QOPT/95997/P PROC
Digital by Design
Printed: 20/10/1998

CRC data
Issue no:
Total pages:
First page:
Last page:
File name: QS8 .TEX
Date required:

Quantum Semiclass. Opt. **10** (1998) 1–10. Printed in the UK PII: S1355-5111(98)95997-8

Instabilities in coupled Nd:YVO_4 microchip lasers

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Received 3 July 1998, in final form 1 October 1998

Abstract. Amplitude instabilities near the threshold of phase locking are investigated in a system of two coupled Nd:YVO_4 microchip lasers. The compact semimonolithic resonator set-up allows a systematic study of the relevant parameter regime, including the observation of synchronous amplitude oscillations and, for the first time in coupled solid-state lasers, localized synchronization. The measurements are compared with numerical simulations.

1. Introduction

Arrays of spatially coupled lasers, especially semiconductor laser arrays, are of considerable technical importance for a number of applications [1]. For some purposes it is desirable that all lasers are synchronized in phase, for example, for the generation of high-power coherent sources operating at a stationary intensity. For other purposes, one may want to operate each element of an array at its own variable power level, for example, for display applications or optical data processing. From a more fundamental point of view, the study of coupled nonlinear oscillators reveals a fascinating wealth of dynamical phenomena; here, coupled lasers—or coupled modes of one laser—are systems well suited for theoretical and experimental investigation of these phenomena. Both experiment and theory have shown that already two single-mode lasers that are stable individually can exhibit a chaotic instability when coupled [2, 3]. Studying only two lasers can thus be considered a preliminary step to the investigation of larger numbers of coupled lasers [4–6].

The time scale of the intensity fluctuations of semiconductor lasers (picoseconds to nanoseconds), however, is much less convenient for precise dynamical measurements than it is in solid-state lasers and CO_2 lasers (microseconds to milliseconds). First studies of two laterally coupled, argon-ion laser pumped Nd:YAG (neodymium-doped yttrium aluminium garnet) lasers [7] revealed the existence of an amplitude–phase instability near the phase-locking threshold of the two lasers in a parameter range agreeing well with theoretical predictions [8]. Measurements, however, were complicated by a substantial amount of fluctuations.

Neodymium-doped yttrium orthovanadate (Nd:YVO_4) is a fairly novel laser material that can provide much higher gain per unit volume than the well established materials like Nd:YAG. Using very thin laser crystals (down to 100 μm), so-called *microchip* or *microcavity* lasers can be created. Owing to the large mode spacing, these lasers can operate on a single frequency without any additional selection elements [9]. In conjunction with intra- or extracavity frequency doubling and laser-diode pumping very compact visible lasers can be realized. Arrays of these lasers, pumped by diode laser arrays, have already been studied [10], and there are suggestions to use these in laser display applications.

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In this paper we report a number of results obtained from a system of two coupled lasers in a semimonolithic Nd:YVO_4 microchip resonator, pumped by a Ti:sapphire laser. We will show that this set-up, due to its high mechanical stability, is very well suited for the systematic investigation of the relevant parameter regime, i.e. especially the regime of small detuning. In addition to the type of amplitude oscillations already reported from the Nd:YAG system [8], a novel type of synchronized oscillation, which has already been predicted as 'localized synchronization' [4] can be observed. The experimental results are compared to numerical computations of the dynamical behaviour to be expected in the particular parameter regimes.

2. Experimental set-up

Nd:YVO_4 has some significant advantages as a laser gain material compared to Nd:YAG, though mostly used at the same pump ($\approx$ 808 nm) and laser ($\approx$1064 nm) wavelengths. The cross sections for pump absorption and for stimulated emission are about eight times larger, and higher neodymium doping concentrations are possible (3% for YVO_4, 1.1% for YAG). Thus, very thin laser crystals can be used. The laser emission will be polarized parallel to the *c*-axis of the uniaxial Nd:YVO_4 crystal, thus a number of lasers in the same crystal will always oscillate with the same polarization.

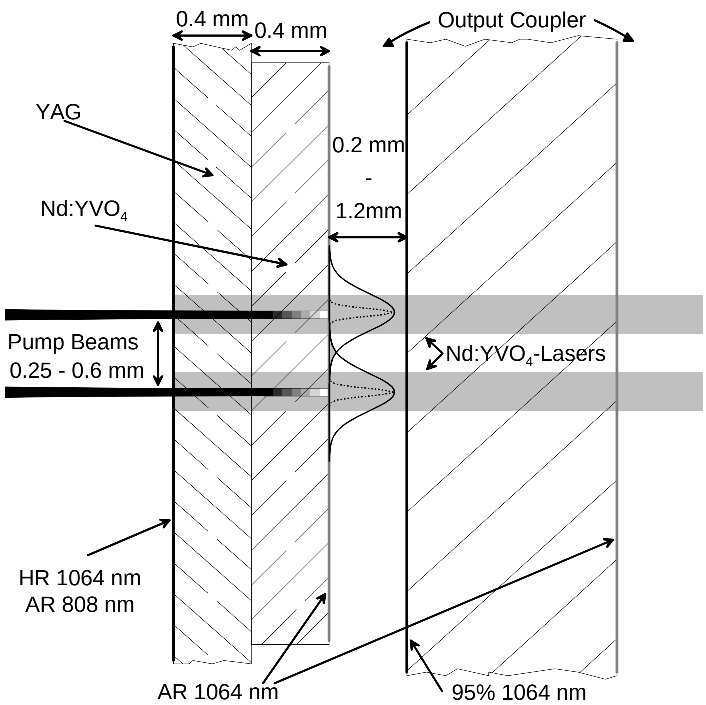

Figure 1. Nd:YVO_4 laser resonator. The 3 × 3 mm Nd:YVO_4 crystal is bonded to an undoped YAG disc of 9 mm diameter which itself is mounted to a copper heat sink. The output coupler can be tilted by very small angles to control the mutual detuning.

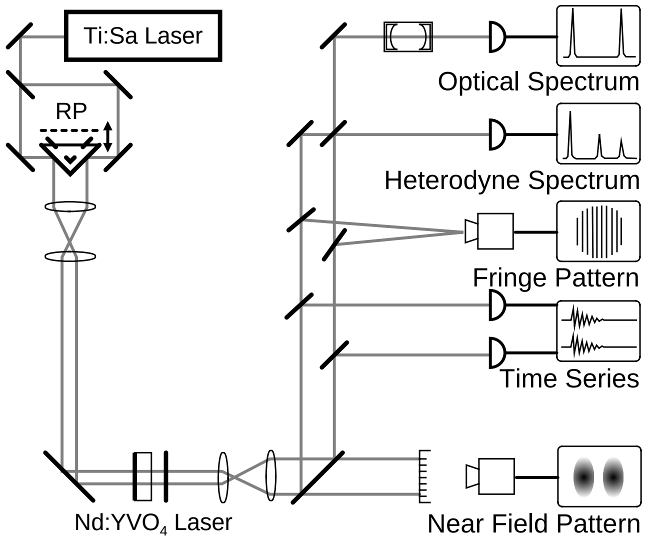

Figure 2. Schematic set-up of the Nd:YVO$_4$ microchip laser experiment.

Here, we use the semimonolithic resonator shown in figure 1. A 0.4 mm thick 3 mm $\times$ 3 mm Nd:YVO$_4$ crystal (3 at. % Nd) is bonded to a 0.4 mm thick undoped YAG disc of 9 mm diameter. The YAG disc serves as a heat sink for the laser crystal, it is itself mounted on a copper heat sink. The outside of the YAG carries a dielectric coating which transmits the 808 nm pump light and has a high reflectance for the 1064 nm laser light, thus forming one mirror of the cavity; the outside of the Nd:YVO$_4$ is antireflection coated for 1064 nm. The only other resonator component is a planar output coupler with 95% reflectivity, placed at a distance between 0.2 and 1.2 mm from the Nd:YVO$_4$ crystal (cf figure 2(*b*)). The optical length of the cavity is thus between 1.7 and 2.7 mm, corresponding to resonator round-trip times τ_c between 11 and 18 ps. The frequency detuning between the lasers can be adjusted by tilting the output coupler using PZT actuators. The detuning control is one of the most crucial issues in this experiment, since, for example, at 0.3 mm laser spacing and 80 GHz free spectral range a detuning increment of 20 MHz corresponds to a tilt of 0.45 μrad.

The system is pumped by two beams from a Ti:sapphire laser at 808 nm with pump waist radii of 20 μm, the spacing between these beams can be adjusted between 0.25 mm and more than 1 mm by a system of beamsplitters and prisms (RP in figure 2) that ensures parallel propagation symmetric to the centre of the crystal.

Thermal lensing induced in the Nd:YVO$_4$ crystal creates two stable, separate cavities emitting TEM$_{00}$ infrared laser beams with waist radii of about 100 μm; the overlap between these two lasers can be changed continuously by varying the spacing. In the investigated range of distances, there is no appreciable overlap of the pump beams. Thus, coupling is entirely due to the spatial overlap of the infrared laser fields.

The individual output intensity time series are recorded with fast photodetectors and a two-channel digitizing oscilloscope. The optical frequency difference and the relaxation oscillation frequencies are measured with a radio-frequency spectrum analyser after combining the two beams on another photodetector. A scanning Fabry–Perot interferometer is used to ensure that both lasers oscillate on a single longitudinal mode each. The plane of the output coupler is imaged onto a screen with a graticule which is monitored by a CCD camera. Phase synchronization between the lasers is observed by monitoring the fringe pattern of the two beams combined under a small angle with another CCD camera (see figure 2).

3. Experimental results

The lasers are operated at pump powers of 60 mW each, which corresponds to 1.4 times the threshold. At these powers, a certain amount of visible fluorescence can be observed, showing the presence of energy transfer upconversion processes that are known to decrease the lifetime of the upper laser level τ_f [9, 11]. This is confirmed by the rather high relaxation oscillation frequency corresponding to $\tau_f = 12\ \mu$s.

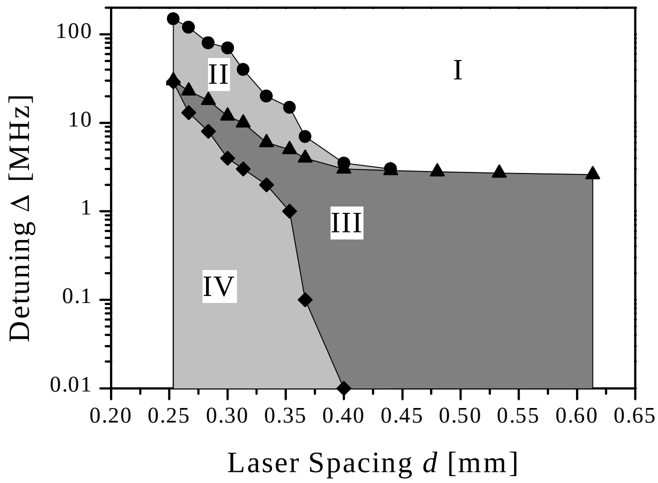

Figure 3. Different regimes of the detuning–spacing parameter space. I, stationary intensity, zero fringe visibility; II, intermediate visibility; III, amplitude instability, high visibility; IV, stationary intensity, maximum visibility.

In the parameter space of laser spacing d, i.e. coupling strength, and laser detuning Δ, four regimes can be distinguished as shown in figure 3:

(I) At large spacings and large detunings both lasers run independently without any visible phase correlation.

(II) At smaller, decreasing detunings a gradual increase of fringe visibility is observed, indicating the onset of phase correlation.

(III) At further decreased detunings, there exists a boundary, below which intensity oscillations appear, correlated with a pronounced increase of fringe visibility. At small spacings ($d < 0.35$ mm), simultaneous intensity oscillations (see figure 4) are observed with almost optimal fringe visibility. At larger spacings, a new type of dynamical behaviour can be observed as well, which is characterized by a spontaneous symmetry breaking: here, there is only a very small degree of modulation in one of the lasers and a phase lag between the small and large amplitude maxima (see figure 6). This

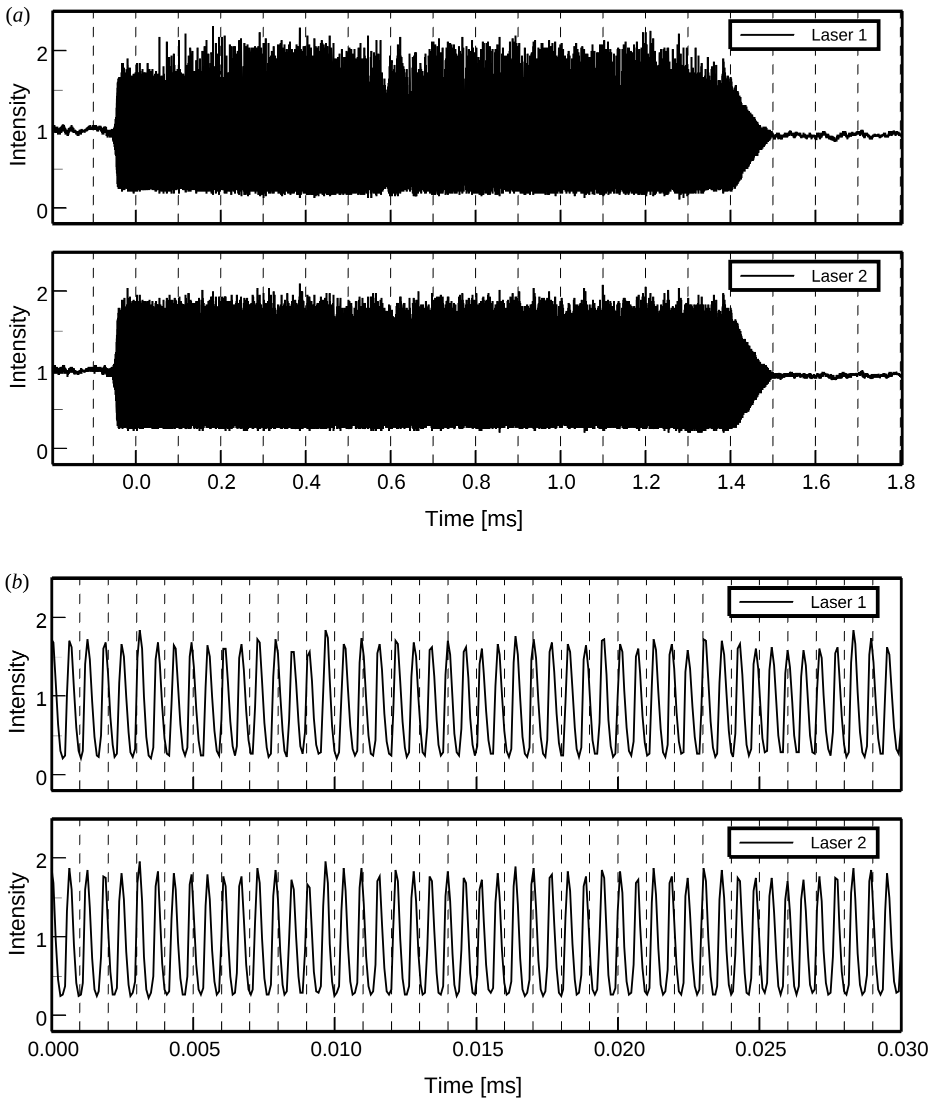

Figure 4. Almost harmonic, synchronized pulsations observed at $d = 0.27$ mm and $\Delta \approx 10$ MHz. (*a*) One oscillation burst; (*b*) detailed view of the same burst. Intensity values are normalized to the stationary value.

type of behaviour has been predicted as 'localized synchronization' [4]. In this range of coupling strengths, the critical detuning is almost of the same order as the detuning fluctuations. This experimental problem causes the burst-like appearance and makes it difficult to give exact values for the detuning thresholds shown in figure 3.

More complex behaviour, like very anharmonic oscillations (figure 5), period-doubling (figure 6(*c*)) or quasi-periodic sequences, can also be observed.

(IV) At very small detuning and small spacings, the intensity instabilities disappear and the lasers remain in a stationary, phase-locked state.

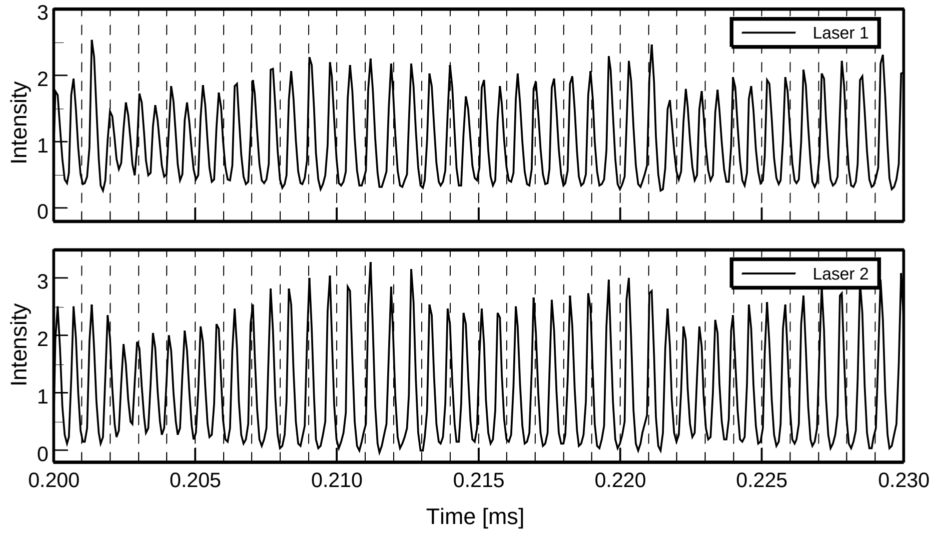

Figure 5. Anharmonic synchronized pulsations observed at $d = 0.36$ mm and $\Delta \approx 5$ MHz (detail).

4. Numerical simulations

The experimentally investigated system is described as a pair of single transverse and longitudinal mode class-B lasers that are coupled to each other through spatial overlap of their electrical fields. The model consists of the coupled differential equations for the complex laser fields $\mathcal{E}_1$, $\mathcal{E}_2$ and their respective gains G_1, G_2 (see [7]):

$$\frac{\mathrm{d}\mathcal{E}_1}{\mathrm{d}t} = \frac{1}{\tau_c}\left[(G_1 - \alpha_1)\,\mathcal{E}_1 - \kappa\mathcal{E}_2\right] + \mathrm{i}\omega_1\mathcal{E}_1 \tag{1a}$$

$$\frac{\mathrm{d}G_1}{\mathrm{d}t} = \frac{1}{\tau_f}\left(p_1 - G_1 - G_1|\mathcal{E}_1|^2\right) \tag{1b}$$

$$\frac{\mathrm{d}\mathcal{E}_2}{\mathrm{d}t} = \frac{1}{\tau_c}\left[(G_2 - \alpha_2)\,\mathcal{E}_2 - \kappa\mathcal{E}_1\right] + \mathrm{i}\omega_2\mathcal{E}_2 \tag{1c}$$

$$\frac{\mathrm{d}G_2}{\mathrm{d}T} = \frac{1}{\tau_f}\left(p_2 - G_2 - G_2|\mathcal{E}_2|^2\right). \tag{1d}$$

In these equations, τ_c is the cavity round-trip time, τ_f is the fluorescence time of the upper lasing level of the Nd^{3+} ion in YVO_4 (1064 nm transition), p_1 and p_2 are the pump coefficients, α_1 and α_2 are the cavity loss coefficients, and ω_1 and ω_2 (angular frequencies) are the detunings of the lasers from a common cavity mode, respectively. The lasers are coupled linearly to each other with strength $\kappa = \exp(-d^2/2w^2)$, as determined by the overlap integral of two Gaussian laser beams with $1/\mathrm{e}^2$ radius w, separated by the distance d. The sign of the coupling terms is chosen to account for the observed stable phase-locked state in which the lasers are 180° out of phase with each other.

Similar equations are found in the investigation of semiconductor laser arrays and multimode lasers; in the latter case a second coupling mechanism via gain sharing has to be taken into account [6].

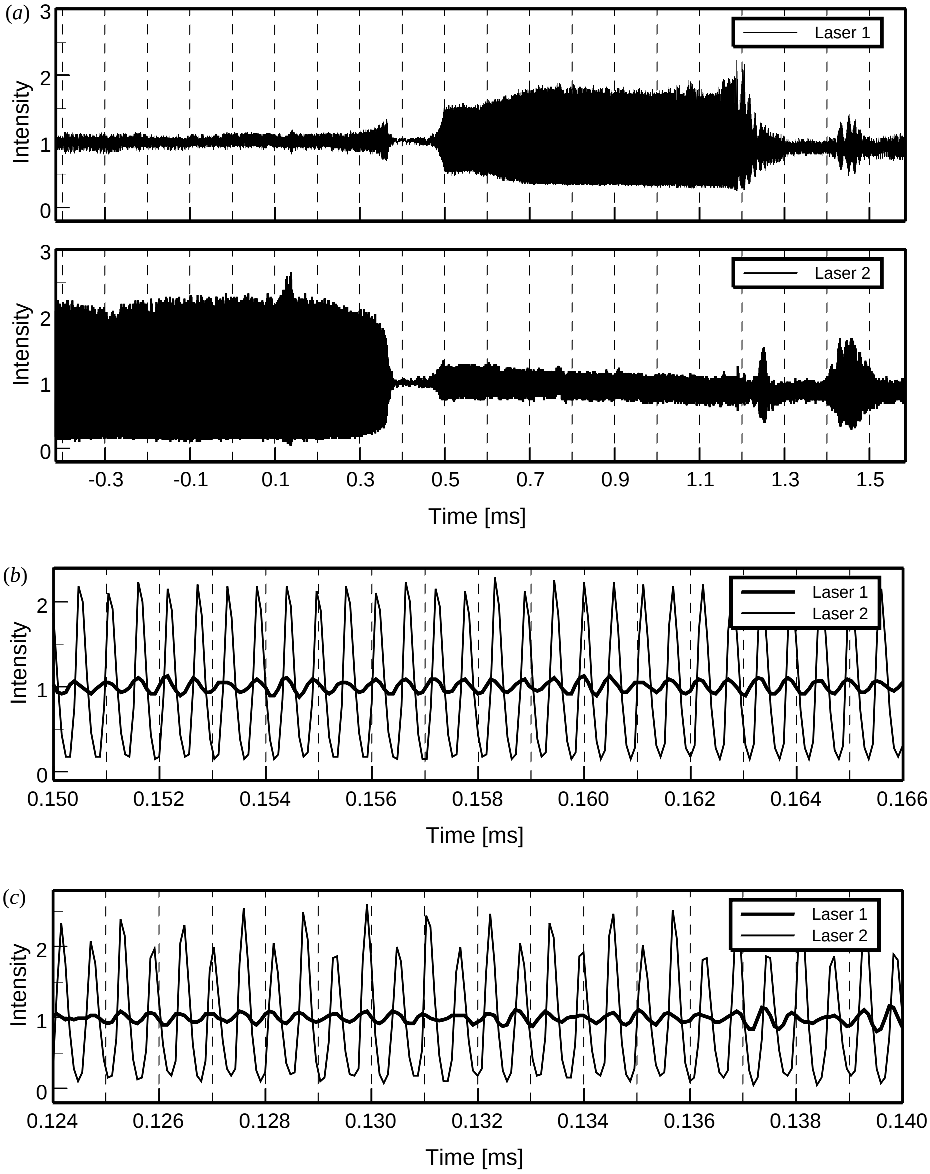

Figure 6. Localized synchronization oscillations measured at $d = 0.48$ mm and $\Delta < 2$ MHz. (*a*) Two bursts of oscillations. Note the symmetry breaking (large or small amplitudes appearing in both lasers). (*b*) Regular oscillation detail. Note the phase lag between the maxima in the two channels (*c*) Period doubling detail.

Equations (1*a*)–(1*d*) can be transformed into equations for the amplitudes and the optical phase difference with $\mathcal{E}_j = E_j \exp(\mathrm{i}\phi_j)$ and $\phi = \phi_2 - \phi_1$:

$$\frac{\mathrm{d}E_1}{\mathrm{d}t} = \frac{1}{\tau_c}[(G_1 - \alpha_1)E_1 - \kappa E_2 \cos\phi] \qquad (2a)$$

$$\frac{\mathrm{d}G_1}{\mathrm{d}t} = \frac{1}{\tau_f}\left(p_1 - G_1 - G_1|E_1|^2\right) \qquad (2b)$$

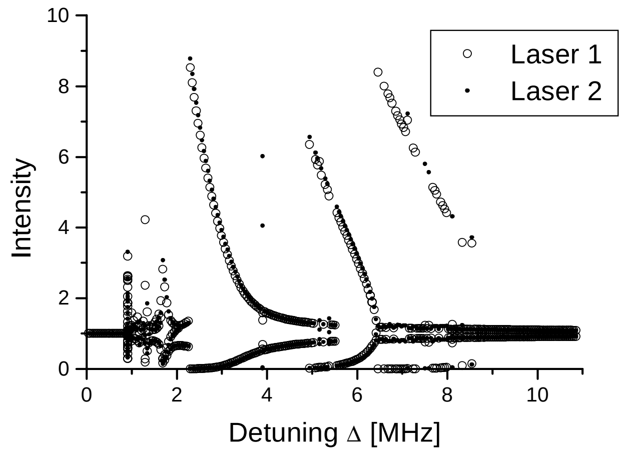

Figure 7. Bifurcation diagram obtained by integrating equations (2*a*)–(2*e*). Parameter values are $\alpha_1 = \alpha_2 = 0.06$, $p_1 = 0.0840$, $p_2 = 0.0846$, $\kappa = 3 \times 10^{-5}$, $\tau_c = 11\,\text{ps}$, $\tau_f = 12\,\mu\text{s}$. Initial conditions are $E_1(0) = 1.1$, $E_2(0) = 0.7$, $G_1(0) = 0.99$, $G_2(0) = 0.98$, $\phi = 4.12$.

$$\frac{\mathrm{d}E_2}{\mathrm{d}t} = \frac{1}{\tau_c}[(G_2 - \alpha_2)E_2 - \kappa E_1 \cos\phi] \tag{2c}$$

$$\frac{\mathrm{d}G_2}{\mathrm{d}t} = \frac{1}{\tau_f}\left(p_2 - G_2 - G_2|E_2|^2\right) \tag{2d}$$

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = \frac{\kappa}{\tau_c}\left(\frac{E_1}{E_2} + \frac{E_2}{E_1}\right)\sin\phi + (\omega_2 - \omega_1). \tag{2e}$$

Equations (2*a*)–(2*e*) were numerically integrated, using parameters corresponding to the situation in the experiment. The bifurcation diagram in figure 7 shows an example for the rich dynamical behaviour. Local maxima and minima of the two laser intensities ($I = E^2$), taken after allowing the system transients to settle down, are plotted as a function of the detuning $\Delta = (\omega_2 - \omega_1)/2\pi$ for a given coupling strength. A single point at a certain detuning indicates a stationary state, two points indicate period-one orbits and more than two extrema represent period-doubled, quasiperiodic or other complex oscillatory states. At a large detuning, both lasers are periodically modulated with a small amplitude. With decreasing detuning the intensity of one laser can describe a periodic orbit with a significantly larger amplitude, while the other laser keeps its small amplitude. This behaviour corresponds to 'localized synchronization' states. At smaller detuning synchronous oscillations appear and both amplitudes begin to grow, until a crisis occurs and the laser intensities are only weakly modulated again. This scenario repeats itself once; for further decreased detuning the lasers show a more complex behaviour such as period doubling with crises and quasiperiodicity. At the critical detuning value $\Delta \approx 1$ MHz the intensities become stationary.

A time trace corresponding to the experimentally recorded synchronous intensity oscillations in the regime of strong coupling is shown in figure 8. The 'localized synchronization' oscillations basically resemble those observed in the experiment (see figure 9); however, the investigation of a larger parameter regime shows that in the simulations the phase lag between the small and large amplitude maxima can cover a larger range, up to almost antiphase oscillations.

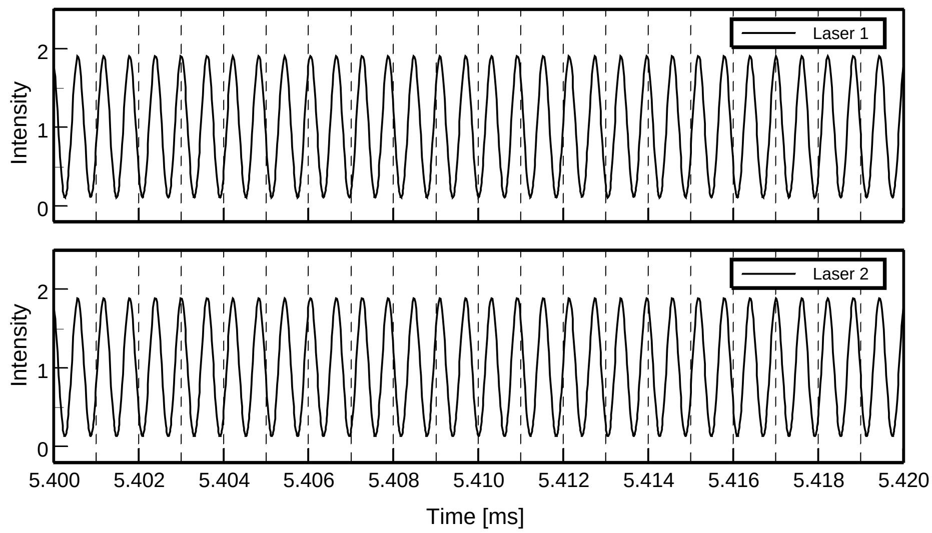

Figure 8. Synchronous oscillations in the regime of strong coupling obtained by integrating equations (2*a*)–(2*e*). Parameter values and initial conditions are the same as in figure 7, except $\kappa = 2 \times 10^{-3}$ and $\Delta = 66.5$ MHz.

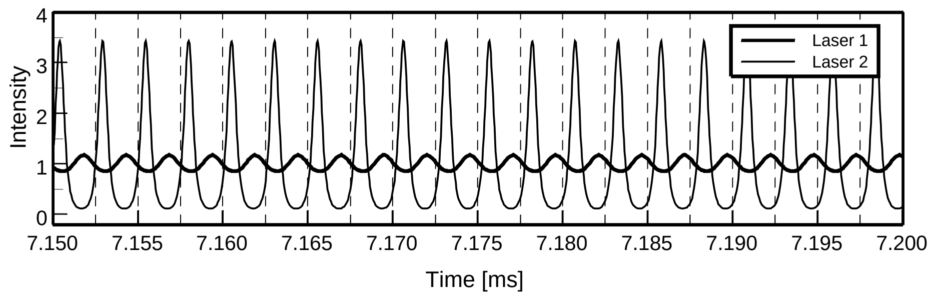

Figure 9. Localized oscillation state obtained by integrating equations (2*a*)–(2*e*). Parameter values and initial conditions are the same as in figure 7, except $\kappa = 5 \times 10^{-6}$, $\Delta = 2.85$ MHz.

In the oscillatory regime, the optical phase difference ϕ is not stationary, but will grow by 2π within one oscillation period. For the oscillations shown in figures 8 and 9 this results in fringe visibilities of 0.8 and 0.45, respectively.

5. Conclusions

A semimonolithic Nd:YVO$_4$ microchip laser was shown to be very well suited for studying the dynamics of coupled lasers, in particular with respect to its insensitivity to mechanical noise. Different types of instabilities could be found, including localized synchronization. Numerical simulations of a model of two coupled single-mode class-B lasers show qualitative agreement with the experimental observations; nevertheless, further theoretical and numerical effort is necessary to gain further insight into the dynamics of the optical phase synchronization—aided by more detailed measurements.

The sensitive dependence on the laser detuning is of considerable importance for applications of monolithic arrays, as differences in the resonator lengths of neighbouring pixels cannot be controlled to the order of less than a thousandth of a wavelength. Future investigations will include more than two lasers, using laser diodes and fibre optics for variable two-dimensional arrangements.

Acknowledgment

We wish to thank VLOC, a division of II–VI Inc. for providing the Nd:YVO_4/YAG crystal.

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