 = 0.03$. Solutions depicted in Fig. 2 with $\lambda_3 < 0.06$ were numerically confirmed to be stable even when they become subject to numerical noise. As discussed earlier, the Gaussian component is

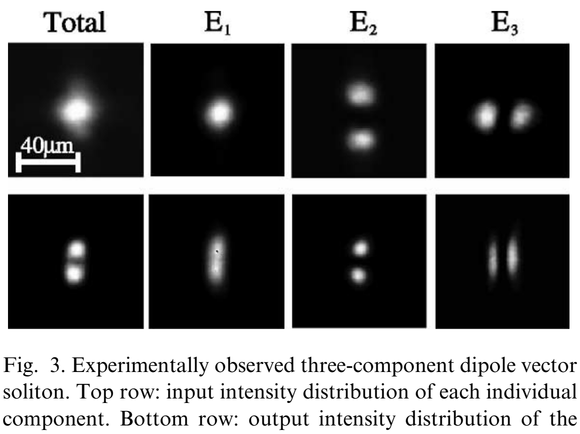

Fig. 3. Experimentally observed three-component dipole vector soliton. Top row: input intensity distribution of each individual component. Bottom row: output intensity distribution of the single components after 10 mm propagation in the crystal. The initial power of the components are: $P_1 = 3.3$, $P_2 = 1.8$, $P_3 = 2.2$ μW, and the biasing field $E_{ext} = 2$ kV/cm.

essential to create a vector soliton that contains two perpendicularly oriented dipole beams. We demonstrate this stabilizing role of the fundamental component in Fig. 4. It shows the output intensity distribution of two co-propagating dipoles immediately after the stabilizing Gaussian component has been blocked in Fig. 4(a). Since the photorefractive material acts noninstantaneously to intensity fluctuations, Fig. 4(a) just represents the contribution of the two dipole constituents to the three-component vector soliton. Fig. 4(b) depicts the situation a couple of seconds after the Gaussian beam has been blocked. Now, the two dipole components propagate separately and do not form a confined light structure any longer. It is evident that the presence of the additional Gaussian beam provides an attractive force which tightly binds both dipoles (and the whole three-component structure).

Although the three-component vector solitons are in general very robust, they develop an instability if their parameters (powers) lie outside the stability region. In Fig. 5 we show an example of such a situation. The propagation constants for the soliton solution depicted in Fig. 5(a) are $\lambda_1 = 0.2$, and $\lambda_2 = \lambda_3 = 0.13$. Fig. 5(a) shows the initial intensity distribution. The numerical propagation of this light structure reveals a symmetry-breaking instability that leads to a breakup into three fragments after five diffraction lengths in Fig. 5(b). For comparison, Fig. 6 presents an experimentally observed unstable propagation of the three-component beam over the distance of 15 mm. The powers of the constituent components are $P_1 = 4.8$, $P_2 = 3.2$, $P_3 = 2$ μW, respectively. The incident light intensity distribution is similar to the one depicted in Fig. 3 (top row), but the relative beam powers have been slightly modified and the propagation length has been extended by making use of an equivalent but longer crystal sample. It is evident that the stable vector soliton does not form but instead the whole structure experiences a symmetry-breaking instability and decays into three beamlets. Notice the very close similarities between numerical and experimental results even though the initial conditions in the experiment differ slightly from those calculated numerically by the Petviashvili method.

In conclusion, we have shown that novel types of multi-component spatial optical solitons, earlier predicted for isotropic nonlinear media, can exist in anisotropic nonlocal media associated with photorefractive nonlinearity. In particular, we

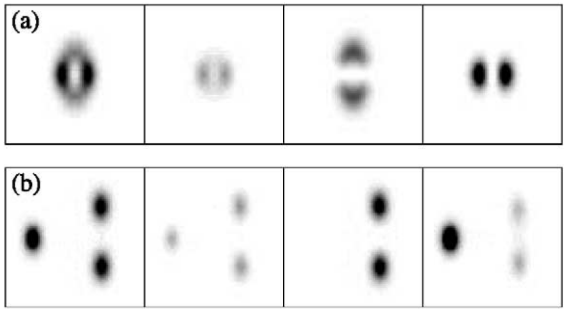

Fig. 5. Example of an unstable three-component soliton. Numerically calculated intensity distribution of a multi-component soliton (a) for $\lambda_1 = 0.2$ and $\lambda_2 = \lambda_3 = 0.13$ and its breakup (b) after propagating five diffraction lengths.

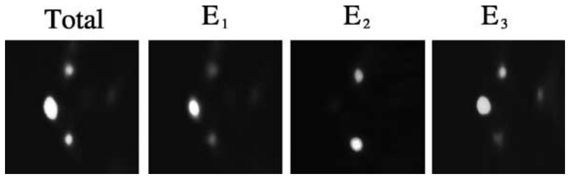

Fig. 6. Example of an unstable three-component soliton. Experimentally observed decay of a three-component beam with beam powers $P_1 = 4.8$, $P_2 = 3.2$, $P_3 = 2$ μW and an external electric field of 2.5 kV/cm.

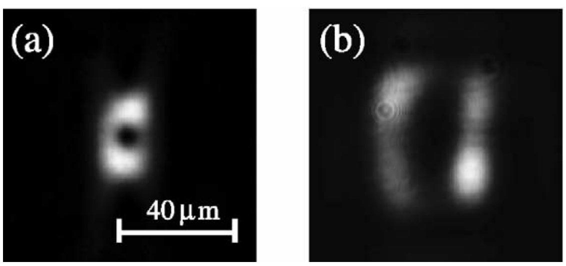

Fig. 4. Experimental demonstration of the stabilization of two orthogonal dipoles by a fundamental beam. Total intensity distribution of two dipole components propagating over a distance of 10 mm in the presence (a) and in the absence of the additional Gaussian beam (b). The initial conditions are the same as in Fig. 3.

have studied in detail the existence and stability of the three-component optical solitons which consist of a fundamental beam co-propagating with two orthogonally oriented dipole components, which represent the lower-order modes of the self-induced waveguide. Numerical simulations show that these solitons are stable in a wide range of their parameter space, but they are eventually subject to a peculiar symmetry-breaking instability. We have observed the formation as well as the breakup of these solitons in experiments with a photorefractive crystal. Our experimental observations are in good agreement with the theoretical predictions.

Acknowledgements

The research in Germany has been supported by the German Academic Exchange Service (DAAD), and Graduiertenkolleg Nichtlineare Kontinuierliche Systeme at Westfälische Wilhelms-Universität Münster. The research in Australia has been supported by the Australian–German Joint Research Co-operation Scheme.

References

[1] S. Trillo, W. Toruellas (Eds.), Spatial Solitons, Springer, Berlin, 2001.
[2] G.I. Stegeman, M. Segev, Science 286 (1999) 1518.
[3] S.V. Manakov, Sov. Phys. JETP 38 (1974) 248.
[4] D.N. Christodoulides, R.I. Joseph, Opt. Lett. 13 (1988) 53.
[5] J.U. Kang, G.I. Stegeman, J.S. Aitchison, N.N. Akhmediev, Phys. Rev. Lett 76 (1996) 3699.
[6] D.N. Christodoulides, S.R. Singh, M.I. Carvalho, M. Segev, Appl. Phys. Lett. 68 (1996) 1763.
[7] A.W. Synder, D.J. Mitchell, L. Poladian, F. Ladouceur, Opt. Lett. 16 (1991) 21.
[8] M. Mitchell, M. Segev, D.N. Christodoulides, Phys. Rev. Lett. 80 (1998) 4657.
[9] Z. Musslimani, M. Segev, D.N. Christodoulides, M. Soljačić, Phys. Rev. Lett. 84 (2000) 1164.
[10] J.J. García-Ripoll, V.M. Pérez-García, E.A. Ostrovskaya, Yu.S. Kivshar, Phys. Rev. Lett. 85 (2000) 82.
[11] W. Krolikowski, E.A. Ostrovskaya, C. Weilnau, M. Geisser, G. McCarthy, Yu.S. Kivshar, C. Denz, B. Luther-Davies, Phys. Rev. Lett. 85 (2000) 1424.
[12] T. Carmon, C. Anastassiou, S. Lan, D. Kip, Z.H. Musslimani, M. Segev, Opt. Lett. 25 (2000) 1113.
[13] A.S. Desyatnikov, D. Neshev, E.A. Ostrovskaya, Yu.S. Kivshar, W. Krolikowski, B. Luther-Davies, J.J. García-Ripoll, V.M. Pérez-García, Opt. Lett 26 (2001) 435.
[14] C. Weilnau, C. Denz, M. Ahles, A. Stepken, K. Motzek, F. Kaiser, Phys. Rev. E 64 (2001) 056601.
[15] A.S. Desyatnikov, D. Neshev, E.A. Ostrovskaya, Yu.S. Kivshar, G. McCarthy, W. Krolikowski, B. Luther-Davies, J. Opt. Soc. Am. B 19 (2002) 586.
[16] A.S. Desyatnikov, Yu.S. Kivshar, Phys. Rev. Lett. 87 (2001) 033901.
[17] M. Ahles, K. Motzek, A. Stepken, F. Kaiser, C. Weilnau, C. Denz, J. Opt. Soc. Am. B 19 (2002) 557.
[18] A.S. Desyatnikov, Yu.S. Kivshar, K. Motzek, F. Kaiser, C. Weilnau, C. Denz, Opt. Lett. 27 (2002) 634.
[19] W. Krolikowski, M. Saffman, B. Luther-Davies, C. Denz, Phys. Rev. Lett. 80 (1998) 3240.
[20] D. Neshev, G. McCarthy, W. Krolikowski, E.A. Ostrovskaya, Yu.S. Kivshar, G.F. Calvo, F. Agullo-Lopez, Opt. Lett. 26 (2001) 1185.
[21] A.A. Zozulya, D.Z. Anderson, Phys. Rev. A 51 (1995) 1520.
[22] V.I. Petviashvili, Fiz. Plazmy 2 (1976) 469;
A.A. Zozulya, D.Z. Anderson, A.V. Mamaev, M. Saffman, Phys. Rev. A 57 (1998) 522.
[23] K. Motzek, A. Stepken, F. Kaiser, M.R. Belić, M. Ahles, C. Weilnau, C. Denz, Opt. Commun. 97 (2001) 161.